

# Impulse and Momentum

## Linear Momentum

From equation of motion

$$\sum \vec{F} = m\vec{a} \quad \rightarrow \quad \sum \vec{F} = m\dot{\vec{v}} = \frac{d}{dt}(m\vec{v})$$

Linear momentum

$$\vec{G} = m\vec{v}$$

$$\sum \vec{F} = \dot{\vec{G}}$$

The resultant of all forces acting on a particle equals its time rate of change of linear momentum.

Scalar components

$$\sum F_x = \dot{G}_x \quad \sum F_y = \dot{G}_y \quad \sum F_z = \dot{G}_z$$

# The Linear Impulse-Momentum Principle

## Linear Impulse and Linear Momentum

From equation of motion

$$\sum \vec{F} = \dot{\vec{G}}$$

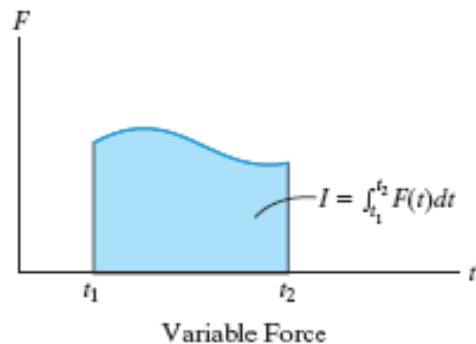


$$\int_{t_1}^{t_2} \sum \vec{F} dt = G_2 - G_1$$

Linear Impulse



$$\int_{t_1}^{t_2} \sum \vec{F} dt = m\vec{v}_2 - m\vec{v}_1$$



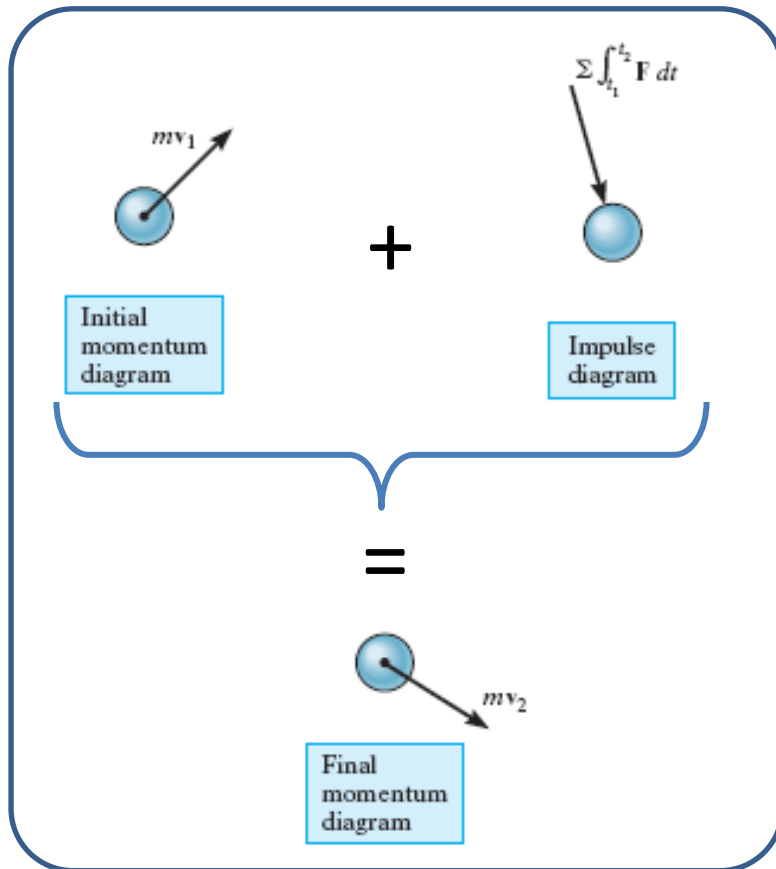
The total linear impulse on  $m$  equals the corresponding change in linear momentum of  $m$ .

