Ch.3 Kinetics of particles

Kinetics: The study of the relations between unbalanced forced and the resulting changes in motion.

Kinetics problems

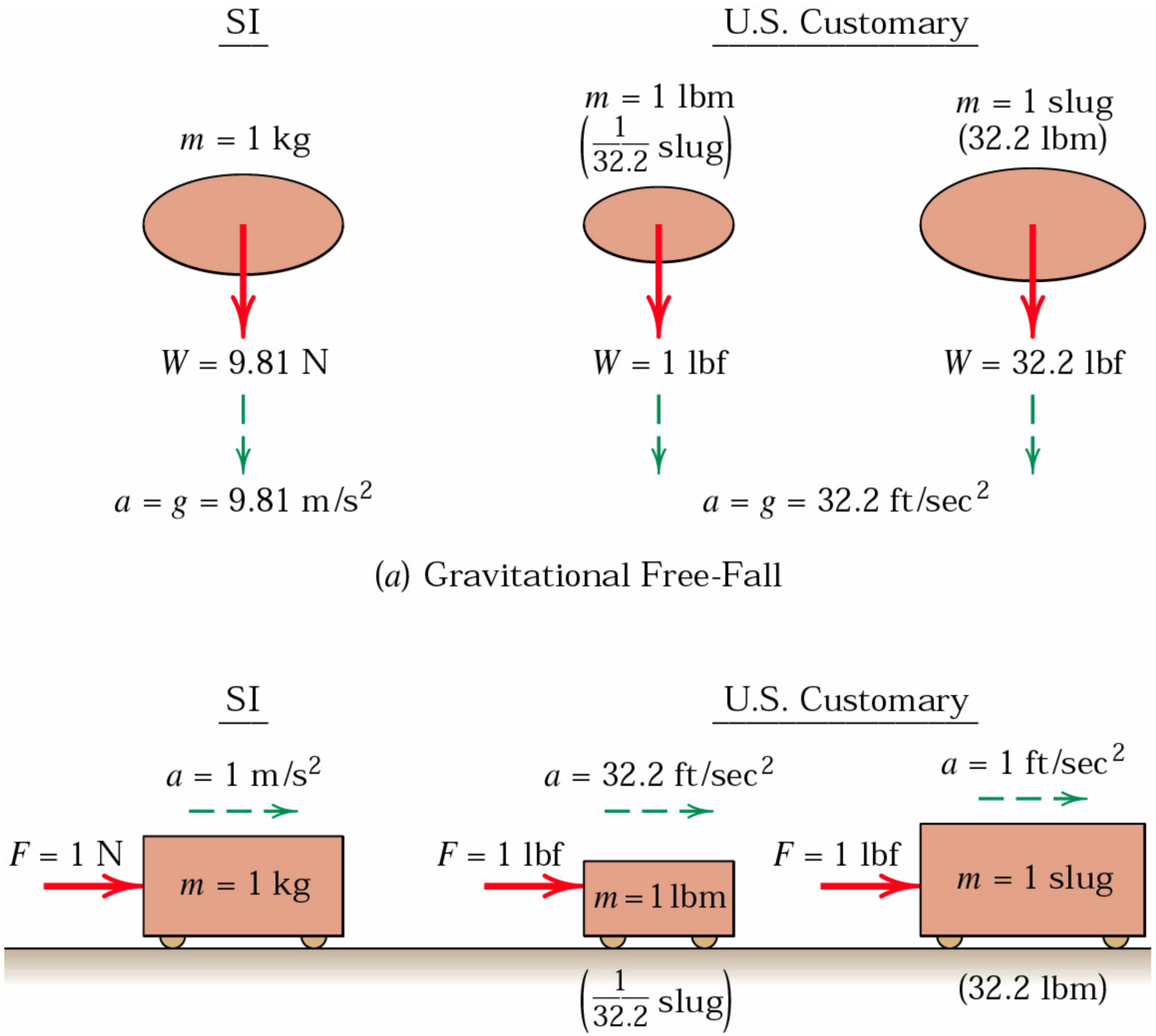
- Force-mass-acceleration method → Study in this class
- Work & energy principles
- Impulse & momentum methods

