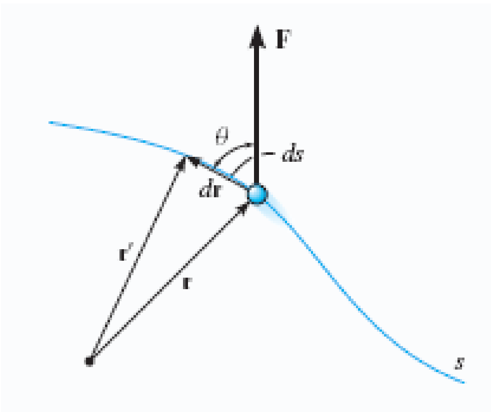
Work and Energy

The work of a Force

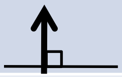
A Force ***F*** will do work on a particle only when the particle undergoes a displacement in the direction of the force.



$$dU = F\,ds\cos\theta$$

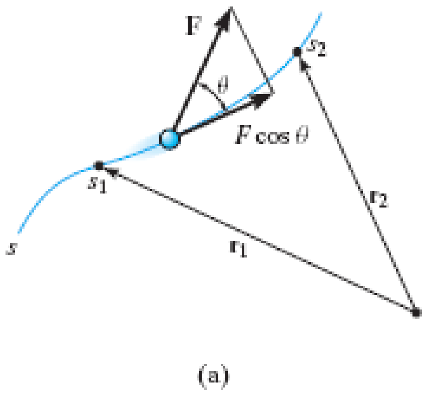
$$dU = \vec{F}\cdot d\vec{r}$$

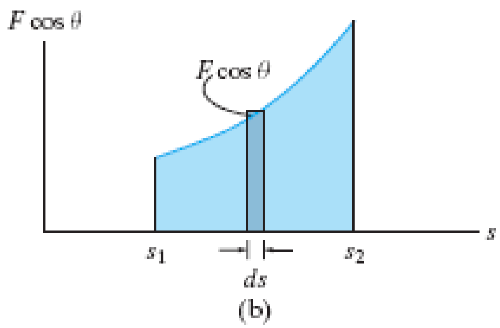
Unit of work: N·m or Joule (J)

Force	Displacement	Work
		Positive
		Negative
		0
	Fixed point (zero disp.)	0

The work of a Force

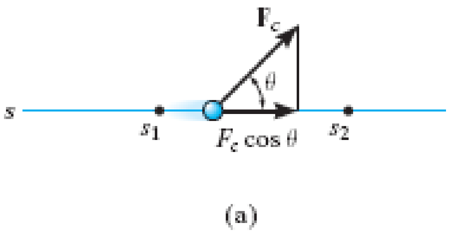
Work of a Variable Force

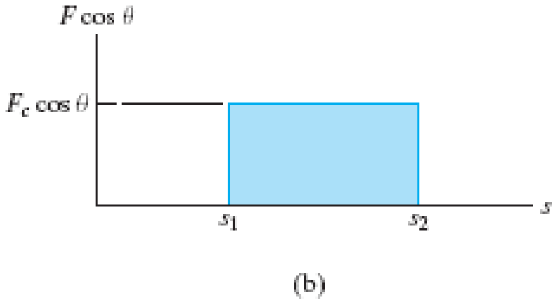




$$U_{1-2} = \int_{r_1}^{r_2} \vec{F} \cdot d\vec{r} = \int_{s_1}^{s_2} F \cos\theta \, ds$$

