CPM

Critical

Path

Method

CPM

Tools for Project Management

- – Minimum Project Duration

- – Scheduling

- – Time-cost Trade-offs

- – Resoruce Leveling (not to be discussed)

Critical Path Method

- Network-based (to be seen why)

- An LP Problem but much more simple that it can be solved by hand

- Deterministic (all the parameters are known or assumed with certainty)

Project Evaluation Review Technique

- **PERT** is an Extension of CPM

- Probability Concept is added to CPM

- Good for a project which has never been done before. Some uncertainty involved
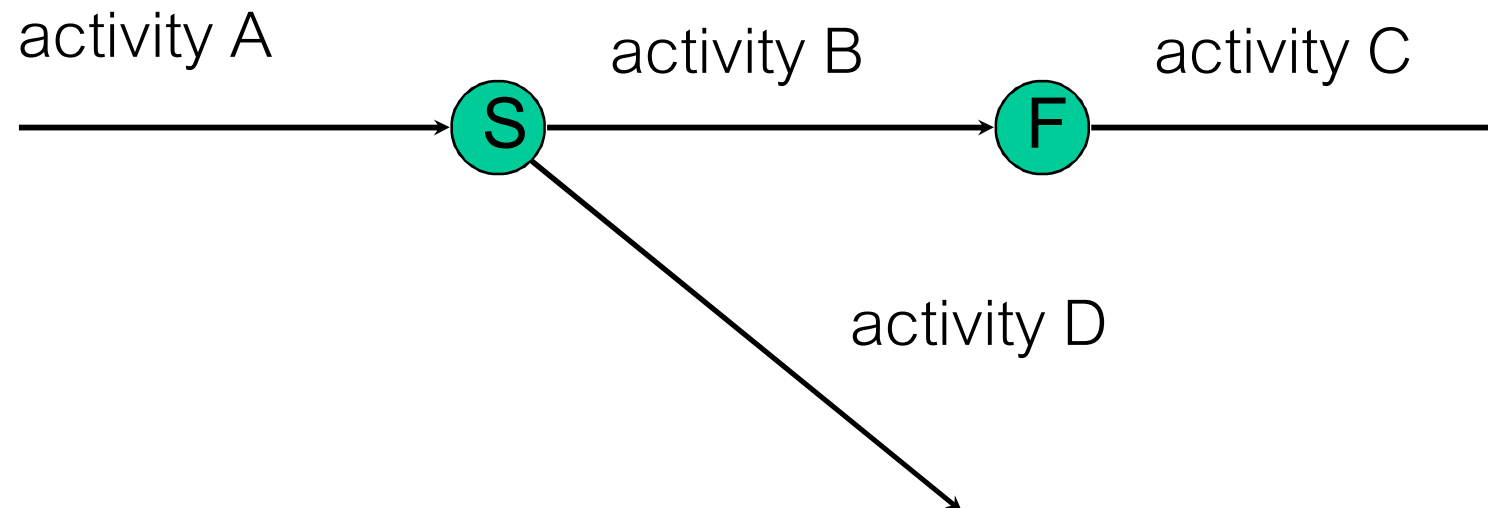
- Not to be discussed

PROJECTS

- There are many activities. Each activity takes time.

- Some activities (successors) cannot start until the other activities (predecessors) finish.

- Can be represented by a directed network

- Examples are construction, scientific project and thesis

Activity-on-Node (AoN)

Activity-on-Arc (AoA)