Newton's second law

A particle will accelerate when it is subjected to unbalanced forces

$$\sum \vec{F} = m\vec{a}$$

Force and mass units



(a) Gravitational Free-Fall

(b) Newton's Second Law

Dynamics problems

Types of dynamics problems

1. Acceleration is known from kinematics conditions
   Determine the corresponding forces
2. Forces acting on the particle are specified
   (Forces are constant or functions $F(t, s, v, \ldots)$)
   Determine the resulting motion

Constrained and unconstrained motion

Unconstrained motion: the particle is free of mechanical guides

Ex. Airplane, rocket

Constrained motion: the path of particle is partially or totally determined by restraining guides.

Ex. A train moving along track, a collar sliding along a shaft

Rectilinear motion

$$\sum \vec{F}_x = m\vec{a}_x \qquad \sum \vec{F}_y = m\vec{a}_y \qquad \sum \vec{F}_z = m\vec{a}_z$$

Acceleration

$$\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$$

$$a = \sqrt{a_x^2 + a_y^2 + a_z^2}$$

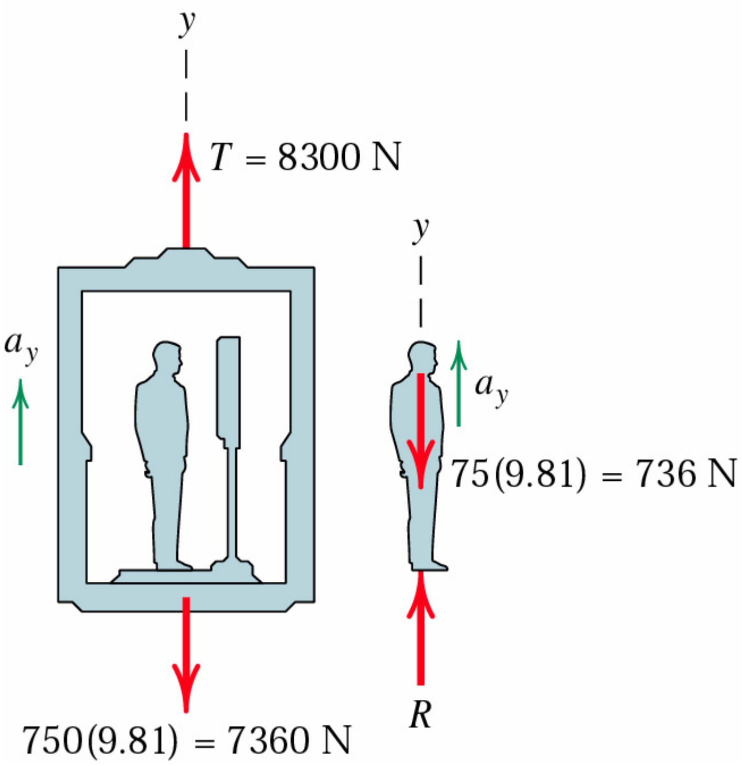
Force

$$\sum \vec{F} = \sum F_x\hat{i} + \sum F_y\hat{j} + \sum F_z\hat{k}$$

$$\left|\sum F\right| = \sqrt{\left(\sum F_x\right)^2 + \left(\sum F_y\right)^2 + \left(\sum F_z\right)^2}$$

There are forces involve, a free body diagram is required!

