Chapter 2 Kinematics of particles

Particle motion

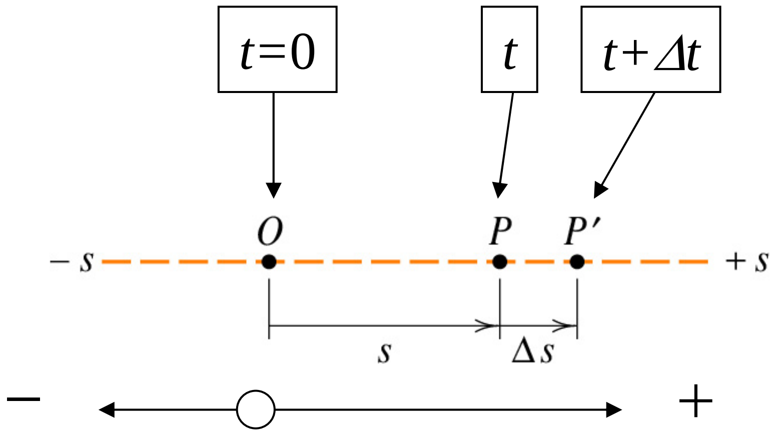
physical dimensions are so small compared with the radius of curvature of its path

⇒

The motion of particle may be treated as that of a point

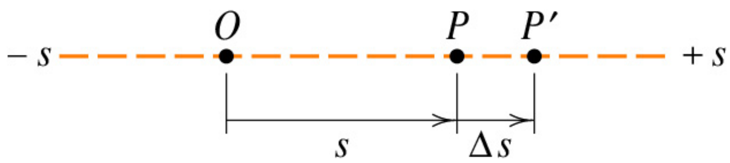
Ex: the motion of airplane, car

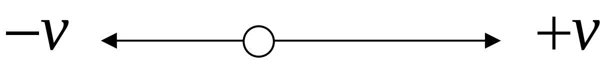
2/2 Rectilinear motion



The change in the position coordinate during the interval time Δt is called the **<u>displacement Δs</u>**

Velocity and Acceleration (1)





The average velocity $v_{av} = \Delta s / \Delta t$

Instantaneous velocity $v = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t}$ ⇨ $v = \frac{ds}{dt} = \dot{s}$

Similarly

Instantaneous acceleration $a = \frac{dv}{dt} = \dot{v}$ or $a = \frac{d^2 s}{dt^2} = \ddot{s}$

Speeding up: $+a$

Slowing down: $-a$ (decelerating)

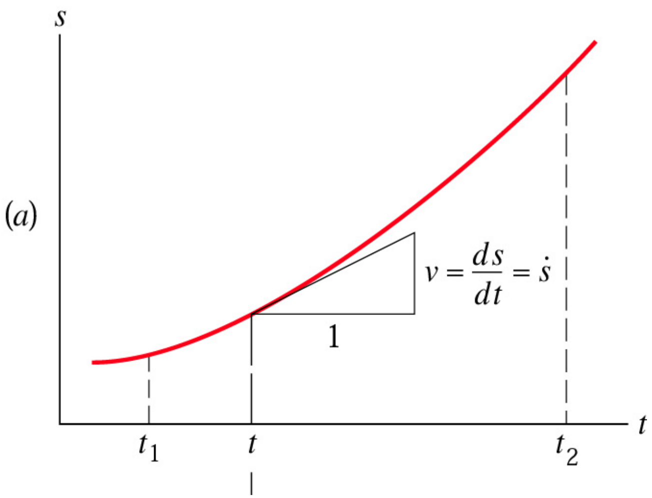
Velocity and Acceleration (2)