activity A   activity B   activity C

S   F
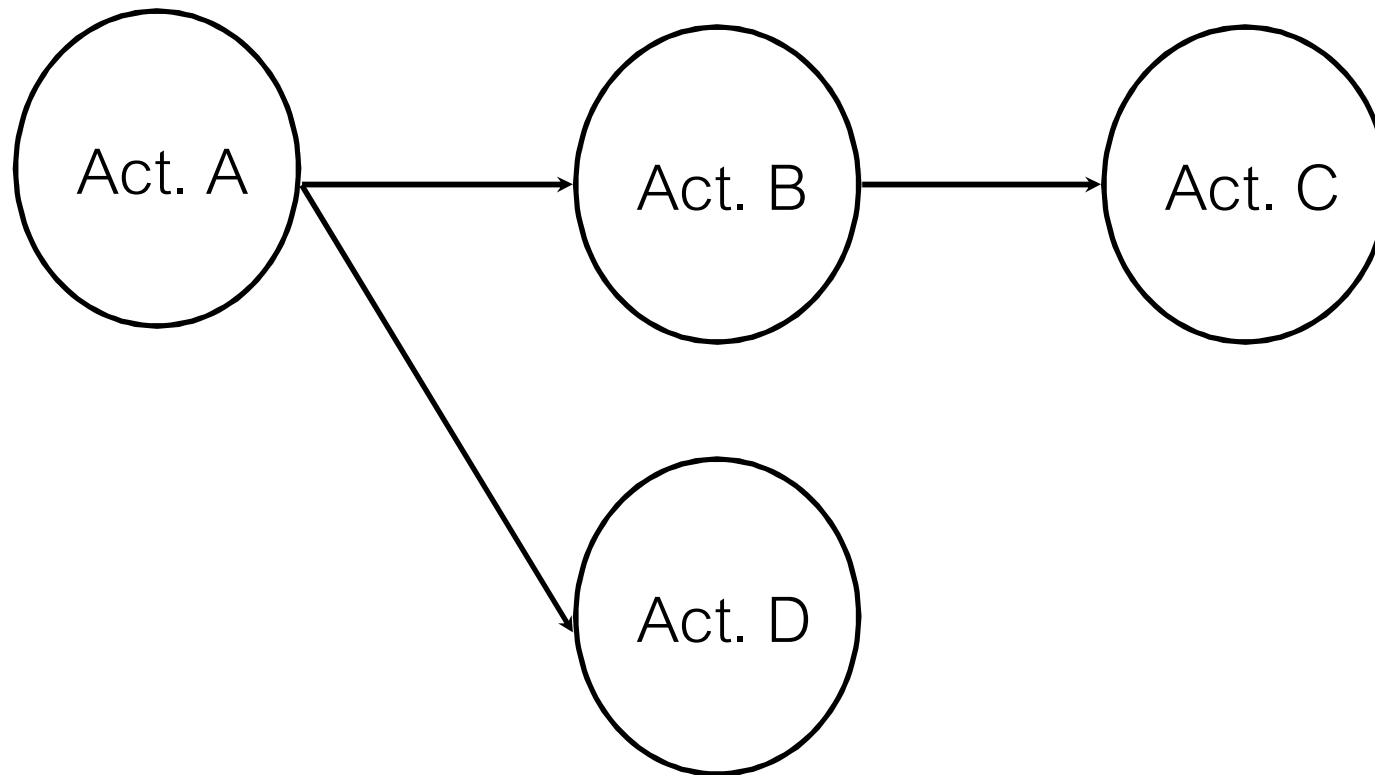
activity D

EXAMPLE OF CPM

Activity	Time	Predecessors
DIG	3	-
FOUND	4	DIG
POURB	2	FOUND
JOISTS	3	FOUND
WALLS	5	FOUND
RAFTERS	3	WALLS, POURB
FLOOR	4	JOISTS
ROUGH	6	FLOOR
ROOF	7	RAFTERS, JOISTS
FINISH	5	ROUGH, ROOF
SCAPE	2	POURB, WALLS

MINIMUM PROJECT DURATION

- Network Method

  - by hand

  - by computer programs,e.g., Microsoft Project (not to be discussed)

- Solving its corresponding LP problem

  - by computer or by hand

Network Method by Hand

- Determine Longest route between start and end

- Performed in two steps.

