Measuring Hospital Efficiency: DEA and Stochastic Frontier Approach

Pongsa Pornchaiwiseskul
Centre for Health Economics
Faculty of Economics
Chulalongkorn University

Abstract

In the year 2002, a new health security policy has been introduced as an effort to provide more affordable secondary health care to the uninsured. Before the new policy, for a small fee the uninsured could voluntarily join the government-run health insurance system which covered the whole family for basic health care services without any co-payment. Due to the abuse and budget balance problems, the system had been abolished. Under the new policy the uninsured can register for the so-called "Gold" card. The approved "Gold" card will entitle the holders to receive the needed treatment at designated government hospitals for only 30 baht (US$ 1=40 baht) out of pocket per visit instead of the full cost. The "Gold" card covers many catastrophic illnesses. For each registered "Gold" card, the hospitals will be allocated the fixed annual capitation fee.of about US$25 baht from the government budget. Government hospitals are the main risk bearers since they cannot deny patients.

There have been big debates about the effectiveness of the new health security policy. A lot of poor uninsured have been claimed to benefit from the "Gold" cards whereas a lot of "Gold" card holding patients also complained about the low quality of health care services due to cost control by government hospitals. High risk by hospitals resulted in budget deficit, low morale of staff and possibly reduced quality in many hospitals. The new policy was meant to increase the efficiency of hospital service. This paper is trying to investigate the technical efficiency of government hospitals after the new policy, assuming the allocative efficiency will not be affected.

The analysis considers health personnel and hospital beds as two major inputs, assuming that the other inputs will be proportionate to the levels of these two inputs. Morbidity of top-ten causes and infant and maternal mortality are used as main hospital inverse outputs. Other hospital outputs are simply ignored. Causes of technical inefficiencies to be investigated are the structure of hospital patients, especially the proportion of uninsured "Gold" cardholders. Hospital data by province (excluding Bangkok) in 2002 has been used in the analysis. The analysis is done in two steps. In step 1, technical efficiency index for each province is generated from hospital outputs and inputs using Data Envelopment Analysis (DEA). In step 2, a technical efficiency index function as structure of hospital patients is analyzed using truncated normal distribution. There seems to be no significant evidence that the new health security policy causes technical inefficiency of the government hospitals.

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I. Background of Thailand's Health System

Thailand's first public hospital was established about 100 years ago in the reign of King Rama V or King Chulalongkorn. Since then, more government-owned hospitals had been established both in Bangkok and provincial towns. Before that, Western-style health care services had been provided by Western missionary group. Owing to the budgetary support by government, public hospitals set the out-of-pocket payment to the minimum. Only drug and laboratory test had been charged to patients. Medical personnel and equipment cost had been paid from annual government budget. Even though they were meant to provide health care services to all people, their services were not easily accessible to the very poor of the society as the services were not generally free. The population groups who enjoyed the service are mostly elite government officials and well-to-do urban people.

The first social welfare scheme is free medical service for the poor in 1975. However, free medical services were not automatically given to all the poor. They were given on case-by-case basis, mostly from hospital revenue. Since then, free medical service under social welfare scheme has been extended to more poor population. In 1981 Low-Income Card (LIC) system has been introduced to identify the poor population before they are given free medical services. Their status must be verified and approved before a "poor" card can be issued. The status will be periodically verified. Even though its coverage is limited to basic medical treatment, the low-income cards have greatly improved the accessibility to medical care for the poor. In 1992 free medical services have been extended to the elderly if they are not covered by any other insurance. In 1993 children up to 12 year old without any coverage have been included into the social welfare scheme.

Health care market was opened up to the private hospitals, especially in Bangkok and urban areas after health care demand outgrew the capacity of government hospitals. Declining service quality and long waiting lines at government hospitals drove well-to-do patients to seek medical services from private hospitals. The very first voluntary health insurance was offerred to general public by a private insurance firm in 1978. Table 1 illustrates the key events in the development of health security in Thailand in the past 30 years.