Another relation

By chain rule $$v = \frac{ds}{dt} = \frac{ds}{dv} \cdot \frac{dv}{dt} = a\frac{ds}{dv}$$

$$\Rightarrow \quad \boxed{vdv = ads}$$

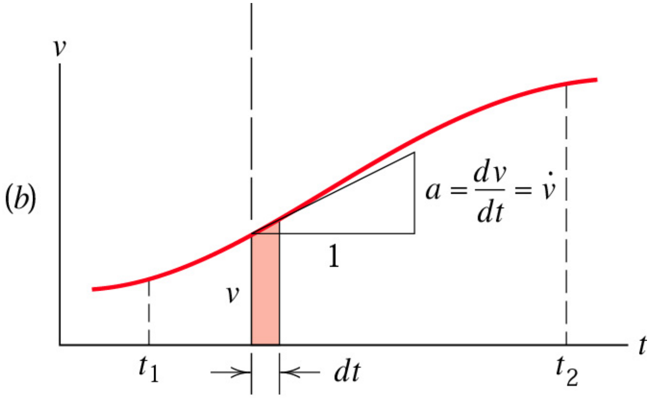
Graphical interpretations



s-*t* graph

- Slope = $\dfrac{ds}{dt} = \dot{s} = v$ (velocity)

Graphical interpretations (1)



v-t graph

- Slope = $\frac{dv}{dt} = \dot{v} = a$ (acceleration)

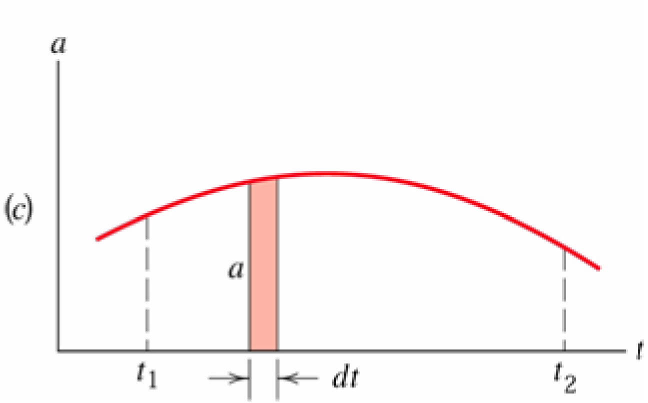
- Area under *v*-*t* curve

$$\frac{ds}{dt} = v \Rightarrow ds = vdt = dA$$

$$A = \int_{s_1}^{s_2} ds = \int_{t_1}^{t_2} vdt \Rightarrow A = s_2 - s_1 = \int_{t_1}^{t_2} vdt$$

Area under *v*-*t* graph between t_1 and t_2 is the net displacement during that interval

Graphical interpretations (2)