Work of a Constant Force Moving Along a Straight Line



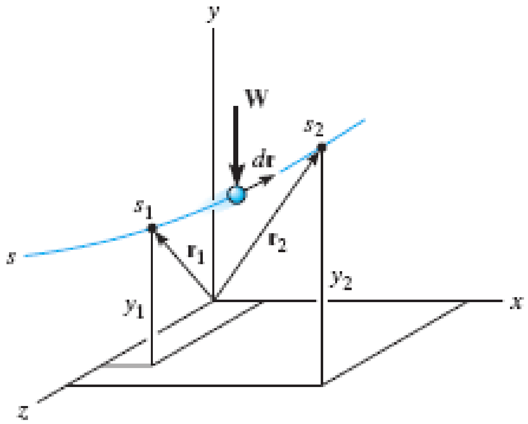


$$U_{1-2} = F_c \cos\theta \int_{s_1}^{s_2} ds$$

$$= F_c \cos\theta (s_2 - s_1)$$

The work of a Force

Work of a Weight



$$U_{1-2} = \int_{r_1}^{r_2} \vec{F} \cdot d\vec{r} = \int_{r_1}^{r_2} (-mg\ \hat{j}) \cdot (dx\hat{i} + dy\hat{j} + dz\hat{k})$$

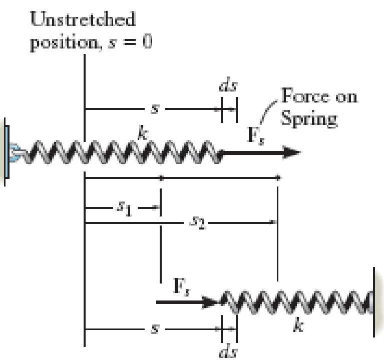
$$= \int_{r_1}^{r_2} -mg\, dy = -mg(y_2 - y_1)$$

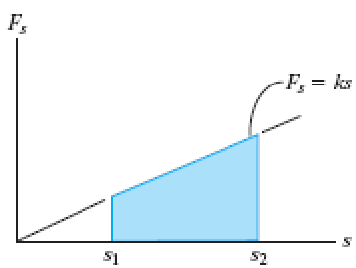
$$U_{1-2} = -mg\Delta y$$

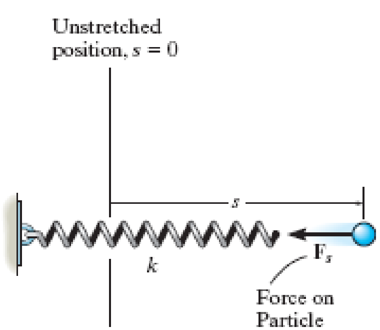
- The work is independent of the path
- Depend on its vertical displacement

The work of a Force

Work of a Spring Force







The work done on the spring

$$U_{1-2} = \int_{s_1}^{s_2} F_s \, ds = \int_{s_1}^{s_2} ks \, ds$$

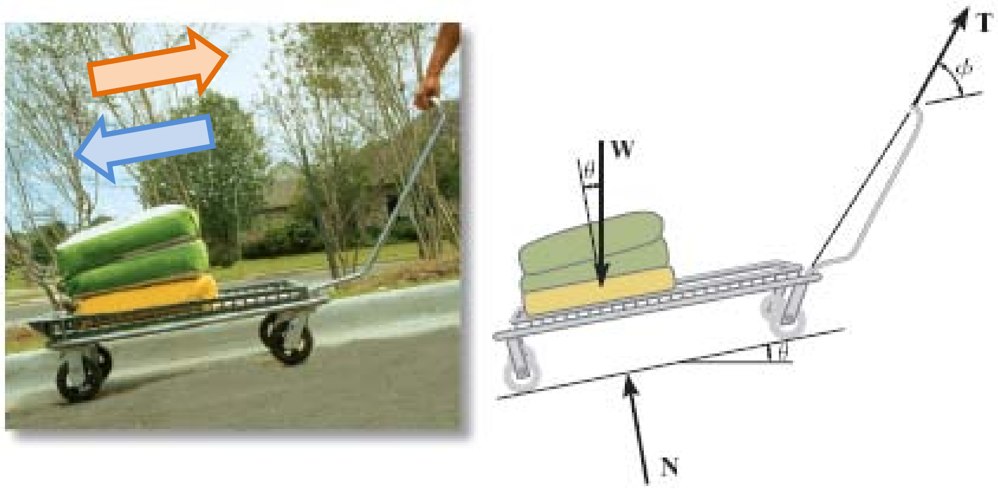
$$= \frac{1}{2} k s_2^2 - \frac{1}{2} k s_1^2 \qquad \text{positive}$$