Table 1 Key Development of Thailand's Health Care Industry

Year	Key Events
1974	Worker compensation fund as a form of compulsory insurance for workers. It covered only work-related illness.
1975	First social welfare. It provided free medical care for the poor.
1978	First private health insurance. The first voluntary health insurance.
1980	Royal decree on Civil Servant Medical Benefit System (CSMBS) was passed. It provides formal

	nationwide health care benefit to government officials, government employees and state enterprises. Before that they had other form of employees' fringe benefit.
1981	Low-income card for the poor was introduced.
1983	Mother and Child Voluntary Health Insurance.
1984	Government-supported Voluntary Health Car. It is covered beyond mother and child.
1990	Social Security Act was passed after 36 years since it was initiated. It first covered enterprises with 20 or more employees. Tripartite contributions.
1992	Free medical care for the elderly.
1993	Compulsary Traffic Accident Victim Protection Insurance. It is included in automobile insurance. Patients must claim from this fund before sorting to the other sources
1994	Social Security Act was extended to enterprises with 10 or more employees.
1994	Free medical care for children up to 12 years old was added to social welfare.
1998	Benefit cut for government employees narrow the benefit gap between SSA and CSMBS. Co-payment, limited hospital stays and essential drug list.
2002	National Gold card scheme was introduced for the uninsured. One who is not covered by any form of health insurance will be covered by the scheme. No joining cost but 30 baht per episode
2003	Social Security Act was extended to all enterprises including self-employed, e.g., farmers

Source: Health System Research Institute, Ministry of Public Health, Thailand

Cheap medical services by government hospitals for all people can be considered as a form of free health insurance with co-payment. The first formal health insurance by government is Civil Servant Medical Benefit System (CSMBS) after the royal decree was passed by the Parliament Assembly in 1980. CSMBS is health fringe benefit for government officials and permanent government employees to compensate for lower income level compared to that in the private sector. No contribution is collected from the insured and co-payment is very small or nil. CSMBS provides quite generous coverage as it covers not only the employees themselves but also their immediate family including children and parents. No other health insurance has ever covered parents of the employees. Government enterprises also provide their employees about the same health benefit. In 1998 CSMBS benefits for government officials and permanent government employees has been cut in terms of co-payment, length of stays and drug cost reimbursement to narrow the benefit gap between CSMBS and the other government-run health insurance schemes. Still, it is considered the most generous health insurance scheme administered by government.

The first compulsory health insurance was dated back to 1974 when Workers' Compensation Fund was established to provide health insurance and compensated income for work-related illnesses and accidents sufferred by the factory workers. It is the first layer of safety net for the factory workers. However, some factory workers are still not covered by this fund. Contributions come from factory workers and their employees.

Social security for private sector employees has been discussed since 1954. Unfortunately, it cannot be realized until 1990 when Social Security Act was passed. It was first to cover only medical services when it was implemented. It has been later on expanded to cover unemployment and retirement benefits. It has been intended for

general population but the first covered individuals are the employees of enterprises with 20 or more employees. Participation is compulsory. Employees, employers and government (Social Security Office) have to equally share the burden of insurance premium payment to the Social Security Fund which is administered by the Social Security Office according to the Act. A universal Social Security Insurance (SSI) scheme was established under the Social Security Act. Its health care coverage also includes maternal medical care and maternity leave pay. Each participating employee must first seek health care from his or her designated hospital before being referred to other providers at the cost of the designated provider. Participating hospitals will receive a fixed capitation fee payment for each participating employee. To attract more participating hospitals, government offer them re-insurance against financial risk of catastrophic illness. When the Social Security scheme started, each employer chose the health care providers (hospitals) for their employees to make it easy to start the scheme. At present, participating employees can freely choose and change their own health care providers. Both public and private hospitals can participate. Willingness-to-participate by private hospitals varies. Some private hospitals eye these social security customers as their main source of profit while others are not much interested.

In 1994 the enforcement of the Social Security Act was extended to employees of enterprises with 10 or more employees. Enterprises of all sizes were under the Social Security Scheme by law in 2002. That is, all individual employees can enjoy the social security benefits. In the same year, self-employed like farmers can join the social security scheme for their savings and health insurance. However, only self-employed with high income have chosen to take part in the social security scheme. Still, many poor self-employed and unemployed are uninsured.

Compulsory insurance for traffic accident victim was enforced in 1993. Vehicle owners have to buy insurance from private insurance companies to cover any possible bodily damage in the traffic accidents. Health care claims for all traffic accidents and factory accidents must be made to the corresponding funds before CSMBS or Social Security funds.