  – Forward Pass(from start to end)

  – Backward Pass (from end back to start)

FORWARD PASS

(1) The project starts at time zero

(2) Every starting activity has an **E**arliest **S**tart(ES) at zero

(3) **E**arliest **F**inish(EF) of an activity is ES + activity time

$$EF_j = ES_j + D_j$$

(4) For an activity j w/ predecessors,

$$ES_j = max\{ \text{ its predecessors' EF}\}$$

FORWARD PASS (cont'd)

(5) The minimum project duration (T)

$$T = \max\{EF \text{ of activities w/o successors}\}$$

Note that

1) The project can earliest finish at time T

2) It can finish later than time T but not before

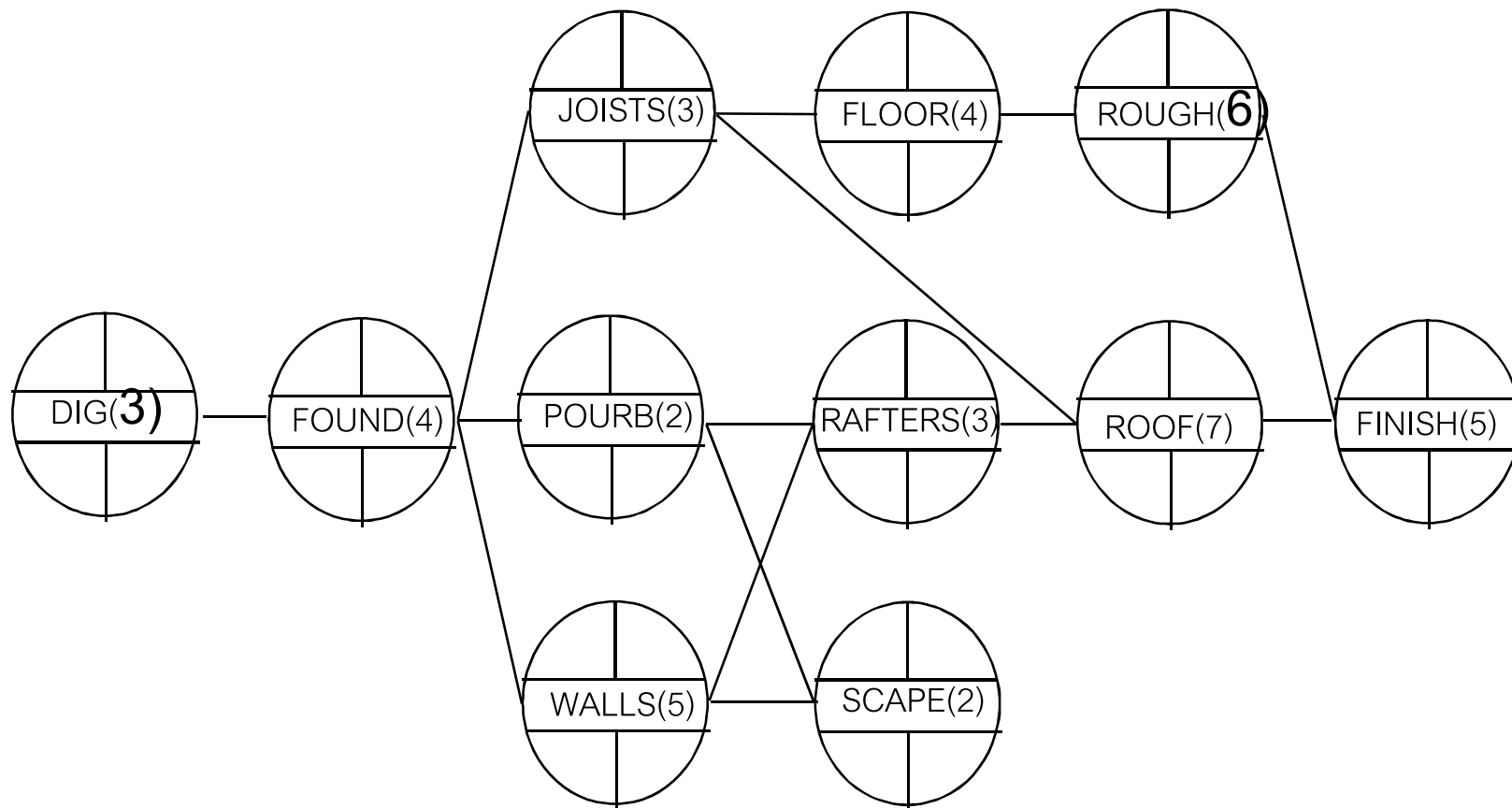# BACKWARD PASS

(1) The project finishes at time T

(2) All the activities w/o successors can Latest Finish(LF) at time T. Their LF = T