The work done on a particle attached to a spring

Force $\boldsymbol{F_s}$ exerted on the particle is <u>opposite</u> to that exerted on the spring

$$U_{1-2} = -\left(\frac{1}{2} k s_2^2 - \frac{1}{2} k s_1^2 \right) \qquad \text{negative}$$

The work of a Force



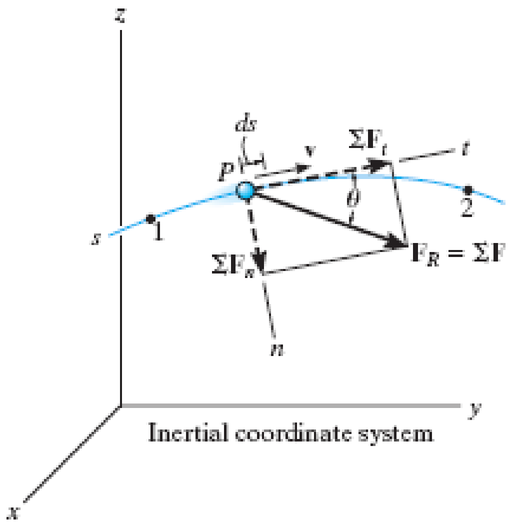
Up Hill

Force	Work (Positive/ Negative)
Towing force T	
Weight W	
Normal force N	
Friction	

Downhill

Force	Work (Positive/ Negative)
Towing force T	
Weight W	
Normal force N	
Friction	

Principle of Work and Energy



Resultant $\vec{F}_R = \sum \vec{F} = \vec{F}_n + \vec{F}_t$

Work done by resultant (all forces)

$= 0, \quad (F_n \perp s)$

$$U_{1-2} = \int_{r_1}^{r_2} \vec{F}_R \cdot d\vec{r} = \int_{r_1}^{r_2} \vec{F}_n \cdot d\vec{r} + \int_{r_1}^{r_2} \vec{F}_t \cdot d\vec{r}$$

$$U_{1-2} = \int_{r_1}^{r_2} \vec{F}_t \cdot d\vec{r} = \int_{s_1}^{s_2} F_t \, ds = \int_{s_1}^{s_2} ma_t \, ds$$

From $vdv = a_t ds$

$$U_{1-2} = \int_{s_1}^{s_2} mvdv = \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2$$

Principle of Work and Energy

Kinetic energy

From $$U_{1-2} = \int_{s_1}^{s_2} mv\,dv = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

If $v_1 = 0$, $v_2 = v$ $$U_{1-2} = \frac{1}{2}mv^2$$ ⇒ **Kinetic Energy (T)**

Kinetic energy: The total work which must be done on the particle to bring it from a state of rest to a velocity *v*.

Principle of Work and Energy

Work done by resultant (all forces)

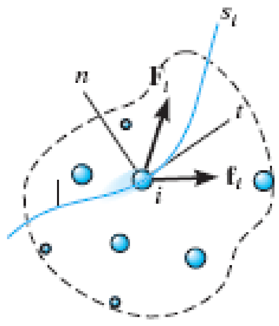
From $$U_{1-2} = \int_{s_1}^{s_2} mv\,dv = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

$$U_{1-2} = T_2 - T_1 = \Delta T$$

or $$T_1 + U_{1-2} = T_2$$

The particle's initial kinetic energy plus the work done by all the forces is equal to the particle's final kinetic energy

Work and Energy for a System of Particles



Inertial coordinate system

Resultant of particle = Ext. force + Int. force

$$\vec{F}_{Ri} = \vec{F}_i + \vec{f}_i$$

From $T_1 + U_{1-2} = T_2$

$$\frac{1}{2} m_i v_{i1}^2 + \int_{s_{i1}}^{s_{i2}} (F_i)_t \, ds + \int_{s_{i1}}^{s_{i2}} (f_i)_t \, ds = \frac{1}{2} m_i v_{i2}^2$$

Work and energy equation for the system

$$\sum \frac{1}{2} m_i v_{i1}^2 + \sum \int_{s_{i1}}^{s_{i2}} (F_i)_t \, ds + \sum \int_{s_{i1}}^{s_{i2}} (f_i)_t \, ds = \sum \frac{1}{2} m_i v_{i2}^2$$

Note Int. forces on adjacent particles are equal and opposite, but the work done will not cancel out since the paths will be different.

