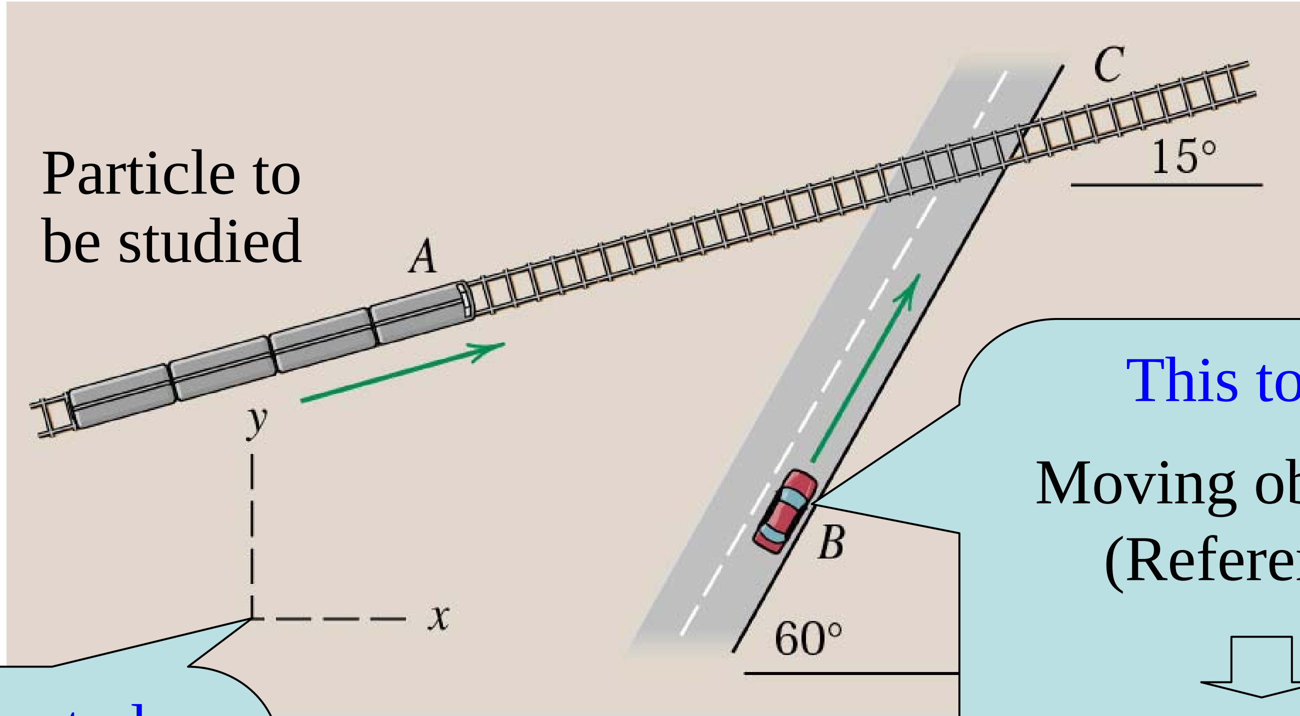
Relative motion (Translating axes)



This topic

Moving observer
(Reference)