# The Linear Impulse-Momentum Principle

$$\int_{t_1}^{t_2} \sum \vec{F} dt = m\vec{v}_2 - m\vec{v}_1$$



$$m\vec{v}_1 + \int_{t_1}^{t_2} \sum \vec{F} dt = m\vec{v}_2$$



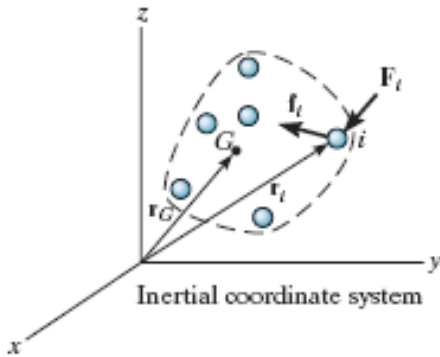
Scalar equations

$$(mv_x)_1 + \int_{t_1}^{t_2} \sum F_x dt = (mv_x)_2$$

$$(mv_y)_1 + \int_{t_1}^{t_2} \sum F_y dt = (mv_y)_2$$

$$(mv_z)_1 + \int_{t_1}^{t_2} \sum F_z dt = (mv_z)_2$$

# Principle for a System of Particles



For particle  $i$

$$\vec{R}_i = \vec{G}_i = m_i \frac{d\vec{v}_i}{dt}$$

$\vec{R}_i$  is the resultant force acting on particle  $i$

$$\vec{R}_i = \vec{F}_i + \vec{f}_i = \text{External force} + \text{Internal force}$$

For all particles in the system

$$\sum \vec{R}_i = \sum \vec{F}_i + \boxed{\sum \vec{f}_i} = \sum m_i \frac{d\vec{v}_i}{dt}$$

**Cancel out,**  
actions and reaction forces  
occur in equal but opposite  
collinear pairs

$\downarrow = 0$



$$\sum \vec{F}_i = \sum m_i \frac{d\vec{v}_i}{dt}$$



**Sum of the external forces  
acting on the system of particles**



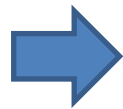
$$\sum m_i (\vec{v}_i)_1 + \sum \int_{t_1}^{t_2} \vec{F}_i dt = \sum m_i (\vec{v}_i)_2$$

# Conservation of Linear Momentum

When the sum of the external impulses acting on a system of particles is zero

$$\sum m_i (\vec{v}_i)_1 + \sum \int_{t_1}^{t_2} \vec{F}_i dt = \sum m_i (\vec{v}_i)_2$$

A red arrow points from the integral term to a red "= 0" above it.



$$\sum m_i (\vec{v}_i)_1 = \sum m_i (\vec{v}_i)_2$$

- The total linear momentum for a system of particles remains constant during the time period
- Linear momentum may be conserved in one direction, but not necessarily in the other directions.

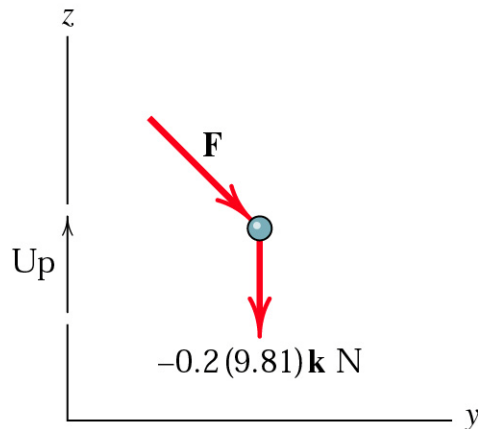
# Example 3/18

A 0.2-kg particle moves in the vertical  $y$ - $z$  plane under the action of its weight and a force  $\mathbf{F}$  which varies with time. The linear momentum of the particle in newton-seconds is given by the expression