Work and Energy for a System of Particles

For the connection among the particles which is frictionless and incapable of any deformation

Ex.

- Translating rigid body
- Particles connected by inextensible cables

⇨ **The work of internal forces cancels.**

Work and energy equation for the system

$$\sum \frac{1}{2} m_i v_{i1}^2 + \sum \int_{s_{i1}}^{s_{i2}} (F_i)_t \, ds = \sum \frac{1}{2} m_i v_{i2}^2$$

Power and Efficiency

Power

The amount of work performed per unit of time.

$$P = \frac{dU}{dt}$$

From $$P = \frac{dU}{dt} = \frac{\vec{F} \cdot d\vec{r}}{dt} = \vec{F} \cdot \frac{d\vec{r}}{dt} \Rightarrow P = \vec{F} \cdot \vec{v}$$

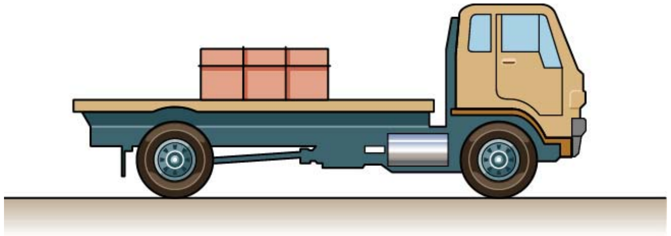
Efficiency

$$\varepsilon = \frac{\text{Power output}}{\text{Power input}} = \frac{\text{Energy output}}{\text{Energy input}}$$

The efficiency is always less than 1.

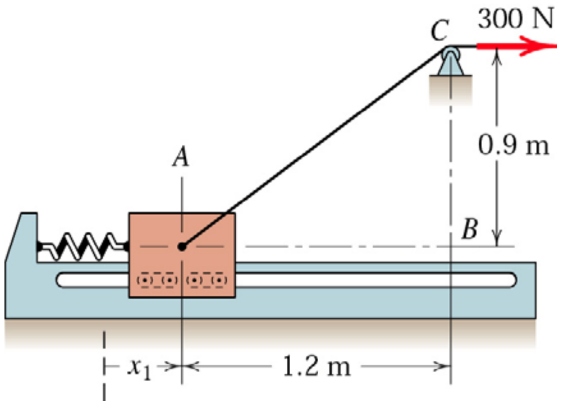
Sample problem 3/12

The flatbed truck, which carries an 80-kg crate, starts from rest and attains a speed of 72 km/h in a distance of 75 m on a level road with constant acceleration. Calculate the work done by the friction force acting on the crate during this interval if the static and kinetic coefficients of friction between the crate and the truck bed are (a) 0.30 and 0.28, respectively, or (b) 0.25 and 0.2 respectively.



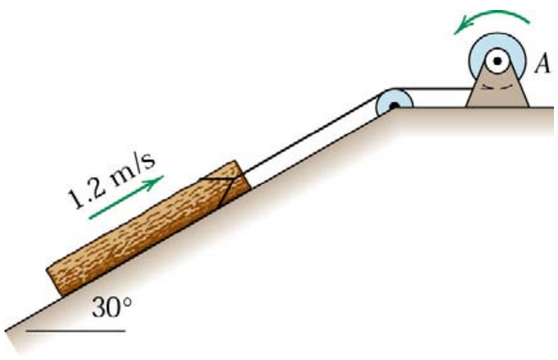
Sample problem 3/13

The 50-kg block at *A* is mounted on rollers so that it moves along the fixed horizontal rail with negligible friction under the action of the constant 300-N force in the cable. The block is released from rest at *A*, with the spring to which it is attached extended an initial amount $x_1 = 0.233$ m. The spring has a stiffness $k = 80$ N/m. Calculate the velocity v of the block as it reaches position *B*.