Relative motion

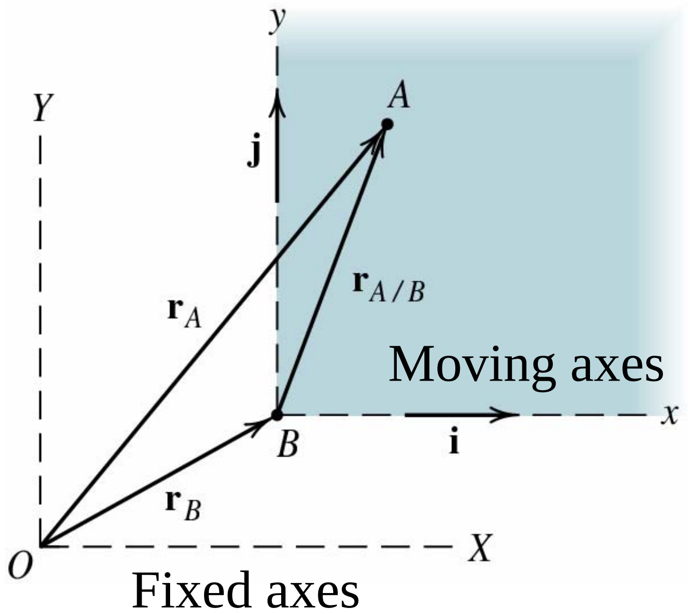
Former study

Observer
(no motion)

Absolute motion

If motion of the reference is known, absolute motion of the particle can also be found

Relative motion (2)

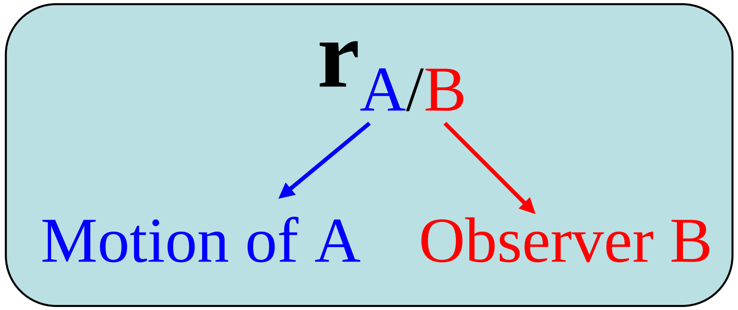


A = a particle to be studied

B = a "(moving) observer" whose *x*-*y* axis is attached to

Motion of *A* measured using *X*-*Y* axis is called the "absolute motion"

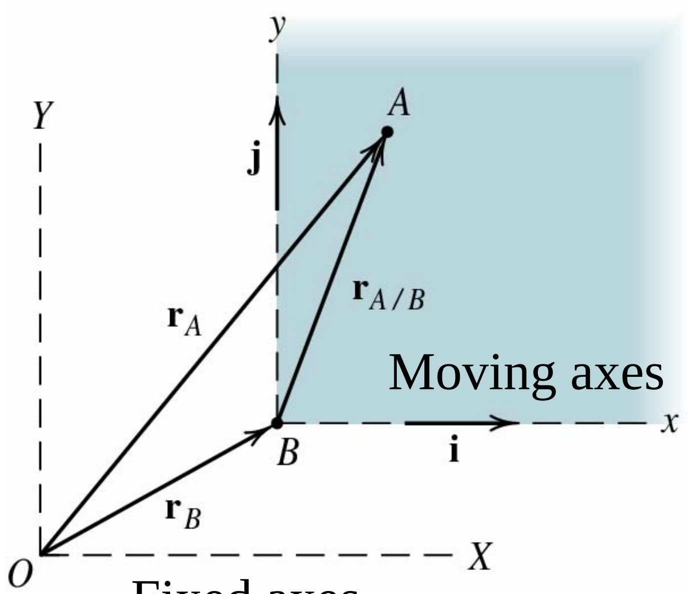
Motion of *A* measured by the observer at *B* is called the "relative motion of *A* with respect to *B*



Note

Axes attached to the earth surface are usually assumed to be fixed

Relative position



Here only the case where the *x*-*y* is <u>not</u> rotating is considered. (only translating)