$$\vec{G} = \frac{3}{2}(t^2 + 3)\hat{j} - \frac{2}{3}(t^3 - 4)\hat{k}$$

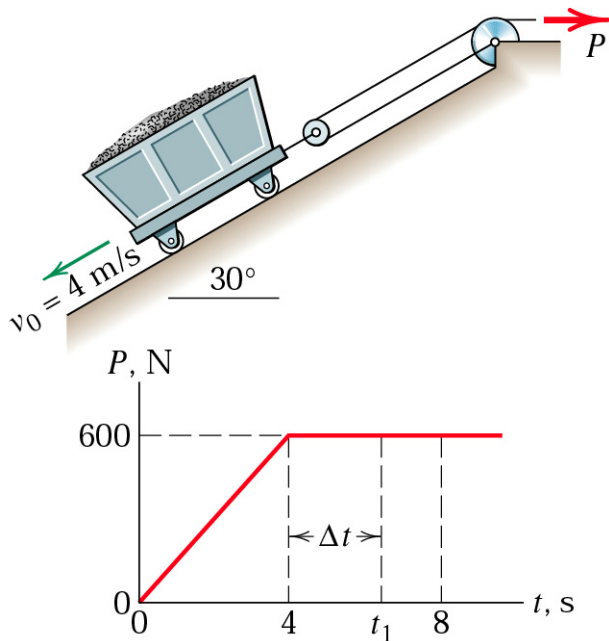
where  $t$  is the time in seconds.

Determine the force  $\mathbf{F}$  and its magnitude for the instant when  $t = 2$  sec.



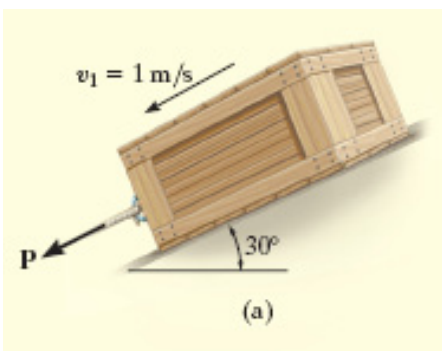
# Example 3/20

The loaded 150-kg skip is rolling down the incline at 4 m/s when a force  $P$  is applied to the cable as shown at time  $t = 0$ . The force  $P$  is increased uniformly with the time until it reaches 600 N at  $t = 4$  sec, after which time it remains constant at this value. Calculate (a) the time  $t_1$  at which the skip reverses its direction and (b) the velocity  $v$  of the skip at  $t = 8$  sec. Treat the skip as a particle.



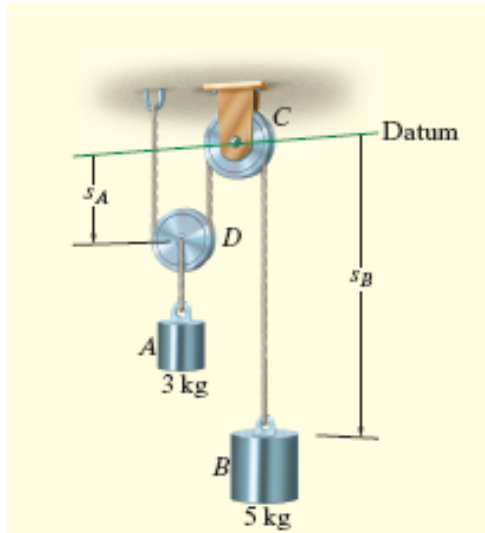
# Example 15.2

The 250-N ( $\approx 25$ -kg) crate shown in the figure is acted upon by force having a variable magnitude  $P = 100t$  N, where  $t$  is in seconds. Determine the crate's velocity 2 s after  $\mathbf{P}$  has been applied. The initial velocity is  $v_1 = 1$  m/s down the plane, and the coefficient of kinetic friction between the crate and the plane is  $\mu_k = 0.3$ .



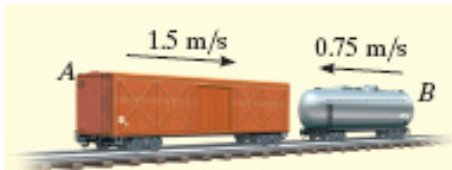
# Example 15.3

Blocks  $A$  and  $B$  shown in the figure have a mass of 3 kg and 5 kg, respectively. If the system is released from rest, determine the velocity of block  $B$  in 6 s. Neglect the mass of the pulleys and cord.