(3) Latest Start(LS) of an activity j is its LF minus activity duration (D),I.e.,

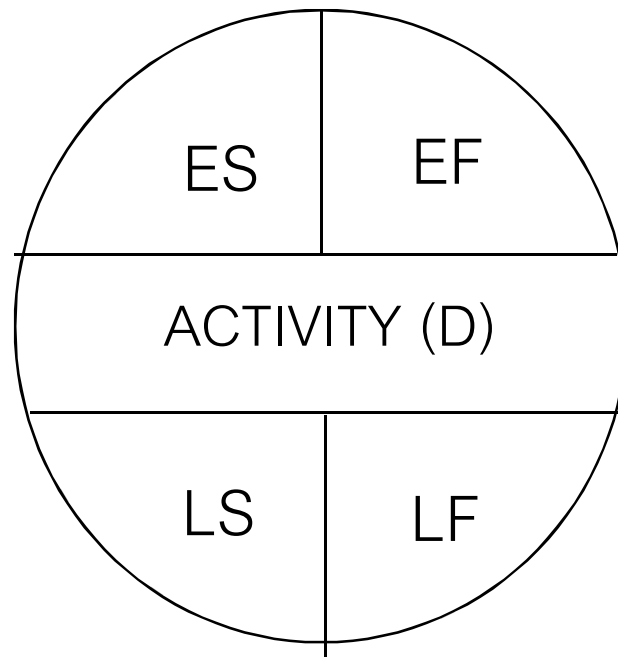$$LS_j = LF_j - D_j$$

