

Transportation Problems

Pongsa Pornchaiwiseskul
Faculty of Economics
Chulalongkorn University

Pongsa Pornchaiwiseskul,

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Drug Distribution

Drugco has three drug factories that supply the need for four provinces. Each drug factory can supply the following numbers of tons of drug per month: factory A, 35; factory B, 50; factory C, 40. The drug demands in these provinces are as follows (in tons per month): province 1, 45; province 2, 20; province 3, 30; province 4, 30. The costs of distribution from factory to province depend on the distance as given in table below.

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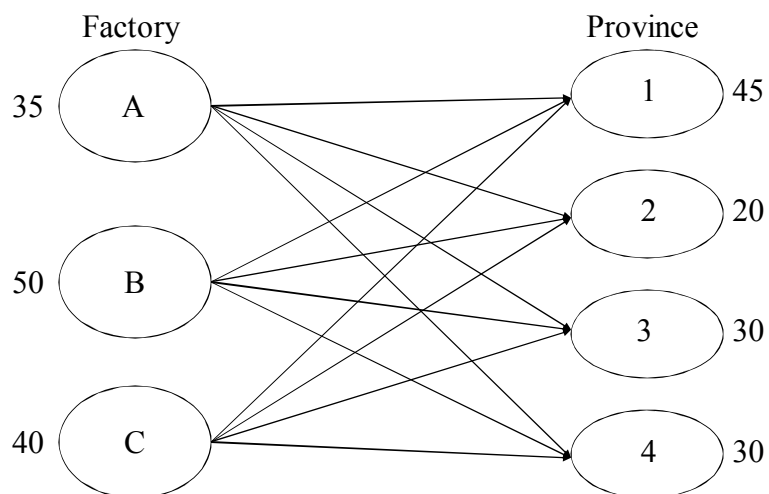
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From	To Province				Supply
	1	2	3	4	tons/mo
Factory A	\$8	\$6	\$10	\$9	35
Factory B	\$9	\$12	\$13	\$7	50
Factory C	\$14	\$9	\$16	\$5	40
Demand tons/mo.	45	20	30	30	125

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Graphical Representation



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LP Formulation

Define

x_{ij} = tons/month of drug shipped
from factory i to province j

$i = A, B, C$

$j = 1, 2, 3, 4$

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LP Problem for Balanced Transportation Problem

$$\begin{aligned} \text{MIN} \quad & 8x_{A1} + 6x_{A2} + 10x_{A3} + 9x_{A4} \\ & + 9x_{B1} + 12x_{B2} + 13x_{B3} + 7x_{B4} \\ & + 14x_{C1} + 9x_{C2} + 16x_{C3} + 5x_{C4} \end{aligned}$$

subject to

$$x_{A1} + x_{A2} + x_{A3} + x_{A4} = 35$$

$$x_{B1} + x_{B2} + x_{B3} + x_{B4} = 50$$

$$x_{C1} + x_{C2} + x_{C3} + x_{C4} = 40$$

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LP Problem (cont'd) for Balanced Transportation Problem

$$x_{A1} + x_{B1} + x_{C1} = 45$$

$$x_{A2} + x_{B2} + x_{C2} = 20$$

$$x_{A3} + x_{B3} + x_{C3} = 30$$

$$x_{A4} + x_{B4} + x_{C4} = 30$$

$$x_{ij} \geq 0 \text{ for all } i, j$$

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LINDO inputs for Balanced Transportation Problem

$$\begin{aligned} \min & 8x_{a1} + 6x_{a2} + 10x_{a3} + 9x_{a4} \\ & + 9x_{b1} + 12x_{b2} + 13x_{b3} + 7x_{b4} \\ & + 14x_{c1} + 9x_{c2} + 16x_{c3} + 5x_{c4} \end{aligned}$$

subject to

$$2) \ x_{a1} + x_{a2} + x_{a3} + x_{a4} = 35$$

$$3) \ x_{b1} + x_{b2} + x_{b3} + x_{b4} = 50$$

$$4) \ x_{c1} + x_{c2} + x_{c3} + x_{c4} = 40$$

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LINDO inputs (cont'd) for Balanced Transportation Problem

$$5) x_{a1} + x_{b1} + x_{c1} = 45$$

$$6) x_{a2} + x_{b2} + x_{c2} = 20$$

$$7) x_{a3} + x_{b3} + x_{c3} = 30$$

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TABLEAU Method

From\To	Prov 1	Prov 2	Prov 3	Prov 4	Supply
Factory A	8	6	10	9	0
	35	0	0	0	35
Factory B	9	12	13	7	
					50
Factory C	14	9	16	5	
					40
Demand	45 8	20	30	30	125

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TABLEAU Method

From\To	Prov 1	Prov 2	Prov 3	Prov 4	Supply
Factory A	8 35	6 0	10 0	9 0	0 35
Factory B	9 10	12	13	7	1 50
Factory C	14 0	9	16	5	40
Demand	45 8	20	30	30	125

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TABLEAU Method

From\To	Prov 1	Prov 2	Prov 3	Prov 4	Supply
Factory A	8 35	6 0	10 0	9 0	0 35
Factory B	9 10	12 20	13	7	1 50
Factory C	14 0	9 0	16	5	40
Demand	45 8	20 11	30	30	125