a-*t* graph

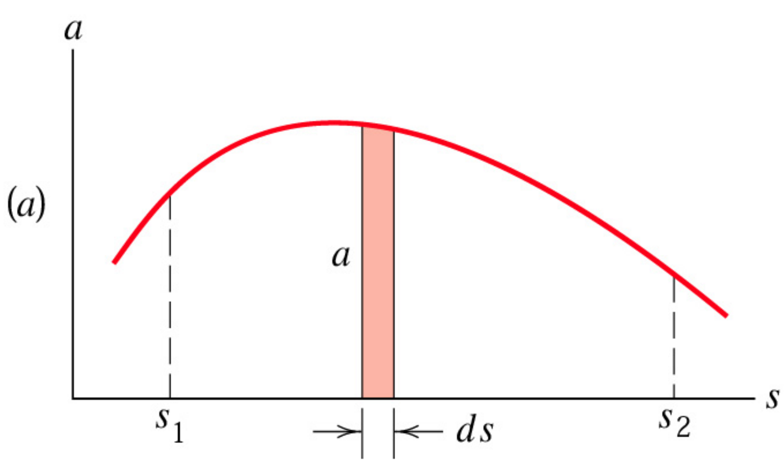
- Area under *a*-*t* curve

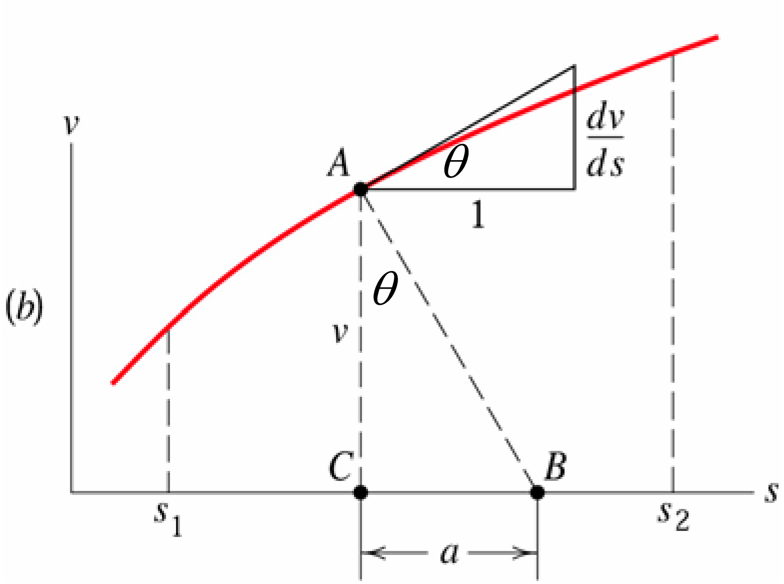
$$\frac{dv}{dt} = a \quad \Rightarrow \quad dv = a\,dt = dA$$

$$A = \int_{v_1}^{v_2} dv = \int_{t_1}^{t_2} a\,dt \quad \Rightarrow \quad A = v_2 - v_1 = \int_{t_1}^{t_2} a\,dt$$

Area under *a*-*t* graph between t_1 and t_2 is the net change in velocity between t_1 and t_2

Graphical interpretations (3)





$$A = \int_{v_1}^{v_2} v\,dv = \int_{s_1}^{s_2} a\,ds$$

$$A = \frac{1}{2}({v_2}^2 - {v_1}^2) = \int_{s_1}^{s_2} a\,ds$$

$$v\,dv = a\,ds \longrightarrow \frac{dv}{ds} = \frac{a}{v} = \frac{CB}{v}$$

Acceleration can be found from distance *CB* (*s* axis)

Analytical integration (1)

Constant acceleration

$$a = \frac{dv}{dt}$$

$$\int_{v_0}^{v} dv = \int_0^t a\,dt = a\int_0^t dt \quad \Rightarrow \quad v = v_0 + at$$

$$v\,dv = a\,ds$$

$$\int_{v_0}^{v} v\,dv = \int_{s_0}^{s} a\,ds = a\int_{s_0}^{s} ds \quad \Rightarrow \quad v^2 = v_0{}^2 + 2a(s - s_0)$$

$$v = \frac{ds}{dt}$$

$$\int_{s_0}^{s} ds = \int_0^t v\,dt = \int_0^t (v_0 + at)\,dt \quad \Rightarrow \quad s = s_0 + v_0 t + \frac{1}{2}at^2$$

Basic relation

Analytical integration (2)

Acceleration given as a function of time, *a = f(t)*

$$a = \frac{dv}{dt}$$

$$vdv = ads$$

$$v = \frac{ds}{dt}$$

Basic relation

$$\int_{v_0}^{v} dv = \int_{0}^{t} adt = \int_{0}^{t} f(t)dt \Rightarrow v = v_0 + \int_{0}^{t} f(t)dt$$

$$v = g(t)$$

$$\int_{s_0}^{s} ds = \int_{0}^{t} vdt \Rightarrow s = s_0 + \int_{0}^{t} vdt$$

Or by solving the differential equation $\ddot{s} = a = f(t)$

Analytical integration (3)

Acceleration given as a function of velocity, *a = f(v)*

$$a = \frac{dv}{dt}$$

$$vdv = ads$$

$$v = \frac{ds}{dt}$$

Basic relation

$$\int_0^t dt = \int_{v_0}^{v} \frac{1}{a} dv = \int_{v_0}^{v} \frac{1}{f(v)} dv$$

$$\int_{s_0}^{s} ds = \int_{v_0}^{v} \frac{v}{a} dv = \int_{v_0}^{v} \frac{v}{f(v)} dv \quad \Rightarrow \quad s = s_0 + \int_{v_0}^{v} \frac{v}{f(v)} dv$$

Analytical integration (4)