In addition to voluntary health insurance offerred by private insurance companies, government also offerred voluntary health insurance (VHI) to cover the uncovered population. Probably the first VHI administered by government is VHI under the mother and child program which started in 1983. Planned mothers can buy insurance to cover medical expense and medical care for infants. VHI was then extended beyond mother and child to general population in 1984. The head of a family can buy VHI for only 500 Baht. VHI can covers the family up to 5 members for one year for medical care at a designated local health care provider. VHI has been subsidized by governmental budget on capitation basis. Referral can be made at the cost of the providers. Even though the services provided by the designated providers is very limited, it has been abused by many already-insured as insurance supplement due to inefficient patient data base system. Many providers ran deficit due to VHI. VHI has been abolished in favor of the new National Health Security (NHS) system which was introduced in 2003.

National Health Security system which is also referred to as “Gold” card system is the most recent addition to Thailand’s health insurance system. It was the

flagship of national health policies by the government under Prime Minister Taksin Shinwatr. Concurrent with the development of national health data base system, NHS system has been established to provide about the same target level of health insurance as that of Social Security insurance to all the remaining uninsured. Opposite to the other health insurance schemes, there is no joining cost for the individual participants but they have to register with National Health Security Office to double-check their eligibility. The eligible will be issued a "Gold" card. The card will entitle its holder to receive medical care with only 30 Baht co-payment per episode at the designated hospital or health center in the area. The scheme has greatly benefited the poor as 30 baht is considered very low compared to the per-episode out-of-pocket payment by an average patient. The number of registered "Gold" cardholders jumped from zero to about 30% of population in just one year. The very poor who can not afford even 30 baht per episode can still seek free medical care under social welfare schemes. The providers will be paid by capitation basis but the capitation fee under NHS is lower than SSI. The providers with NHS patients have higher risk of running deficit than those with SSI patients. Higher capitation fee for NHS patients is being sought to attract hospitals to participate. NHS office tried to invite private hospitals to join the scheme but very few are willing to as the capitation fee is not attractive. Table 2 illustrates shares of different schemes in 1999 and 2002. Decrease in the shares of Government employee can be noticed due to position freeze and early retirement policies for government employees. Proportion of uninsured has decreased and VHI has disappeared since the introduction of NHS.

Table 2 Proportion of Health Insurance by schemes in Thai Population between 1999 and 2002

Scheme	1999	2002
Government Employee	8.90%	5.02%
Social Security	7.10%	8.83%
Social Welfare	32.10%	35.27%
VHI	18.60%	0.01%
Gold Card	0.00%	29.63%
Uninsured	33.30%	21.23%

II. Statement of Problem

Since the introduction of Low-income cards and Social Security Insurance, there have been deficit problems in many government-run hospitals, especially at the tertiary care level. Cost containment becomes the priority policy in all hospitals under budgetary pressure from the government. Reimbursement is generally slow. Patient discrimination was becoming more obvious. Patients of different types were handled separately despite the claim that patients of all types will be equally treated. Many low-income patients and SSI patients have complained that the medical care they received was not up to the satisfactory level. It has been claimed that many death cases were due to low quality treatment, lack of proper treatment or lack of attendance by medical staff under cost containment pressure. The problem became more serious when the National Health Security scheme or "Gold" card scheme started in 2003. The government effort to provide cheap health care services to all the remaining uninsured has put even more cost containment pressure on the government hospitals as the capitation fee for gold cardholders is set at a much lower level than that of SSI

holders. Gold card scheme can be regarded as a form of social welfare as the gold cardholders don't have to make any contribution. Out-of-pocket has been forced to as low as 30 Baht per episode. The capitation fee must be wholly paid out of tax revenue. The policy has significantly increased welfare for the poor in general. The question is "Does it harm the health industry?". Private hospitals at the low end of the market have been affected by this policy. They have lost many customers to government hospitals as they cannot compete with the "30 Baht" scheme in government hospitals. To survive the competition, private hospitals have to change their marketing strategies to aim at the higher end of the market-higher income customers and foreign customers. Even though the competition is good for economic efficiency, there has been a lot of distortion in the "price" of health care and policy-induced inefficiency. Many medical staff in government hospitals decided to move to private hospitals to avoid cost containment pressure and heavy workload as government hospitals cannot deny patients. The shrinking staff causes even more pressure to the existing staff and more congestion in government hospitals. Those moved to the private sector can produce less output with better pay. Do the existing pool of health care resource produce the same level of output in terms of both quality and quantity? Is the health industry as a whole less efficient than before? If it is, what could be the causes of inefficiency? Is inefficiency related to capitation payment? Quality complaints seem to inversely vary with the capitation payment.