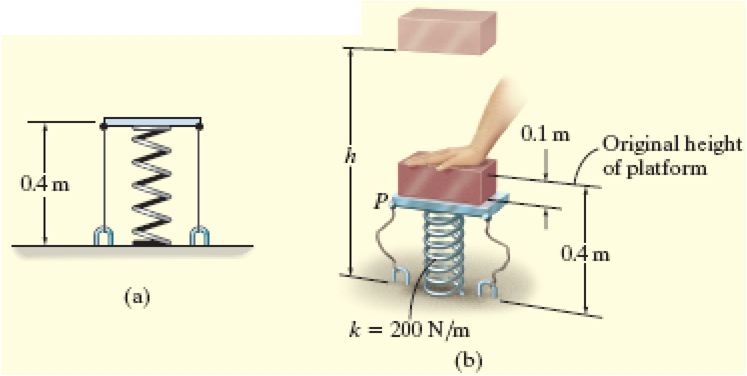
Sample problem 3/14

The power winch *A* hoists the 360-kg log up the 30° incline at a constant speed of 1.2 m/s. If the power output of the winch is 4 kW, compute the coefficient of kinetic friction μ_k between the log and the incline. If the power is suddenly increased to 6 kW, what is the corresponding instantaneous acceleration a of the log.



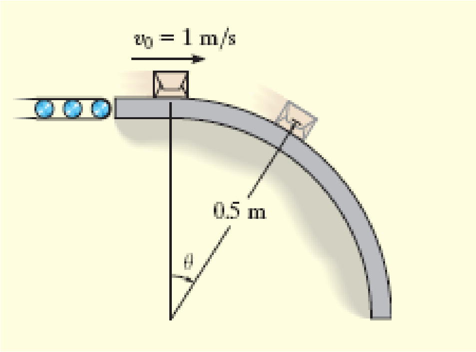
Sample problem 14.4

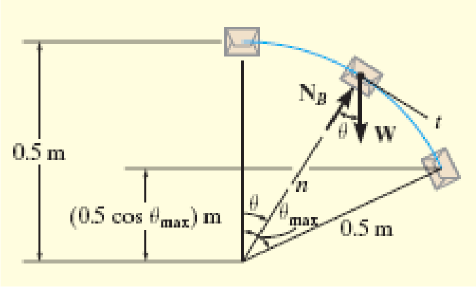
The platform *P* has negligible mass and is tied down so that the 0.4-m-long cords keep a 1-m-long spring compressed 0.6 m when nothing is on the platform. If a 2-kg block is placed on the platform and released from rest after the platform is pushed down 0.1 m, determine the maximum height *h* the block rises in the air, measured from the ground.



Sample problem 14.5

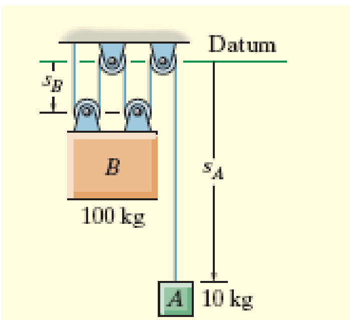
Packages having a mass of 2 kg are delivered from a conveyor to a smooth circular ramp with a velocity of v_0 = 1 m/s. If the radius of the ramp is 0.5 m, determine the angle $\theta = \theta_{max}$ at which each package begins to leave the surface.

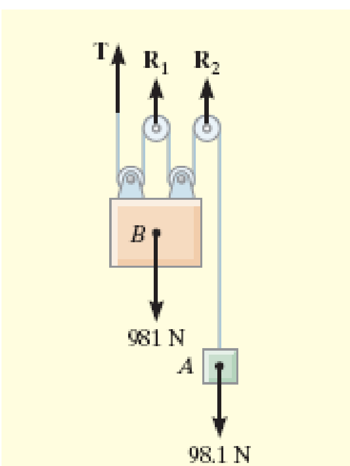




Sample problem 14.6

The blocks *A* and *B* have a mass of 10 kg and 100 kg, respectively. Determine the distance *B* travels from the point where it is released from rest to the point where its speed becomes 2 m/s.