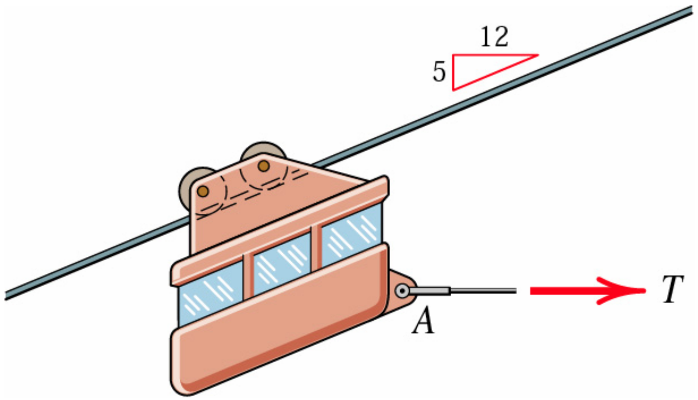
Sample problem 3/1



A 75-kg man stands on a spring scale in an elevator. During the first 3 seconds of motion from rest, the tension *T* in the hoisting cable is 8300 N. Find the reading *R* of the scale in newtons during this interval and the upward velocity *v* of the elevator at the end of the 3 seconds. The total mass of the elevator, man, and scale is 750 kg.

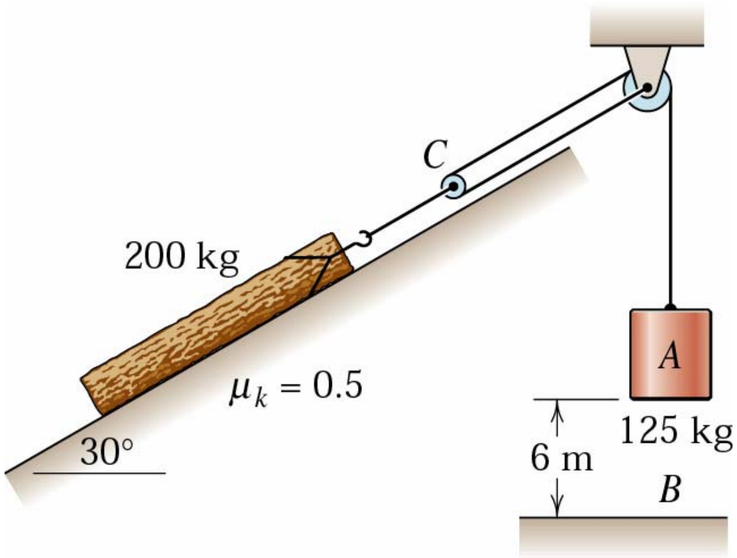
Sample problem 3/2

A small inspection car with a mass of 200 kg runs along the fixed overhead cable and is controlled by the attached cable at *A*. Determine the acceleration of the car when the control cable is horizontal and under a tension $T = 2.4$ kN. Also find the total force *P* exerted by the supporting cable on the wheels.



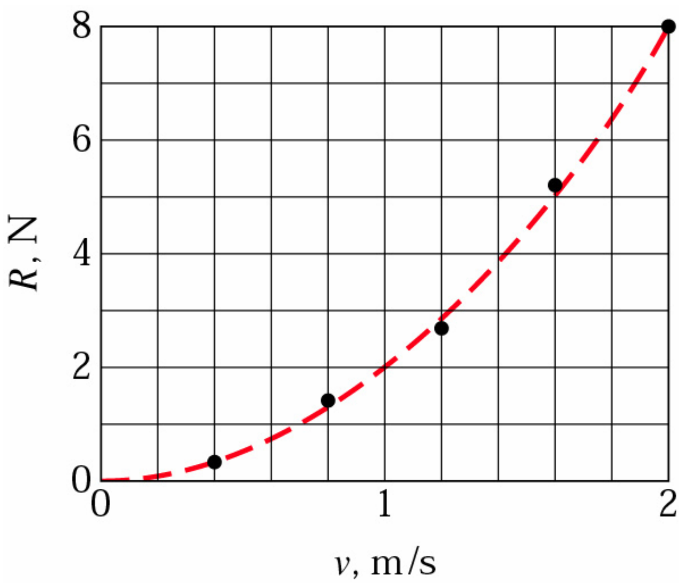
Sample problem 3/3

The 125-kg concrete block *A* is released from rest in the position shown and pulls the 200-kg log up the 30° ramp. If the coefficient of kinetic friction between the log and the ramp is 0.5, determine the velocity of the block as it hits the ground at *B*.