III. Research Objectives

The main objective of this paper is to determine the effects of customer/patient types on the efficiency or inefficiency of the hospital system as different patient types mean different pressure on cost containment. Since the hospital sector of the health industry produce multiple outputs and require multiple inputs, this paper has developed the tool for formulation of an ideally efficient (benchmark) hospital with multiple outputs and inputs.

IV. Scope of the Study

Since the question to be asked is about the efficiency of the hospital system not each individual hospital, this study will look at the aggregate hospital services. Government and private hospitals in each area are aggregated as a representative hospital for the area. In this study, 72 cross-sectional data by province in 2002 has been used. There are 76 provinces in the country. Bangkok Metropolitan was excluded. So are other 3 provinces with incomplete data. Lower district level data is more preferred but not available at the time of study.

V. Methodology

If all the inputs and outputs are going to be included and inefficiency of a multiple output firm is not caused by uncertainty, a Data Envelopment Analysis (DEA) model will be very ideal for this study. Unfortunately, there will be some ignored input factors and there are some unavoidable risks involved even for a very ideally efficient firm. The ignored and unexplainable factors could be major sources for variation of efficiency among provinces. Since it is not possible to accommodate all the possible efficiency determinants in the analysis, the ignored and unexplained determinants will be simply treated as a part of random variation among provinces. The more ignored and unexplained determinants the more importance of uncertainty. A Stochastic Frontier Analysis (SFA) model can accommodate uncertainty but adaptation of an SFA model to multiple output firms is rather complex. Its

econometric estimation is not yet well developed. To make estimation simple, it is necessary to aggregate multiple outputs as a single output before an SFA model can be used. Accordingly, the estimation approach taken in this study is divided into two steps.

In the first step, a DEA model is used to estimate the production frontier with only some selected inputs and outputs. It is not the production efficiency index that is being sought in this step. The frontier is needed only to estimate the slopes of its tangents which is regarded as the competitive price ratio for the outputs if prices are known and market is assumed to be price competitive.