Acceleration given as a function of displacement, *a = f(s)*

$$a = \frac{dv}{dt}$$

$$v\,dv = a\,ds$$

$$v = \frac{ds}{dt}$$

Basic relation

$$\int_{v_0}^{v} v\,dv = \int_{s_0}^{s} a\,ds = \int_{s_0}^{s} f(s)\,ds \quad \Rightarrow \quad v^2 = v_0^{\,2} + 2\int_{s_0}^{s} f(s)\,ds$$

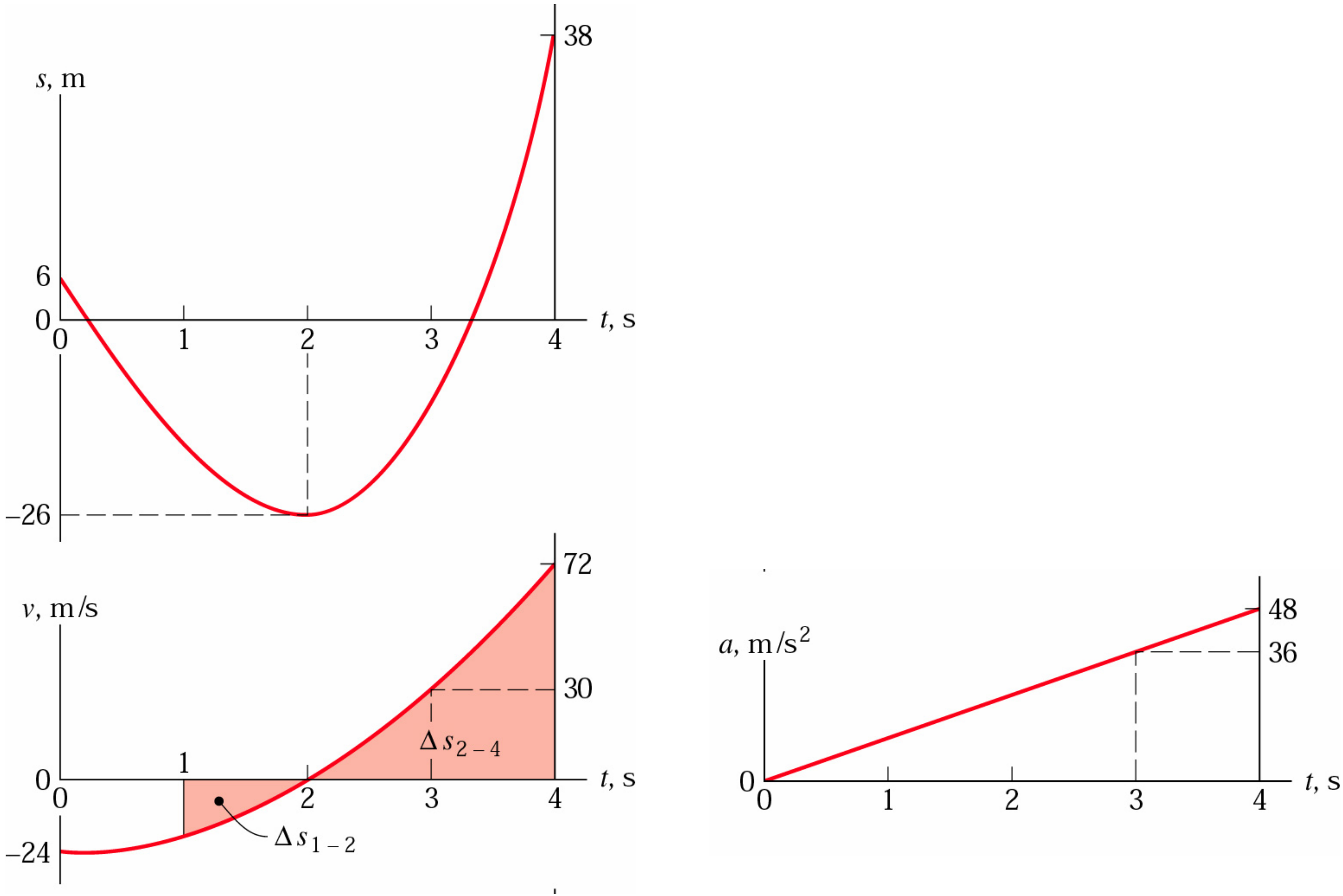
$$v = g(s)$$

$$\int_{0}^{t} dt = \int_{s_0}^{s} \frac{1}{v}\,ds = \int_{s_0}^{s} \frac{1}{g(s)}\,ds$$