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TABLEAU Method

From\To	Prov 1	Prov 2	Prov 3	Prov 4	Supply
Factory A	8 35	6 0	10 0	9 0	0 35
Factory B	9 10	12 20	13 20	7 0	1 50
Factory C	14 0	9 0	16	5	40
Demand	45 1	20 11	30 12	30	125

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TABLEAU Method

From\To	Prov 1	Prov 2	Prov 3	Prov 4	Supply
Factory A	8 35	6 0	10 0	9 0	0 35
Factory B	9 10	12 20	13 20	7 0	1 50
Factory C	14 0	9 0	16 10	5 30	4 40
Demand	45 8	20 11	30 12	30 1	125

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TABLEAU Method

From\To	Prov 1		Prov 2		Prov 3		Prov 4		Supply
Factory	8	0	6	-5	10	-2	9	8	0
A	35		0		0		0		35
Factory	9	0	12	0	13	0	7	5	1
B	10		20		20		0		50
Factory	14	2	9	-6	16	0	5	0	4
C	0		0		10		30		40
Demand	45	8	20	11	30	12	30	1	125

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TABLEAU Method

From\To	Prov 1		Prov 2		Prov 3		Prov 4		Supply
Factory	8	0	6	-5	10	-2	9	8	0
A	35		0		0		0		35
Factory	9	0	12	0	13	0	7	5	1
B	10		20-p		20+p		0		50
Factory	14	2	9	-6	16	0	5	0	4
C	0		p		10-p		30		40
Demand	45	8	20	11	30	12	30	1	125

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TABLEAU Method

From\To	Prov 1		Prov 2		Prov 3		Prov 4		Supply
Factory	8		6	-5	10	-2	9	2	0
A	35-p		0+p		0		0		35
Factory	9		12		13		7	-1	1
B	10+p		10-p		30		0		50
Factory	14	8	9		16	6	5		-2
C	0		10		0		30		40
Demand	45	8	20	11	30	12	30	7	125

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TABLEAU Method

From\To	Prov 1		Prov 2		Prov 3		Prov 4		Supply
Factory	8	0	6	0	10	-2	9	7	0
A	25		10		0		0		35
Factory	9	0	12	5	13	0	7	4	1
B	20		0		30		0		50
Factory	14	3	9	0	16	1	5	0	3
C	0		10		0		30		40
Demand	45	8	20	6	30	12	30	2	125

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TABLEAU Method

From\To	Prov 1		Prov 2		Prov 3		Prov 4		Supply
Factory	8	0	6	0	10	-2	9	7	0
A	25-p		10		0+p		0		35
Factory	9	0	12	5	13	0	7	4	1
B	20+p		0		30-p		0		50
Factory	14	3	9	0	16	1	5	0	3
C	0		10		0		30		40
Demand	45	8	20	6	30	12	30	2	125

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TABLEAU Method

From\To	Prov 1		Prov 2		Prov 3		Prov 4		Supply
Factory	8	2	6	0	10	0	9	7	0
A	0		10		25		0		35
Factory	9	0	12	3	13	0	7	2	3
B	45		0		5		0		50
Factory	14	5	9	0	16	3	5	0	3
C	0		10		0		30		40
Demand	45	6	20	6	30	10	30	2	125

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Unbalanced Transportation Problem

From	To Province				Supply
	1	2	3	4	tons/mo
Factory A	\$8	\$6	\$10	\$9	45
Factory B	\$9	\$12	\$13	\$7	50
Factory C	\$14	\$9	\$16	\$5	40
Demand tons/mo.	45	20	30	30	125\135

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LP Problem for Unbalanced Transportation Problem

$$\begin{aligned} \text{Min } & 8x_{A1} + 6x_{A2} + 10x_{A3} + 9x_{A4} \\ & + 9x_{B1} + 12x_{B2} + 13x_{B3} + 7x_{B4} \\ & + 14x_{C1} + 9x_{C2} + 16x_{C3} + 5x_{C4} \end{aligned}$$

subject to

$$x_{A1} + x_{A2} + x_{A3} + x_{A4} + d_A = 45$$

$$x_{B1} + x_{B2} + x_{B3} + x_{B4} + d_B = 50$$

$$x_{C1} + x_{C2} + x_{C3} + x_{C4} + d_C = 40$$

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LP Problem (cont'd) for UnBalanced Transportation Problem

$$x_{A1} + x_{B1} + x_{C1} = 45$$

$$x_{A2} + x_{B2} + x_{C2} = 20$$

$$x_{A3} + x_{B3} + x_{C3} = 30$$

$$x_{A4} + x_{B4} + x_{C4} = 30$$

$$x_{ij}, d_i \geq 0 \text{ for all } i, j$$

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Unbalanced Transportation Problem

From	To Province					Supply tons/mo
	1	2	3	4	dummy	
Factory A	\$8	\$6	\$10	\$9	\$0	45
Factory B	\$9	\$12	\$13	\$7	\$0	50
Factory C	\$14	\$9	\$16	\$5	\$0	40
Demand tons/mo.	45	20	30	30	10	135

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