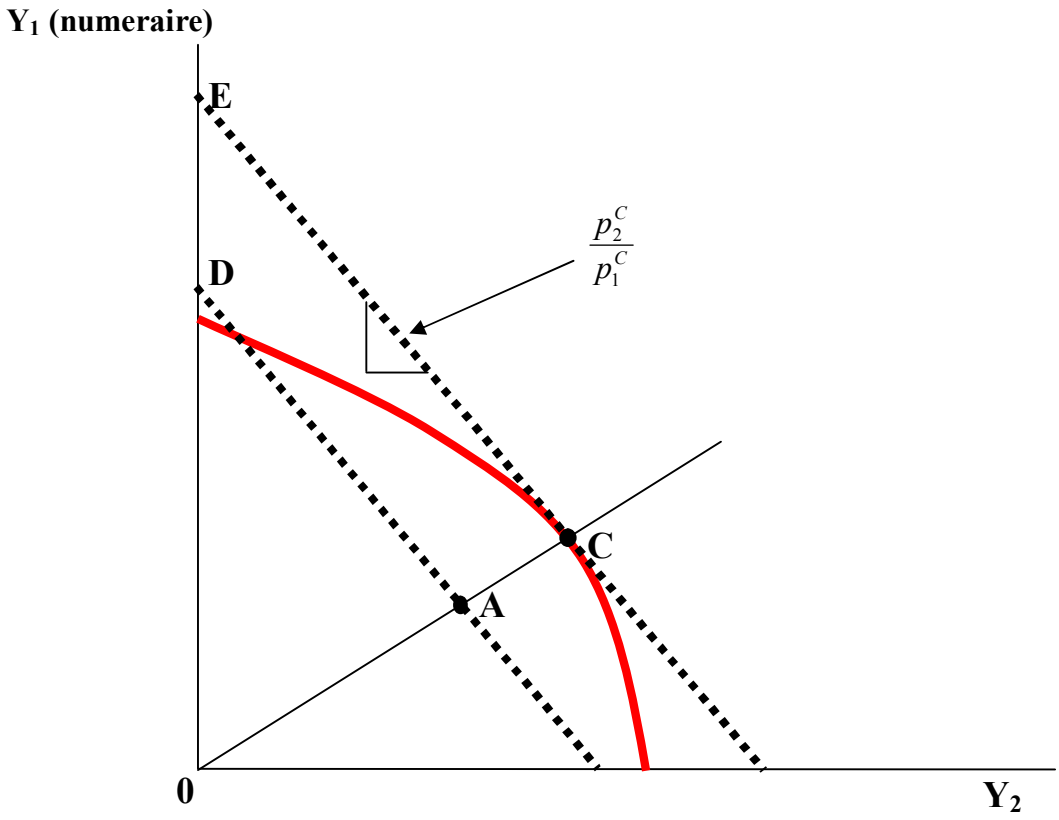


Figure 1 Production Frontier for 2 outputs given a fixed set of inputs

In order to aggregate multiple outputs, one of the outputs will be chosen as the numeraire and all the other outputs will be converted into the numeraire equivalence. The conversion could be made by the price ratio. Figure 1 illustrates the production frontier between 2 outputs with a given set of inputs. Output bundle C represents an efficient production set as point C is right on the frontier. Any output bundle on line 0C will be less efficient compared to output bundle C. Value of output bundle C is equivalent to that of output bundle E which can be quantified in terms of output#1 by using the price ratio as follows:

$$Y_{eq}^C = Y_1^E = Y_1^C + \frac{p_2^C}{p_1^C} Y_2^C$$

where Y_1^X and Y_2^X are the quantity of the two outputs at any point X

Y_{eq}^X

is equivalent amount of output-1 for any bundle X.

$\frac{p_2^C}{p_1^C}$ is the price ratio or slope of the tangent at point C on the frontier

Using the same price ratio, output bundle A which is considered inefficient compared to bundle C is equivalent to output bundle D which is a single output.

$$Y_{eq}^A = Y_1^D = Y_1^A + \frac{p_2^C}{p_1^C} Y_2^A$$

Based on Aigner, Lovell and Schmidt (1977), all the rational firms choose their production set to maximize the combined value of their outputs given a set of inputs. Production choices of all the pareto-efficient firms will form a production frontier. Given that all the hospitals behave like a rational firm, the DEA model for the estimation of price ratios can be expressed a linear programming problem as follows:

$$\max_{\boldsymbol{\mu},\boldsymbol{\nu}} \quad \mu_1 IPD_i + \mu_2 BRTH_i + \mu_3 OPD_{1i} + \ldots + \mu_{23} OPD_{21i}$$

Subject to

$$\nu_1 MD_i + \nu_2 BED_i = 1$$

$$\mu_1 IPD_j + \mu_2 BRTH_j + \mu_3 OPD_{1j} + \ldots + \mu_{23} OPD_{21j} - \nu_1 MD_j - \nu_2 BED_j \leq 0, \quad j=1,2,\ldots,72$$

$$\boldsymbol{\mu},\boldsymbol{\nu} \geq \mathbf{0}$$

where

$\boldsymbol{\mu},\boldsymbol{\nu}$	are the set of output and input prices
IPD_j	is log of number of the inpatient-days of province *i*
$BRTH_j$	is log of the number of birth cases of province *i*
OPD_j	is log of number of out-patient-visits of province *i* categorized by 21 causes of illness
MD	is log of number of medical doctors (input#1) of province *i*
BED	is log of number of overnight beds (input#2) of province *i*

The list of causes of illness is shown in Table A1 in the appendix. The above DEA model is based on the following assumptions:

1) All the hospitals follow a constant return-to-scale Cobb-Douglas production function so that hospitals in the same area can be aggregated to a representative hospital
2) Medical doctors (MD) and patient beds (BED) are two substitutable inputs to be considered. Other inputs are assumed to be complement of these two inputs or randomly varied.
3) The twenty three outputs are as follows:

   birth cases (BRTH)

   inpatient-days (IPD)

   numbers of outpatient visits by 21 causes of illness (OPD_i)