Sample 1

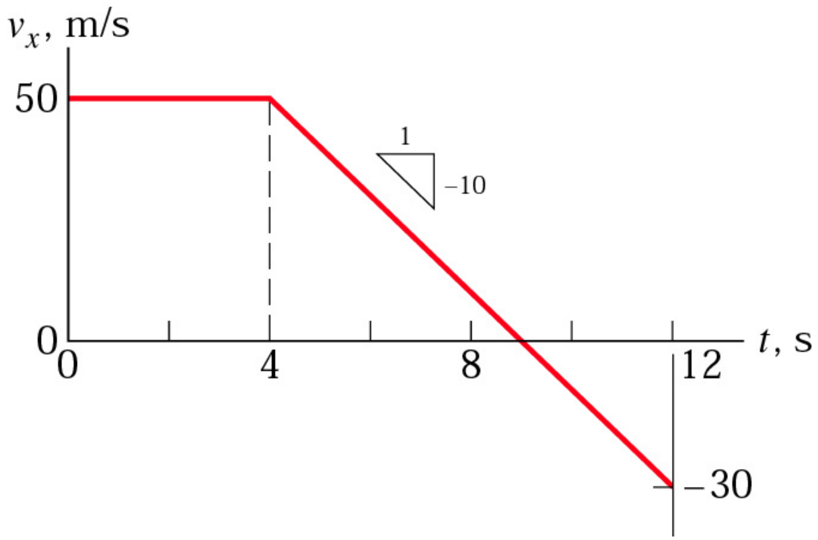
The position coordinate of a particle which is confined to move along a straight line is given by $s = 2t^3 - 24t + 6$, where s is measured in meters from a convenient origin and t is in seconds. Determine *(a)* the time required for the particle to reach a velocity of 72 m/s from its initial condition at $t = 0$, *(b)* the acceleration of the particle when $v = 30$ m/s, and *(c)* the net displacement of the particle during the interval from $t = 1$ s to $t = 4$ s.

Sample 1



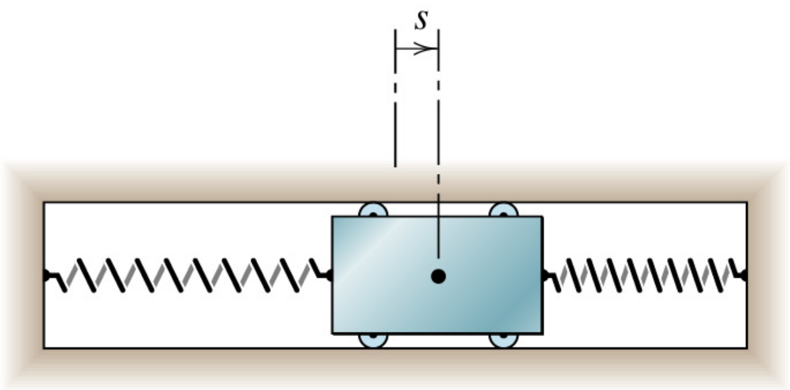
Sample 2

A particle moves along the x-axis with an initial velocity $v_x = 50$ m/s at the origin when $t = 0$. For the first 4 seconds it has no acceleration, and thereafter it is acted on by a retarding force which gives it a constant acceleration $a_x = -10$ m/s^2. Calculate the velocity and the x-coordinate of the particle for the conditions of $t = 8$ s and $t = 12$ s and find the maximum positive x-coordinate reached by the particle.