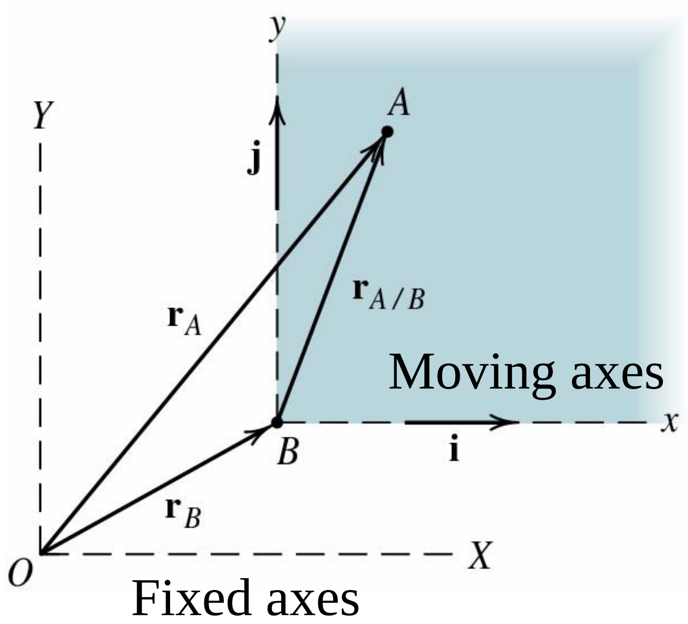
If the observer at *B* use the *x*-*y* coordinate system to describe the position vector of *A* we have

$$\vec{r}_{A/B} = x\hat{i} + y\hat{j}$$

- $\hat{i}$ and $\hat{j}$ are the unit vectors along *x* and *y* axes
- (x, y) is the coordinate of *A* measured in *x*-*y* frame

<u>*Note*</u>: Other coordinate systems can also be used

Absolute and relative motion



$$\vec{r}_A = \vec{r}_B + \vec{r}_{A/B}$$

$\vec{r}_A$: absolute position vector of A

$\vec{r}_B$: absolute position vector of B

$\vec{r}_{A/B}$: relative position vector of A with respect to B

Velocity

$$\dot{\vec{r}}_A = \dot{\vec{r}}_B + \dot{\vec{r}}_{A/B}$$

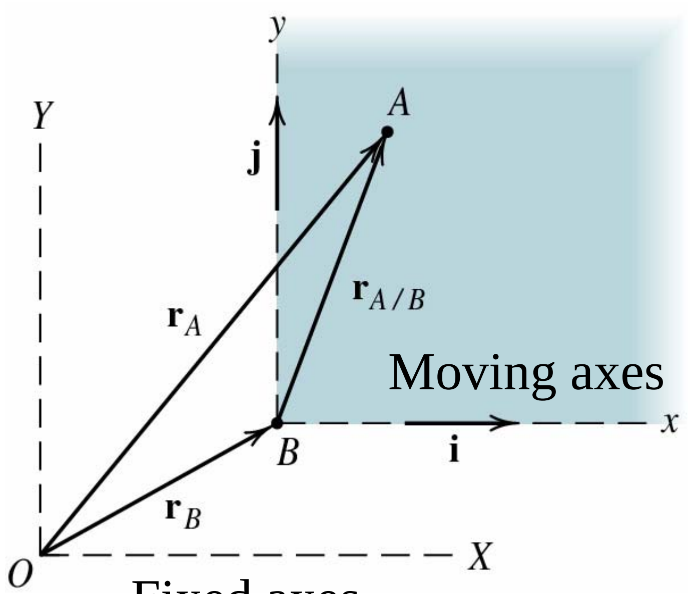
$$\vec{v}_A = \vec{v}_B + \vec{v}_{A/B}$$

Acceleration

$$\ddot{\vec{r}}_A = \ddot{\vec{r}}_B + \ddot{\vec{r}}_{A/B}$$

$$\vec{a}_A = \vec{a}_B + \vec{a}_{A/B}$$

Absolute and relative motion (2)