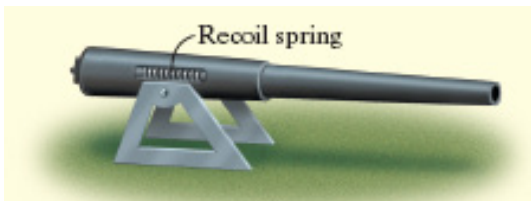
# Example 15.4

The 15-Mg boxcar  $A$  is coasting at 1.5 m/s on the horizontal track when it encounters a 12-Mg tank car  $B$  coasting at 0.75 m/s toward it as shown in the figure. If the cars meet and couple together, determine (a) the speed of both cars just after the coupling, and (b) the average force between them if the coupling takes place in 0.8 s.



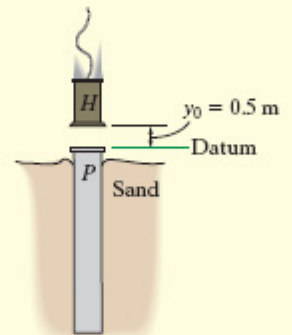
# Example 15.5

The 600-kg cannon shown in the figure fires an 4-kg projectile with a muzzle velocity of 450 m/s relative to the ground. If firing takes place in 0.03 s, determine (a) the recoil velocity of the cannon just after firing, and (b) the average impulsive force acting on the projectile. The cannon support is fixed to the ground, and the horizontal recoil of the cannon is absorbed by two springs.



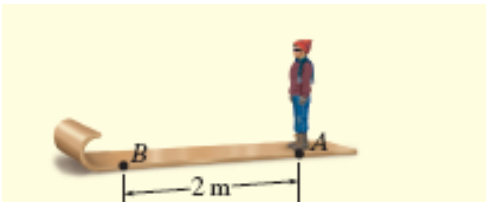
# Example 15.7

An 800-kg rigid pile  $P$  shown in the figure is driven into the ground using a 300-kg hammer  $H$ . The hammer falls from rest at a height  $y_0 = 0.5$  m and strikes the top of the pile. Determine the impulse which the hammer imparts on the pile if the pile is surrounded entirely by loose sand so that after striking, the hammer does not rebound off the pile.



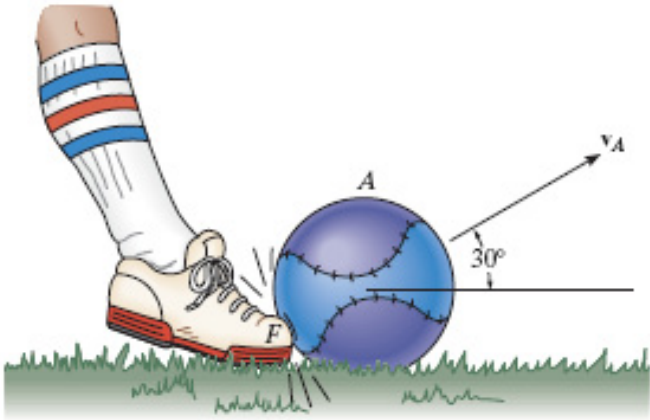
# Example 15.8

A boy having a mass of 40 kg stands on the back of a 15-kg toboggan which is originally at rest. If he walks to the front B and stops, determine the distance the toboggan moves. Neglect friction between the bottom of the toboggan and the ground (ice).



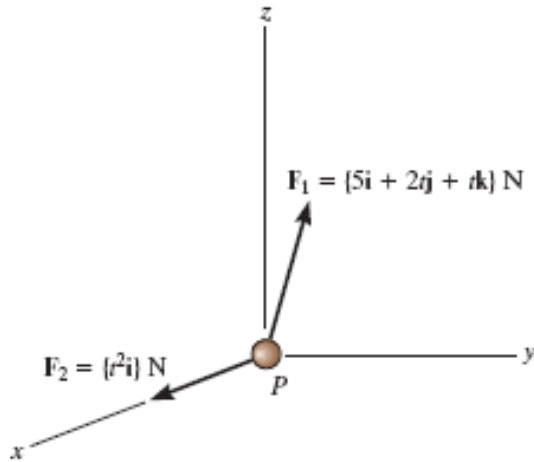
# Problem 15-10

A man kicks the 200-g ball such that it leaves the ground at an angle of  $30^\circ$  with the horizontal and strikes the ground at the same elevation a distance of 15 m away. Determine the impulse of his foot  $F$  on the ball. Neglect the impulse caused by the ball's weight while its being kicked.