Conservative Forces and Potential Energy

Conservative forces

The work done by a conservative forces is independent of the path.

- Weight and spring are conservative forces (depend on positions)
- Frictional forces are nonconservative forces (The longer path, the greater the work.)

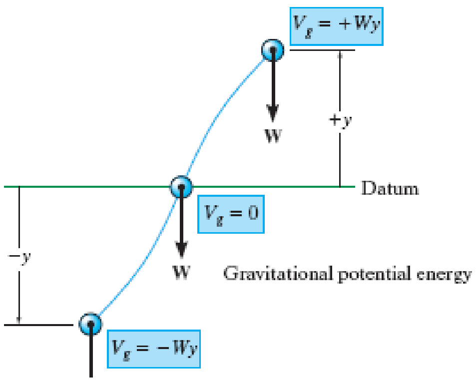
Potential energy

Energy: the capacity for doing work.

Potential energy : measure of the amount of work of a conservative force will do when it moves from a given position to the datum.

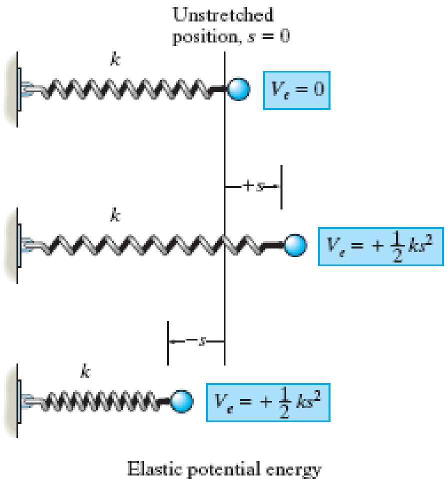
Potential Energy

Gravitational P.E.



$$V_g = Wy = mgy$$

Elastic P.E.



$$V_e = \frac{1}{2}ks^2$$ (always positive)

PE is the work of a force will do when it moves from a given position to the datum.

P.E. = – (the work of a weight)

= – (the work of a spring force exerted on the particle)

Conservation of Energy

Work-Energy Equation

From $$U_{1-2} = T_2 - T_1 = \Delta T$$

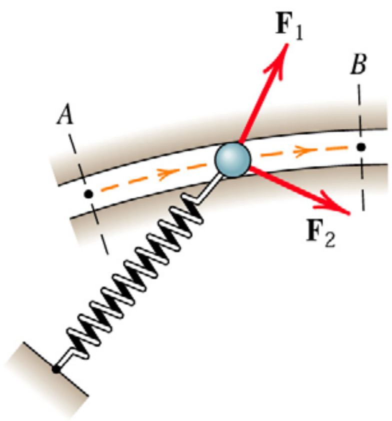
U'_{1-2} is the work of all external forces other than gravitational forces and spring forces

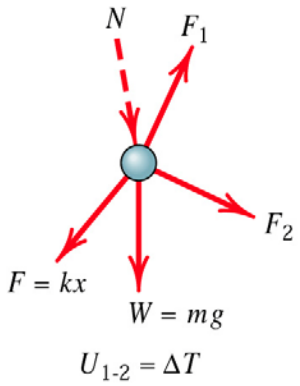
$$U'_{1-2} + (-\Delta V_g) + (-\Delta V_e) = \Delta T$$