For rectangular coordinate

$$\vec{r}_{A/B} = x\hat{i} + y\hat{j}$$

$$\vec{v}_{A/B} = \dot{\vec{r}}_{A/B} = \dot{x}\hat{i} + \dot{y}\hat{j}$$

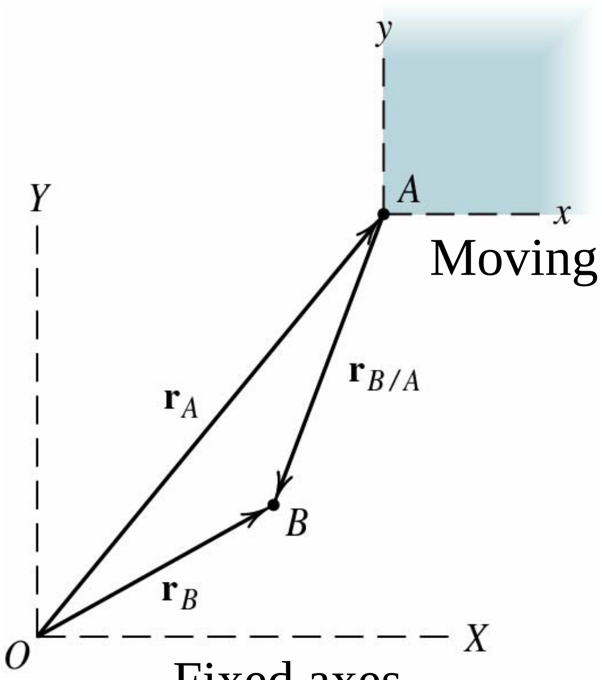
$$\vec{a}_{A/B} = \ddot{\vec{r}}_{A/B} = \ddot{x}\hat{i} + \ddot{y}\hat{j}$$

$$\vec{v}_A = \vec{v}_B + \vec{v}_{A/B} = \vec{v}_B + \dot{x}\hat{i} + \dot{y}\hat{j}$$

$$\vec{a}_A = \vec{a}_B + \vec{a}_{A/B} = \vec{a}_B + \ddot{x}\hat{i} + \ddot{y}\hat{j}$$

Similarly any coordinate system may be used for the absolute motion

Choice of observer



Moving axes

Fixed axes

Particle *A* can also be used as the origin of the reference coordinate

$$\vec{r}_B = \vec{r}_A + \vec{r}_{B/A}$$

$$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A}$$

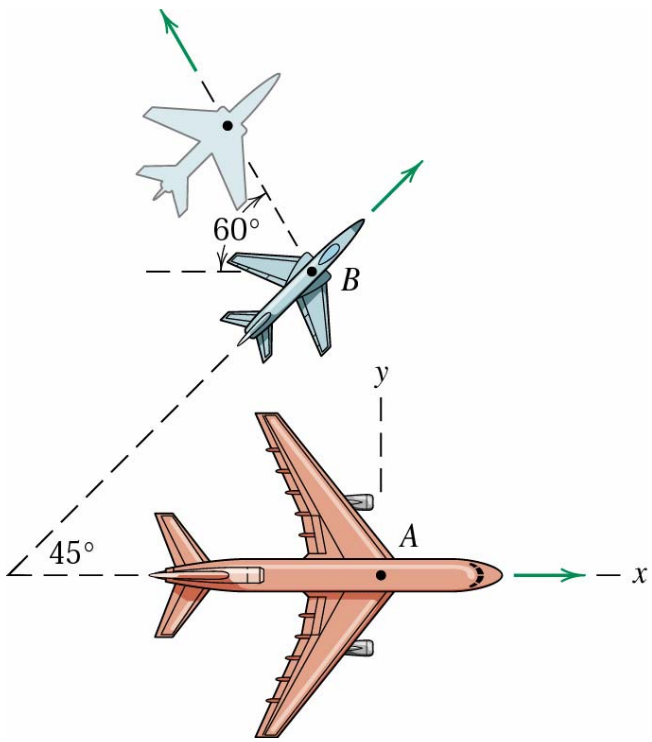
$$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$$

