Applied Data Analysis (Operations Research)

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DECISION MAKING TOOLS

- Statistics
- operations research (a mathematical <u>optimization</u> model of a complex system)
  - deterministic, e.g., Linear Programming, transportation problem
  - stochastic, e.g., decision tree, queuing theories

Coverage

- Linear/Integer/Mixed Programming
- Network system
- Multiple objective
- Inter-temporal (dynamic) system
- uncertainty (stochastic)

MATHEMATICAL PROBLEMS

- Most decision making problems could be formulated as a <u>mathematical</u> problem.
- Decision making processes can be represented with mathematical functions and constraints.
- Some decision making problems involve optimization of an objective function. Others try to simulate the result of various decisions before the decision can be made.

OPTIMIZATION PROBLEMS

- maximize or minimize an objective function
- subject to various constraints.

EXAMPLES OF OBJECTIVE FUNCTIONS

- net profit
- output or throughput
- cost or input
- time
- likeliness or probability
- variance or uncertainty

EXAMPLES OF CONSTRAINTS

- staffing
- space
- budget
- time
- technology

LINEAR PROGRAMMING(LP) PROBLEM

- mathematical optimization problem
- its objective function and constraints are all linear

EXAMPLE OF AN LP PROBLEM

A hypothetical hospital of 30 beds has two service departments (OPD and IPD). The In-Patient Department (IPD) needs 2 doctor-hours and 4 nurse-hours for every patient-day. In Out-Patient Department(OPD), each doctor-hour must be supported by 4 nurse-hours. The hospital can offer no more than 50 doctor-hours of OPD service in a day. The hospital can earn $120 for every patient-day in IPD and $80 for every doctor-hour in OPD. The hospital has a limited resource of 80 doctor-hours and 240 nurse-hours per day. Determine the resource allocation between OPD and IPD so that income of the hospital is maximized.

STRUCTURE OF THE LP
(for the example)

Objective = maximize income

Constraints

1) # of doctor-hours

2) # of nurse-hours

3) # of beds

4) # of OPD service hours

DEFINE THE VARIABLES

X_1 = # of in-patients per day

X_2 = # of OPD doctor-hours per day

LP REPRESENTATION

$$\max_{X_1, X_2} \quad Z = 120X_1 + 80X_2$$

s.t.

$2X_1 + X_2 <= 80$ ---------(1)

$4X_1 + 4X_2 <= 240$ ---------(2)

$X_1 <= 30$ ---------(3)

$X_2 <= 50$ ---------(4)

$X_1, X_2 >= 0$

NOTE You will get a different LP representation if you define the variables differently. Anyway, all the LPs still represent the same decision-making problem

Let's define

Y_1 = # of IPD doctor-hours per day

Y_2 = # of OPD nurse-hours per day

ANOTHER LP REPRESENTATION

max $Z = 60Y_1 + 20Y_2$

Y_1, Y_2

s.t.

$Y_1 + 0.25Y_2 <= 80$ ---------(1')

$2Y_1 + Y_2 <= 240$ ---------(2')

$Y_1 <= 60$ ---------(3')

$Y_2 <= 200$ ---------(4')

$Y_1, Y_2 >= 0$

Replace X_1 with $0.5Y_1$ and X_2 with $0.25Y_2$

HOW TO SOLVE AN LP PROBLEM

- Graphical Method
- Simplex method
  - by hand (not discussed)
  - by computer

GRAPHICAL METHOD

- good for LPs with two variables
- make it easy to understand solving an LP

SIMPLEX METHOD

- algebraic method developed by George Dantzig
- need to understand it very well if you want to do it by hand or develop a computer program to do it
- quite a few computer programs can do this,e.g.,
  - LINDO (Windows versions)
  - Microsoft Excel
  - MPSX (IBM mainframe)

LP SOLUTIONS OR RESULTS

- Unique Solution
  - no other values of LP variables will yield the same optimized value of objective function
- Alternative Solution (non-unique)
  - infinitely many feasible solutions
- Infeasible solution
  - No values of LP variables satisfying the constraints

LINDO Inputs

```
MAX     120 x1 + 80 x2
SUBJECT TO
      2)  2 x1 + x2 <= 80
      3)  4 x1 + 4 x2 <=240
      4)  x1 <= 30
      5)  x2 <= 50
END
```

LINDO Report

LP OPTIMUM FOUND AT STEP 3

OBJECTIVE FUNCTION VALUE

1) 5600.000

VARIABLE	VALUE	REDUCED COST
X1	20.000000	0.000000
X2	40.000000	0.000000

LINDO Report (cont’d)

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	0.000000	40.000000
3)	0.000000	10.000000
4)	10.000000	0.000000
5)	10.000000	0.000000

NO. ITERATIONS= 3

DUAL SOLUTIONS

- For each constraint, there exist a dual variable sometimes called dual price
- Define

  m_i = dual variable for constraint i

  = change in Z / change in RHS value
- Graphical method cannot give a dual solution (value of dual variables) but Simplex can

MEANING OF DUAL SOLUTION
(First Representation)

m_1 = opportunity cost of a doctor-hour

m_2 = opportunity cost of a nurse-hour

m_3 = opportunity cost of a bed-day

m_4 = opportunity cost of the OPD limit