(4) LF of an activity w/ successors

= min {LS of its successors}

AoN Representation
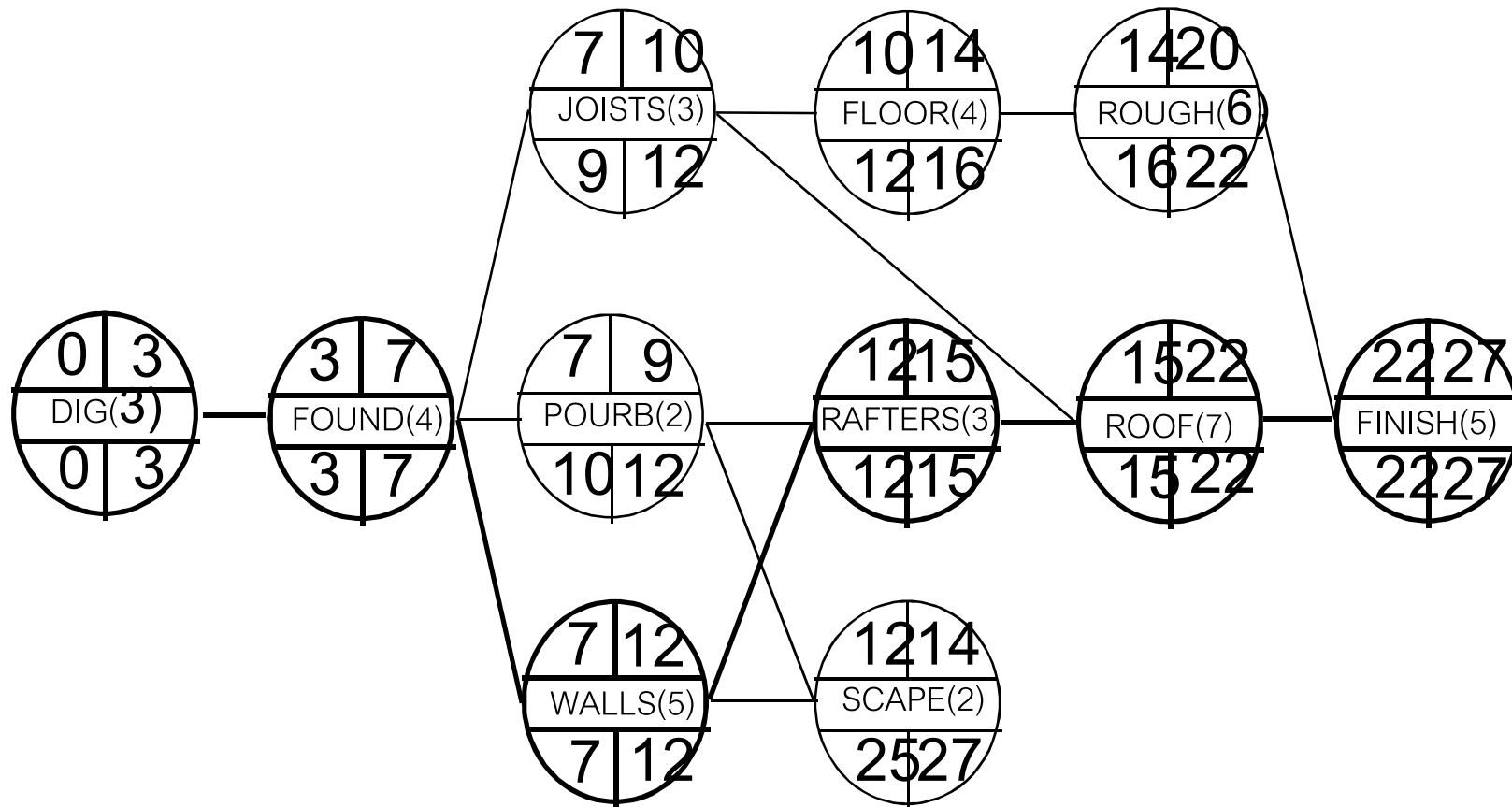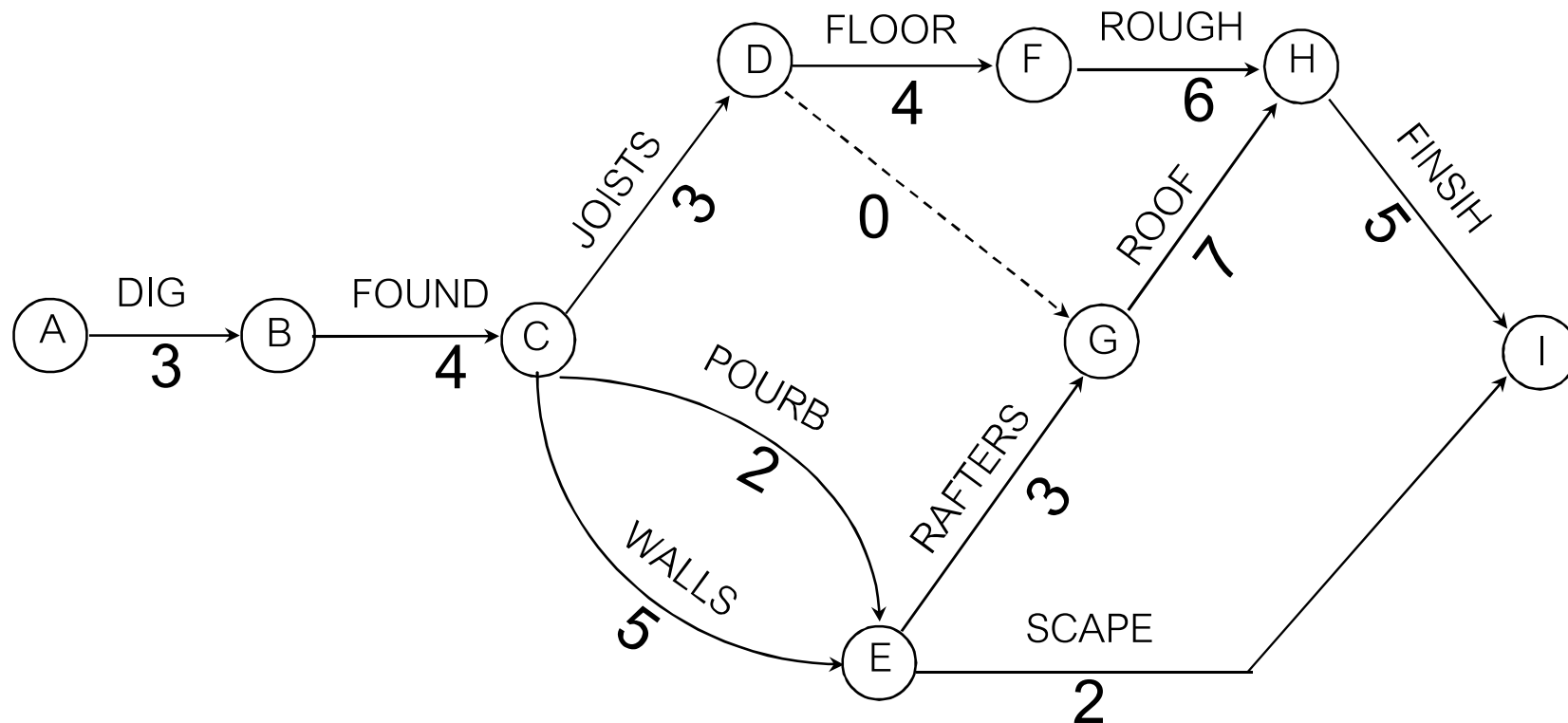
AON Legend