It is seen that

$$\vec{r}_{A/B} = -\vec{r}_{B/A} \qquad \vec{v}_{A/B} = -\vec{v}_{B/A} \qquad \vec{a}_{A/B} = -\vec{a}_{B/A}$$

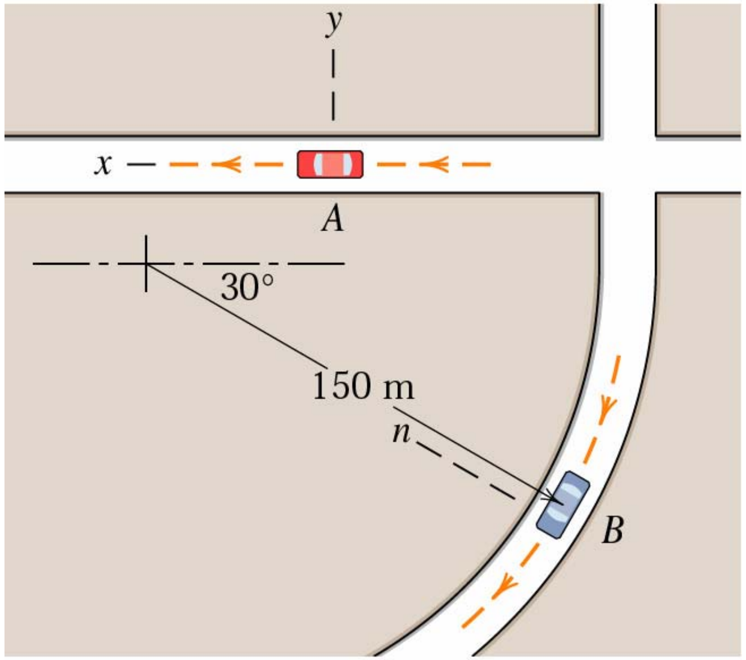
Sample problem 2/12

Passengers in the jet transport *A* flying east at a speed of 800 km/h observe a second jet plane *B* that passes under the transport in horizontal flight. Although the nose of *B* is pointed in the 45° northeast direction, plane *B* appears to the passengers in *A* to be moving away from the transport at the 60° angle as shown. Determine the true velocity of *B*.



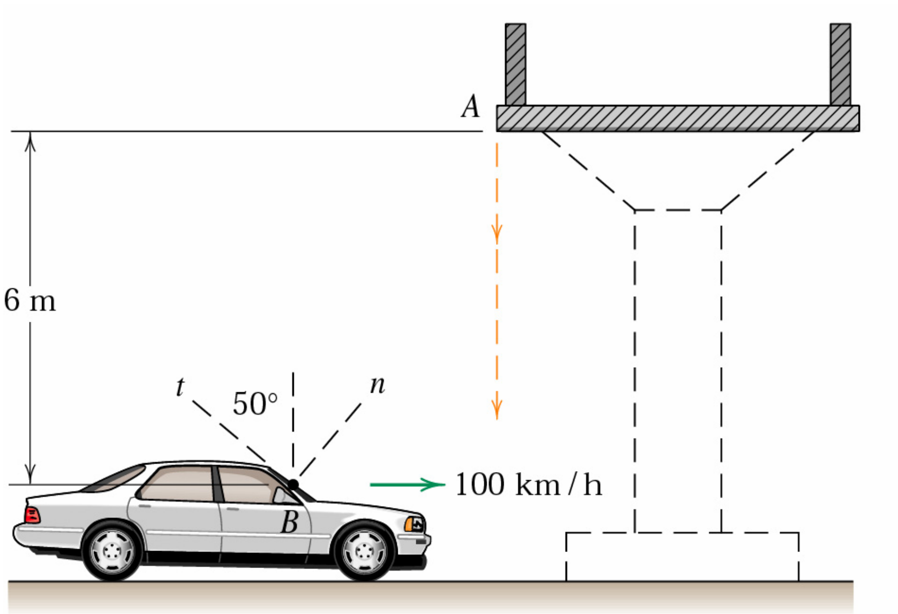
Sample problem 2/13

Car *A* is accelerating in the direction of its motion at the rate of 1.2 m/s^2. Car *B* is rounding a curve of 150-m radius at a constant speed of 54 km/h. Determine the velocity and acceleration which car *B* appears to have to an observer in car *A* if car *A* has reached a speed of 72 km/h for the positions represented.