4) Inpatient-days is treated as the output numeraire. All the other outputs will be converted to numeraire equivalence

Cross-sectional data of 2 input factors and 23 outputs by province for 2002 are shown in Table A1 and Table A2. Given hospital output and input data for each of the chosen provinces, optimization of the above DEA model for province *i* will yield all the output and input prices (**μ,ν**). However, only the output prices will be used. The twenty three outputs as described above can be aggregated into a single output in terms of IPD equivalent.

$$Y_i = IPD_i + \frac{\mu_2}{\mu_1} BRTH_i + \frac{\mu_3}{\mu_1} OPD_{1i} + \frac{\mu_4}{\mu_1} OPD_{2i} + \ldots + \frac{\mu_{23}}{\mu_1} OPD_{21i}$$

where

Y_i is the equivalent output in terms of IPD output or in terms of inpatient-days

The equivalent output of each province reflects the productivity of the two included input factors (MD and BED) and the productivity of the other ignored factors and inefficiency of the province.

In the second step, the equivalent hospital output and the two chosen input factors (MD and BED) of the provinces will be fed into an SFA model. The idea of Stochastic Frontier Analysis model dated back to 1957 when Farell published his article titled "The Measurement of Productive Efficiency" in Journal of the Royal Statistical Society. There were many additions to Farell's work. Determinants of the technical efficiencies of firms were included into SFA in Battese and Coelli (1988) which is the basis of the stochastic model in this study. To be consistent with the DEA model adopted in the first step, the following assumptions are made:

1) Hospital inefficiency could be due to patient types.

2) Allocative efficiency is assumed. Inefficiency is purely technical.

The SFA model to be adopted in this study can be described as follows:

$$\begin{aligned}
Y_i &= \beta_0 + \beta_1 MD_i + (1-\beta_1) BED_i + \varepsilon_i - u_i \\
\varepsilon_i &\sim N(0, \sigma_\varepsilon^2) \\
u_i &\sim \text{truncated } N(Z_i, \sigma_u^2) \\
Z_i &= \delta_0 + \delta_1 GC_i + \delta_2 WC_i + \delta_3 SS_i + \delta_4 GV_i
\end{aligned}$$

where i = province index, i=1,2,…72

ε_i – uncertainty component for province *i*'s output

u_i – non-negative measure of technical inefficiency of province *i*

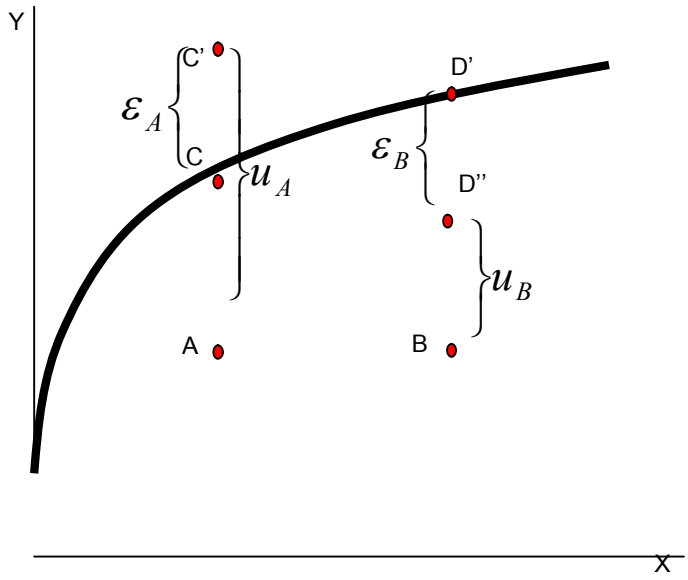
Z_i – unobserved determinant for technical inefficiency

GC_i – log of the number of Gold Card patients in province *i*

WC_i – log of the number of social welfare card patients in province *i*

SS_i – log of the number of social security patients in province *i*

GV_i – log of the number of government employee patients in province *i*



According to Battese and Coelli (1988), the uncertainty (ε_i) could happen to any firm even the most efficient one while technical inefficiency (u_i) is firm-specific characteristics which depends on various determinants. Intuition behind the SFA model can be explained by Figure 2 above in which the horizontal axis represents the input vector and the vertical axis represents the aggregate output. The curve is the fully efficient frontier. Deviation from the frontier is due to uncertainty and technical inefficiency.