AoN Representation

AoA Representation

RESULT

Activity	Time	Predecessors	ES	EF
LS	LF			
DIG	3	-	0	
3	0	3		
FOUND	4	DIG	3	7
3	7			
POURB	2	FOUND	7	9
10	12			
JOISTS	3	FOUND	7	10
10	12			
WALLS	5	FOUND	7	12
7	12			
RAFTERS	3	WALLS, POURB	12	15
12	15			
FLOOR	4	JOISTS	10	14
12	16			

LP Representation

MAX

3DIG+4FOUND+2POURB+3JOISTS+5WALLS

+3RAFTERS+4FLOOR+6ROUGH+7ROOF+5FINISH

+2SCAPE

SUBJECT TO

2) DIG <=1

3) FOUND - DIG = 0

4) JOISTS + POURB + WALLS - FOUND =0

LP Representation(cont'd)

5) FLOOR + DUMMY - JOISTS $= 0$

6) RAFTERS + SCAPE - POURB - WALLS $= 0$

7) ROUGH - FLOOR $= 0$

8) ROOF - RAFTERS - DUMMY $= 0$

9) FINISH - ROUGH - ROOF $= 0$

END

SCHEDULING WITH BAR CHART

Activity	5	10	15	20	25
DIG					
FOUND					
POURB					
JOISTS					
WALLS					
RAFTERS					
FLOOR					