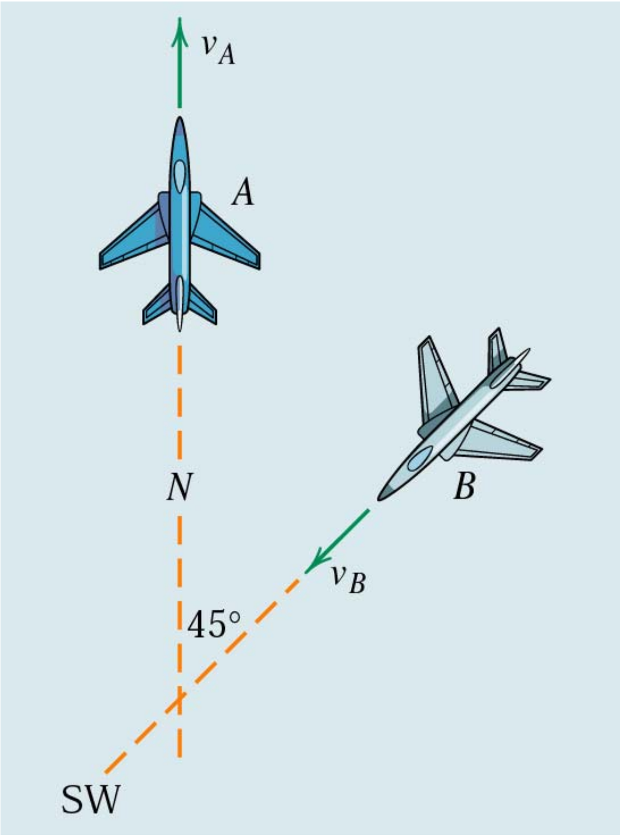
Sample 3 (2/195)

A drop of water falls with no initial speed from point *A* of a highway overpass. After dropping 6 m, it strikes the windshield at point *B* of a car which is traveling at a speed of 100 km/h on the horizontal road. If the windshield is inclined 50 from the vertical as shown, determine the angle θ relative to the normal *n* to the windshield at which the water drop strikes.



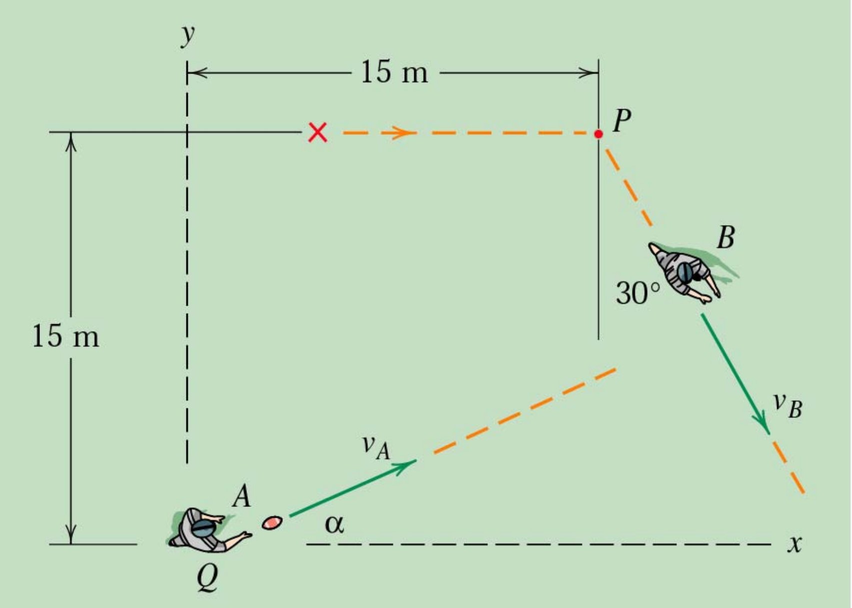
Sample 4 (2/201)