$$U'_{1-2} = \Delta T + \Delta V_g + \Delta V_e$$

or $$T_1 + V_{g1} + V_{e1} + U'_{1-2} = T_2 + V_{g2} + V_{e2}$$

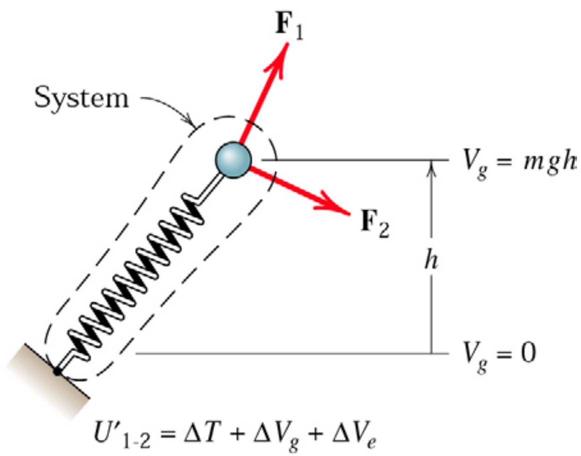
Conservation of Energy





$$U_{1-2} = T_2 - T_1 = \Delta T$$

- All forces must be considered
- $N \perp$ path $\Rightarrow$ work = 0



$$U'_{1-2} = \Delta T + \Delta V_g + \Delta V_e$$

- F_1 and F_2 are considered
- V_g and V_e are added in calculation

Conservation of Energy

From $$U'_{1-2} = \Delta T + \Delta V_g + \Delta V_e = \Delta E$$

where $$E = T + V_g + V_e$$ ⇨ The total mechanical energy of the particle

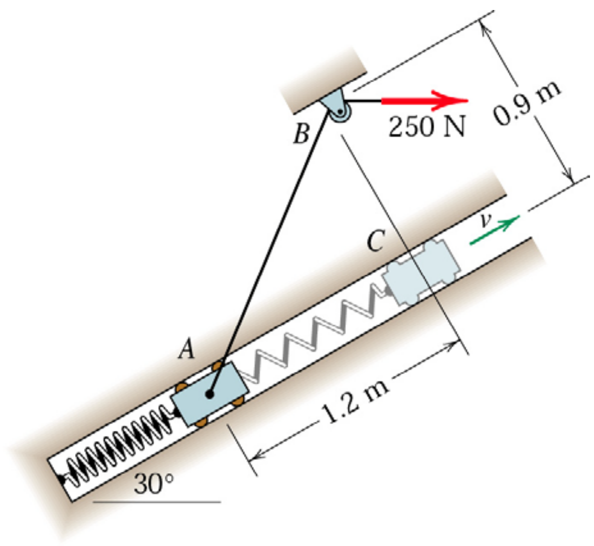
For problems where the only forces are gravitational, elastic, and nonworking constraint forces,

$$\Delta E = 0 \quad \text{or} \quad E = \text{constant}$$ ⇨ Conservation of Energy

- The sum of particle’s K.E. and P.E. remains const.
- K.E. must be transformed into P.E. and vice versa.

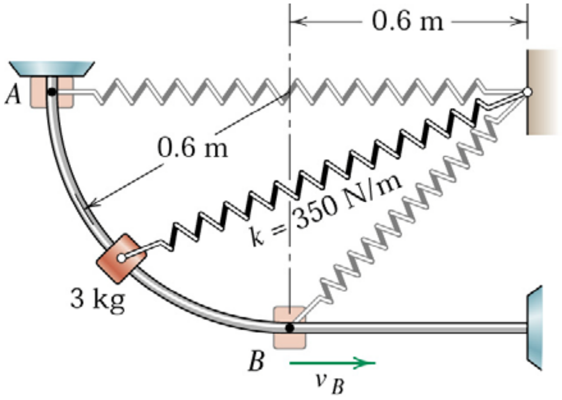
Sample problem 3/16

The 10-kg slider *A* moves with negligible friction up the inclined guide. The attached spring has a stiffness of 60 N/m and is stretched 0.6 m in position *A*, where the slider is released from rest. The 250-N force is constant and the pulley offers negligible resistance to the motion of the cord. Calculate the velocity v of the slider as it passes point *C*.



Sample problem 3/17

The 3-kg slider is released from rest at point *A* and slides with negligible friction in a vertical plane along the circular rod. The attached spring has a stiffness of 350 N/m and has an unstretched length of 0.6 m. Determine the velocity of the slider as it passes position *B*.



Sample problem 14.11

A smooth 2-kg collar *C* fits loosely on the vertical shaft. If the spring is unstretched when the collar is in the position *A*, determine the speed at which the collar is moving when $y = 1$ m, if (a) it is released from rest at *A*, and (b) it is released at *A* with an upward velocity $v_a = 2$ m/s.