The SFA model is of econometric type. It can be estimated with the maximum likelihood method. In this study the suspected (hypothesized) determinants are the number of patients according to types. It has been hypothesized that some patient types could cause inefficiencies as already explained in the above paragraphs. Parameter δs are the focal point of the study. If they are significantly different from zero, it can be claimed that the number of patients by type determines technical efficiency of the hospital system of the province. According to the above discussion, parameter δs are expected to have the sign as follows:

Parameter	Expected sign
δ_1	+
δ_2	+
δ_3	+
δ_4	-

VI. Estimation Results, Discussion and Conclusion

Given hospital data for 72 provinces in 2002, aggregate output by province estimated from the DEA model is shown in Appendix A2. The Maximum Likelihood estimated SFA model can be shown as follows:

$$Y_i = -2.096 + 1.000\, MD_i + 0.000\, BED_i + \varepsilon_i - u_i$$

se	0.0661	0.0012
z-stat	-31.71	833.3

$$\varepsilon_i \sim N(0, 0.6471)$$

$$u_i \sim \text{truncated } N(Z_i, 0.0345)$$

$$Z_i = 0.7246 + 0.0858\, GC_i + 0.2098\, WC_i + 0.0060\, SS_i - 0.4008\, GV_i$$

se	0.5217	0.0975	0.0762	0.0654	0.1138
z-stat	1.389	0.8800	2.753	0.0917	-3.522

log likelihood = 18.60

LR stat = 21.84 (df = 6)

N = 72

The derived technical efficiencies by provinces according to the SFA model were listed in Table A3 in the Appendix. Three significant conclusions can be made out of the results as follows:

1) Deviation from the production frontier is mainly due to uncertainty not the technical inefficiency.
2) BED cost share or β_1 is zero. This simply implies that BED is a complement rather than a substitute to MD.
3) Sign of the coefficient estimates of all the technical inefficiency determinants is the same as expected but only those for social welfare patients and government officials are significantly different from zero. They are on the opposite ends of health benefit spectrum. Generous health benefit of government officials makes them very welcome to all the hospitals as it covers more than the other schemes and the reimbursement is more flexible. Social welfare patients seem to be a burden to all the hospitals even though the expense is reimbursable. Hospitals are reluctant to welcome these group of patients. Slow reimbursement or non-reimbursability could be the reason. As for capitation fee patients like SSI and NHS patients, their marginal effect on technical inefficiency does not seem to be significant. Capitation fee patients which are in the middle of health insurance spectrum tend to be less welcome compared to government official patients but more welcomed than social welfare patients as can be seen from the z-statistic values for the coefficient estimates. However, the inefficiency variation among provinces does not seem to confirm it. Pair-wise comparison between SSI patients and NHS patients did not reveal any significant different in their marginal contribution to technical inefficiency of the hospitals.

H_0: $\beta_1 \leq \beta_3$
H_1: $\beta_1 > \beta_3$

z-stat=0.0609

In conclusion, it is not surprising to find the significant marginal effects of social welfare scheme and government employee health benefit on the hospital technical efficiency. It is not the major research question in this study. Finding that NHS patients have no significant effect on technical inefficiency is quite against expectation. The selected year of analysis might be inappropriate as NHS was introduced in 2002 but symptoms of inefficiency came after that. The real effect may have not taken place in the year of analysis. It is suggested that similar analyses should be done on later years to confirm the results.

VIII. Suggested Future Studies

The two-step approach taken in this study made it easy for calculation and estimation but its consistency is still questionable. The DEA model will be consistent if there is no uncertainty and all the major input factors are considered. If all the ignored factors are treated as random variation among provinces, they can be effectively regarded as uncertainty. Since uncertainty is responsible for about 90% of the deviation from the mean frontier, further improvement can be made by subtracting the original level of outputs with uncertainty component determined by the SFA model and re-running the DEA model. The calculation can be made in iteration to reduce the uncertainty proportion to improve the estimation quality.

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