Airplane *A* is flying north with a constant horizontal velocity of 500 km/h. Airplane *B* is flying southwest at the same altitude with a velocity of 500 km/h. From the frame of reference of *A* determine the magnitude v_r of the apparent or relative velocity of *B*. Also find the magnitude of the apparent velocity v_n with which *B* appears to be moving sideways or normal to its centerline. Would the results be different if the two airplanes were flying at different but constant altitude.



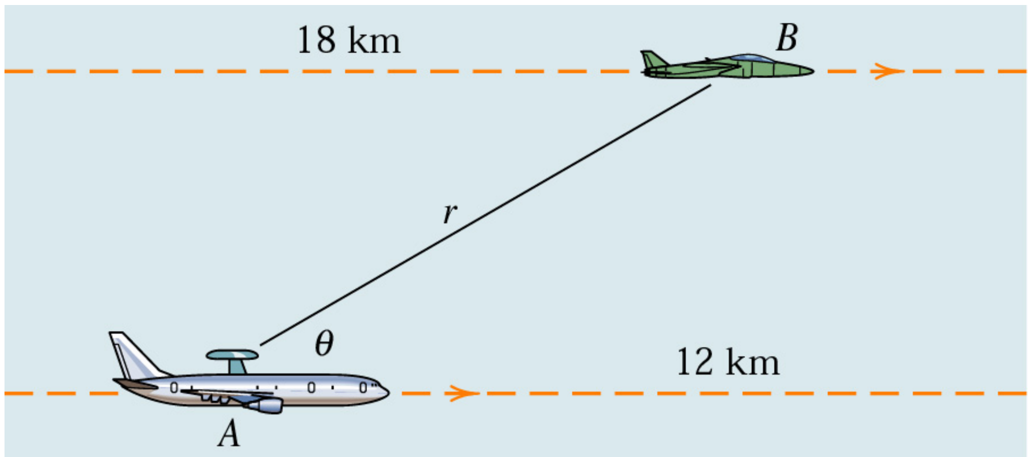
Sample 5 (2/203)

After starting from the position marked with the “x”, a football receiver *B* runs the slant-in pattern shown, making a cut at *P* and thereafter running with a constant speed v_b = 7 m/s in the direction shown. The quarterback releases the ball with a horizontal velocity of 30 m/s at the instant the receiver passes point *P*. Determine the angle α at which the quarterback must throw the ball, and the velocity of the ball relative to the receiver when the ball is caught. Neglect any vertical motion of the ball.



Sample 6 (2/204)

The aircraft *A* with radar detection equipment is flying horizontally at an altitude of 12 km and is increasing its speed at the rate of 1.2 m/s each second. Its radar locks onto an aircraft *B* flying in the same direction and in the same vertical plane at an altitude of 18 km. If *A* has a speed of 1000 km/h at the instant when $\theta = 30°$, determine the values of $\ddot{r}$ and $\ddot{\theta}$ at the same instant if *B* has a constant speed of 1500 km/h.



Sample 7 (2/205)

A batter hits the ball A with an initial velocity of $v_0 = 30$ m/s directly toward fielder *B* at an angle of 30° to the horizontal; the initial position of the ball is 0.9 m above ground level. Fielder *B* requires 0.25 s to judge where the ball should be caught and begins moving to that position with constant speed. Because of great experience, fielder *B* chooses his running speed so that he arrives at the "catch position" simultaneously with the ball. The catch position is the field location with the ball altitude 2.1 m. Determine the velocity of the ball relative to the fielder at the instant the catch is made.