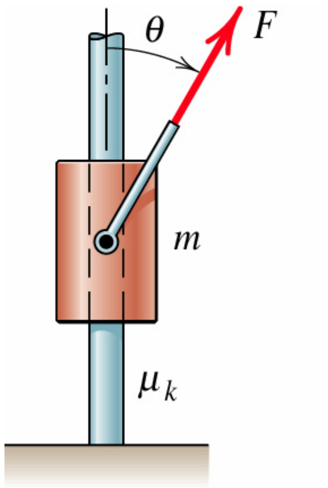
Sample problem 3/4

The design model for a new ship has a mass of 10 kg and is tested in an experimental towing tank to determine its resistance to motion through the water at various speeds. The test results are plotted on the accompanying graph, and the resistance *R* may be closely approximated by the dashed parabolic curve shown. If the model is released when it has a speed of 2 m/s, determine the time *t* required for it to reduce its speed to 1 m/s and the corresponding travel distance *x*.



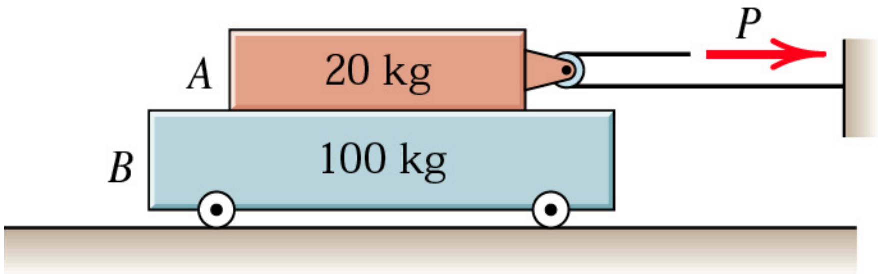
Sample problem 3/5

The collar of mass m slides up the vertical shaft under the action of a force F of constant magnitude but variable direction. If $\theta = kt$ where k is a constant and if the collar starts from rest with $\theta = 0$, determine the magnitude F of the force which will result in the collar coming to rest as θ reaches $\pi/2$. The coefficient of kinetic friction between the collar and shaft is μ_k.



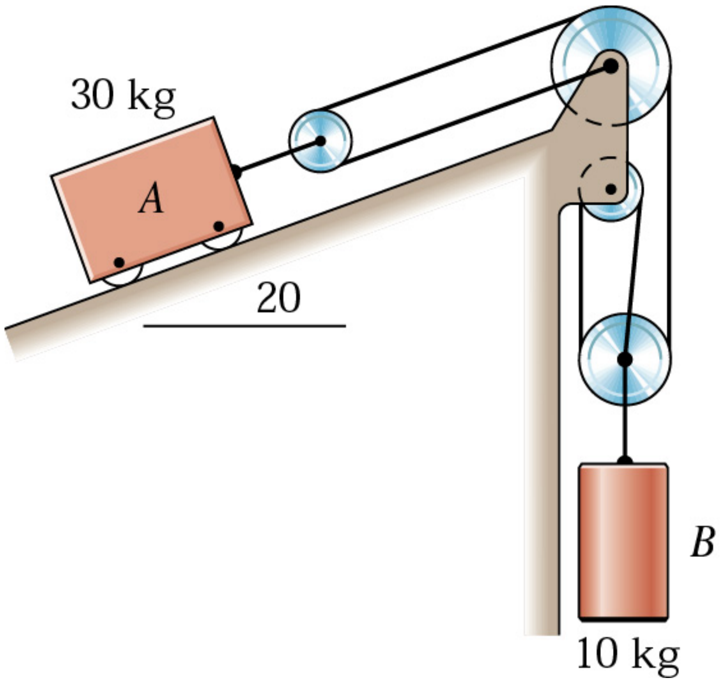
Sample 6 (3/23)

If the coefficients of static and kinetic friction between the 20-kg block *A* and the 100-kg cart *B* are both essentially the same value of 0.5, determine the acceleration of each part for (*a*) *P* = 60 N and (*b*) *P* = 40 N.