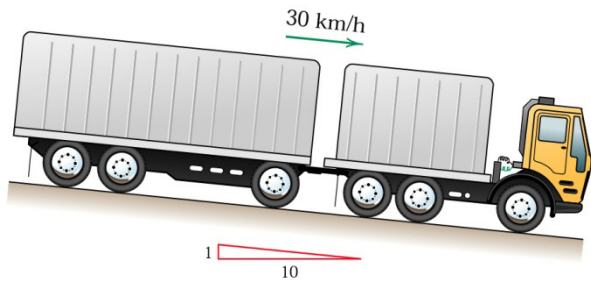
# Problem 15-11

The particle P is acted upon by its weight of 30 N ( $\approx 3$  kg) and forces  $F_1$  and  $F_2$ , where  $t$  is in seconds. If the particle originally has a velocity of  $\vec{v}_1 = 3\hat{i} + 1\hat{j} + 6\hat{k}$  m/s, determine its speed after 2 s.



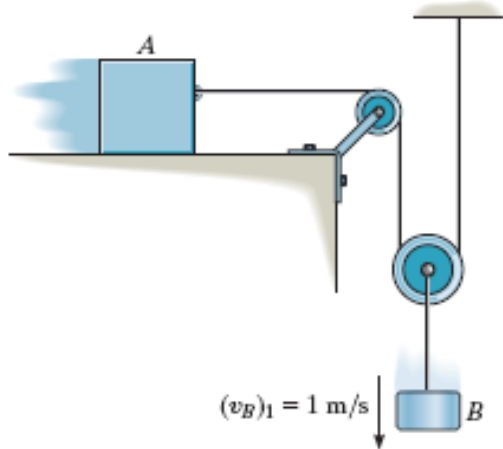
# Problem 3/191

The hydraulic braking system for the truck and trailer is set to produce equal braking forces for the two units. If the brakes are applied uniformly for 5 s to bring the rig to a stop from a speed of 30 km/h down the 10-percent grade, determine the force  $P$  in the coupling between the trailer and the truck. The mass of the truck is 10 Mg and that of the trailer is 7.5 Mg.



# Problem 15-27

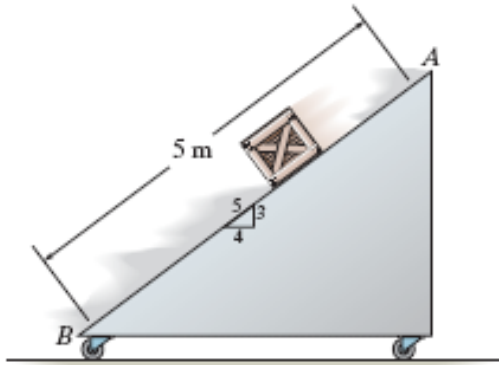
Block  $A$  weight  $100\text{ N}$  ( $\approx 10\text{ kg}$ ) and block  $B$  weight  $30\text{ N}$  ( $\approx 3\text{ kg}$ ). If  $B$  is moving downward with a velocity  $(v_B)_1 = 1\text{ m/s}$  at  $t = 0$ , determine the velocity of  $A$  when  $t = 1\text{ s}$ . Assume that the horizontal plane is smooth. Neglect the mass of the pulleys and cords.





# Problem 15-51

The free-rolling ramp has a weight of 600 N ( $\approx 60$  kg). If the 400 N ( $\approx 40$  kg) crate is released from rest at  $A$ , determine the distance the ramp moves when the crate slides 5 m down the ramp and reaches the bottom  $B$ .



# Problem 15-53

The free-rolling ramp has a mass of 40 kg. A 10-kg crate is released from rest at  $A$  and slides down 3.5 m to point  $B$ . If the surface of the ramp is smooth, determine the ramp's speed when the crate reaches  $B$ . Also, what is the velocity of the crate?