ROUGH					
ROOF					
FINISH					
SCAPE					
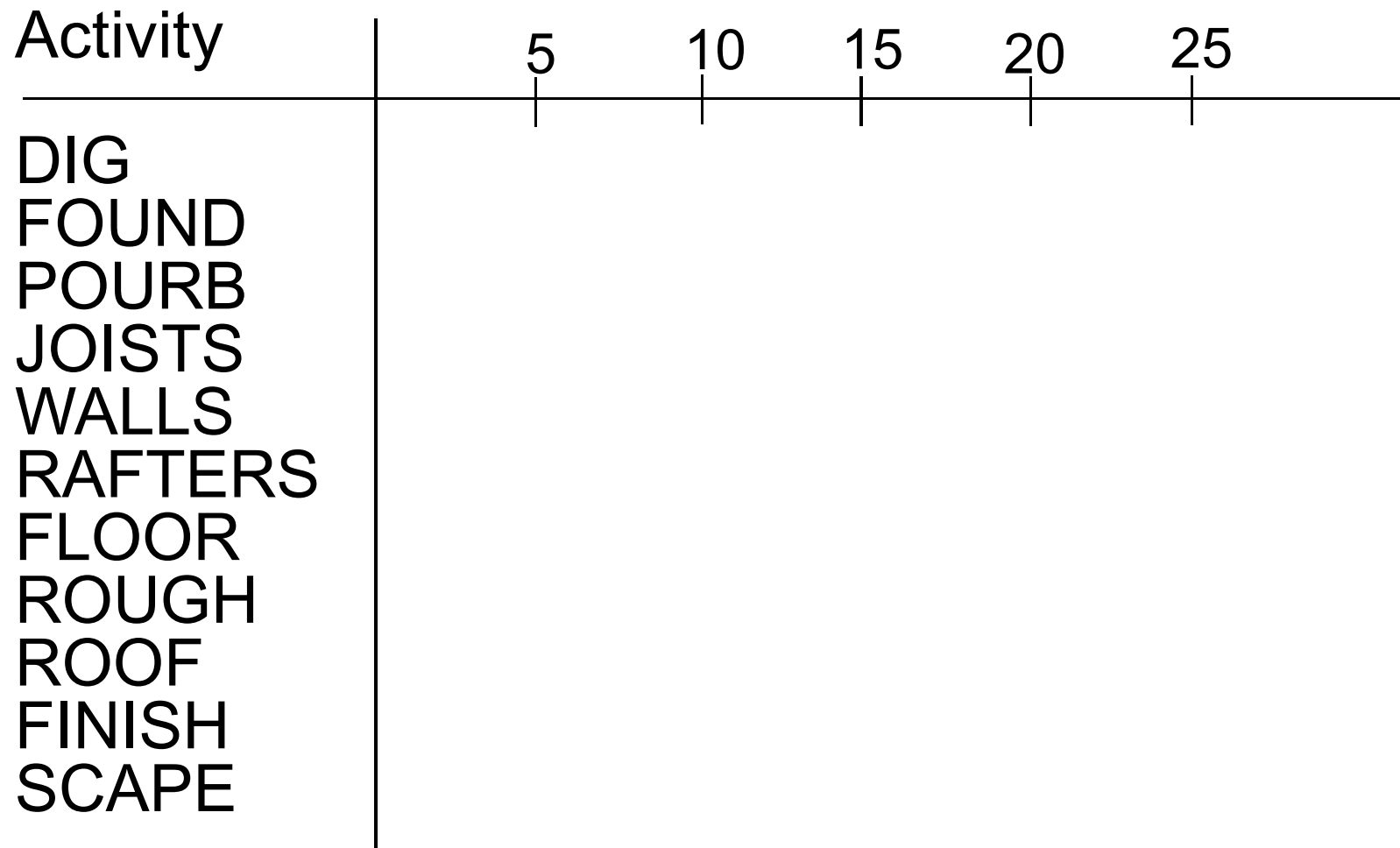
CRITICAL ACTIVITIES

- activities with zero slack.

  slack = LS - ES

  or = LF - EF

- critical activities form a Critical Path

TIME COST TRADE-OFFS

Choose to shorten the critical activity with lowest cost until the activity becomes non-critical.

CRASHING THE PROJECT

Decrease the project duration by shortening the activities.

Activity	Normal Duration	Max. Crash	$/day
DIG	3	1	50
JOIST	3	1	30
WALLS	5	3	40
FINISH	5	2	80

RESULT

Activity LF	Time	Predecessors	ES	EF	LS
DIG	3	-			
FOUND	4	DIG			
POURB	2	FOUND			
JOISTS	3	FOUND			
WALLS	5	FOUND			
RAFTERS	3	WALLS, POURB			
FLOOR	4	JOISTS			
ROUGH	6	FLOOR			
ROOF	7	RAFTERS, JOISTS			
FINISH	5	ROUGH, ROOF			
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