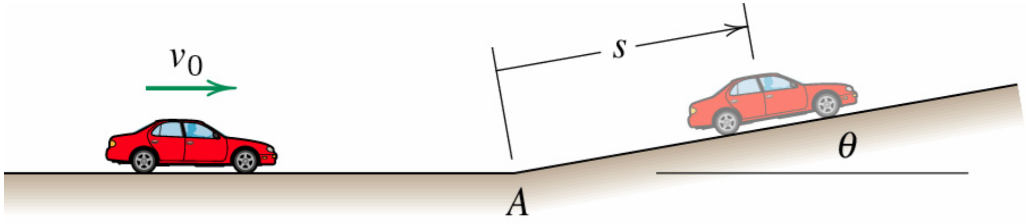
Sample 3

The spring-mounted slider moves in the horizontal guide with negligible friction and has a velocity v_0 in the s-direction as it crosses the mid-position where $s = 0$ and $t = 0$. The two springs together exert a retarding force to the motion of the slider, which gives it an acceleration proportional to the displacement but oppositely directed and equal to $a = -k^2 s$, where k is constant. Determine the expressions for the displacement s and velocity v as functions of the time t.



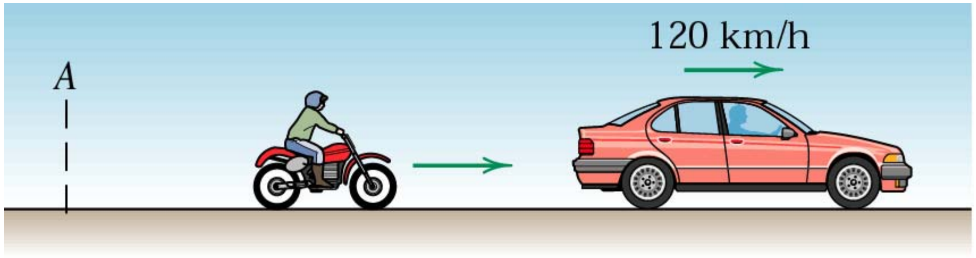
Sample 4

The car is traveling at a constant speed v_0 = 100 km/h on the level portion of the road. When the 6-percent ($\tan\theta = 6/100$) incline is encountered, the driver does not change the throttle setting and consequently the car decelerates at the constant rate $g\sin\theta$. Determine the speed of the car (*a*) 10 seconds after passing point A and (*b*) when s = 100 m.

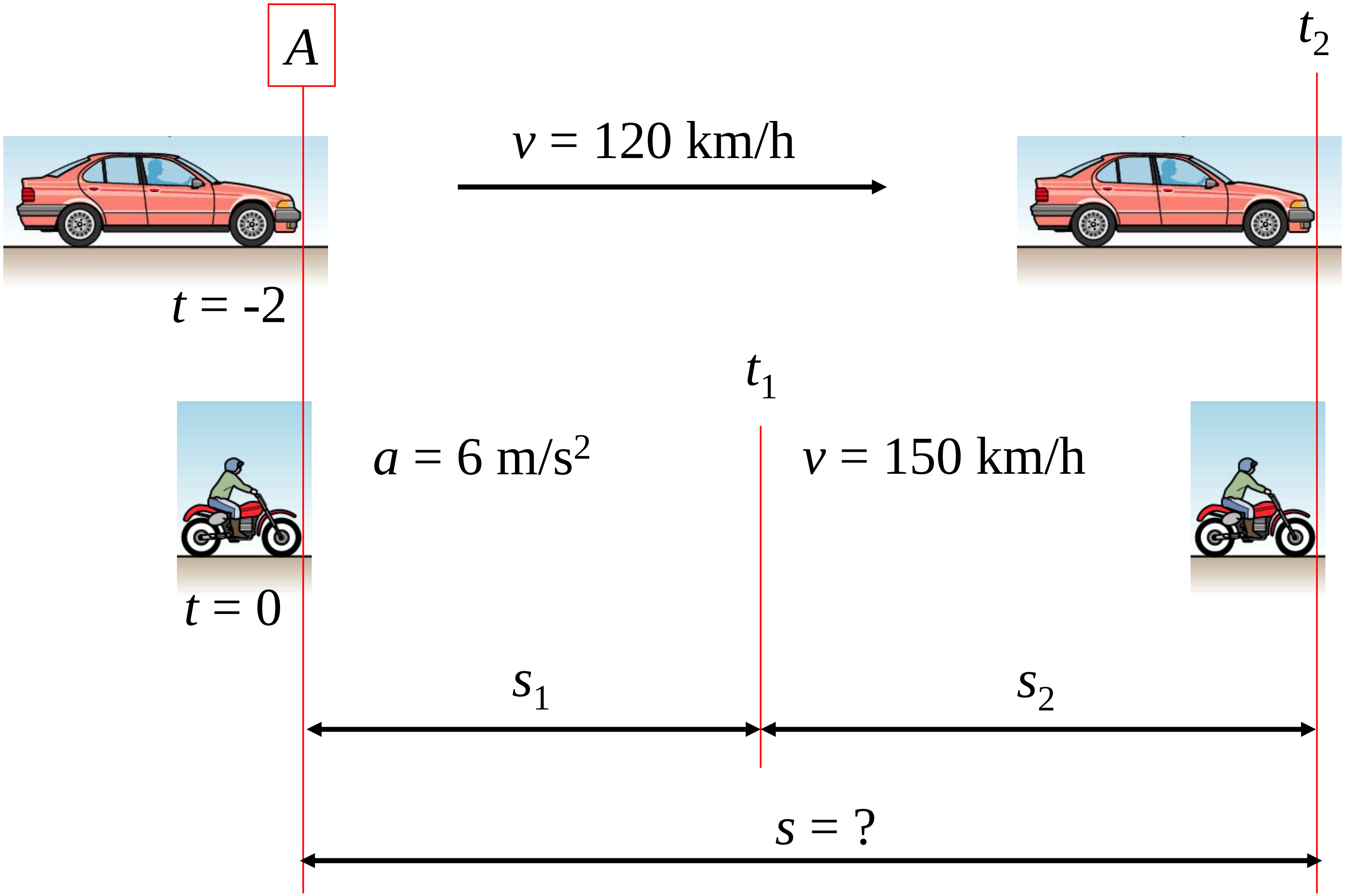


Sample 5

A motorcycle patrolman starts from rest at *A* two seconds after a car, speeding at the constant rate of 120 km/h, passes point *A*. If the patrolman accelerates at the rate of 6 m/s^2 until he reaches his maximum permissible speed of 150 km/h, which he maintains, calculate the distance *s* from point *A* to the point at which he overtakes the car.

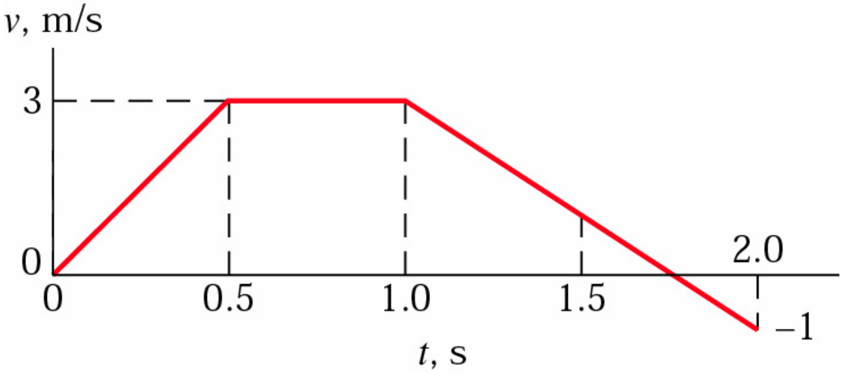


Sample 5



Sample 6

A particle starts from rest at x = -2 m and moves along the x-axis with the velocity history shown. Plot the corresponding acceleration and the displacement histories for the 2 seconds. Find the time t when the particle crosses the origin.



Sample 7

When the effect of aerodynamic drag is included, the y-acceleration of a baseball moving vertically upward is $a_u = -g - kv^2$, while the acceleration when the ball moving downward is $a_d = -g + kv^2$, where k is a positive constant and v is the speed in meters per second. If the ball is thrown upward at 30 m/s from essentially ground level, compute its maximum height h and its speed v_f upon impact with the ground. Take k to be 0.006 m-1 and measure that g is constant.