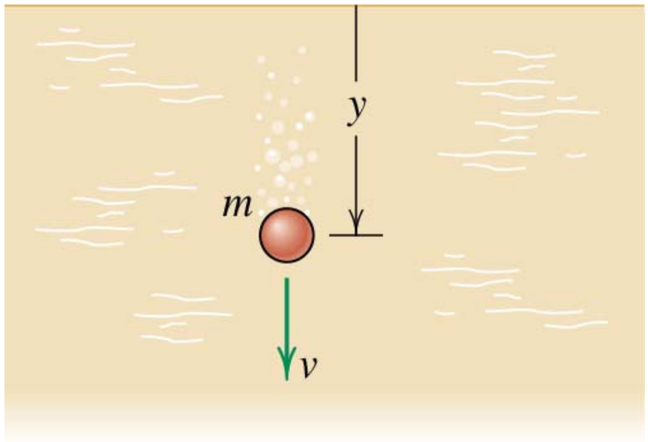
Sample 7 (3/27)

From the figure, neglect all friction and the mass of the pulleys , determine the accelerations of bodies *A* and *B* upon release from rest.



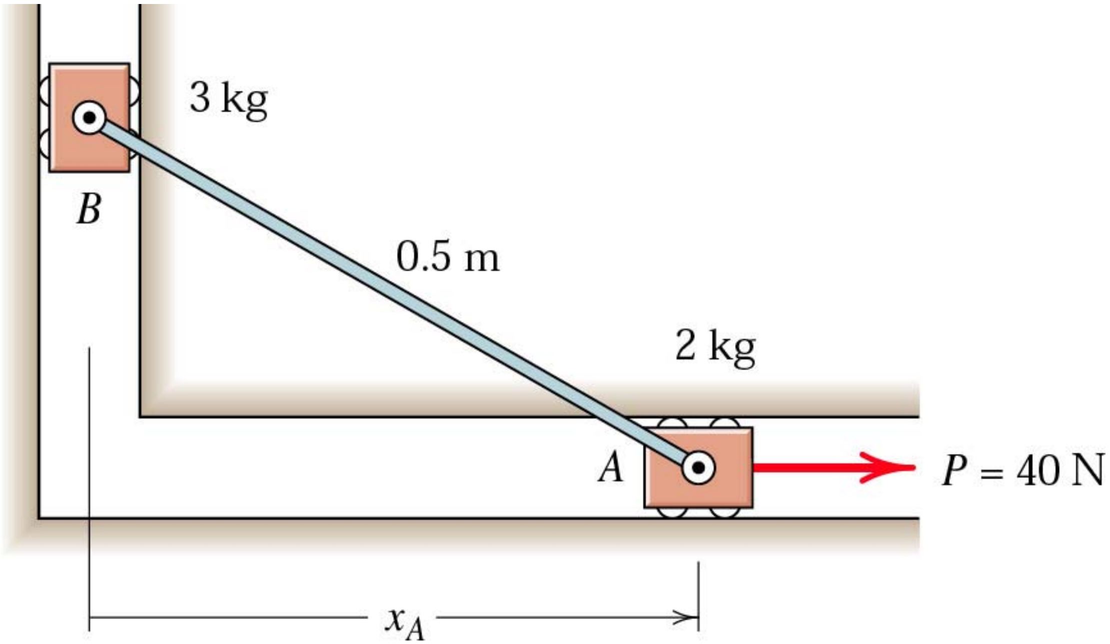
Sample 8 (3/39)

In a test of resistance to motion in an oil bath, a small steel ball of mass m is released from rest at the surface ($y = 0$). If the resistance to motion is given by $R = kv$ where k is a constant, derive an expression for depth h required for the ball to reach a velocity v.



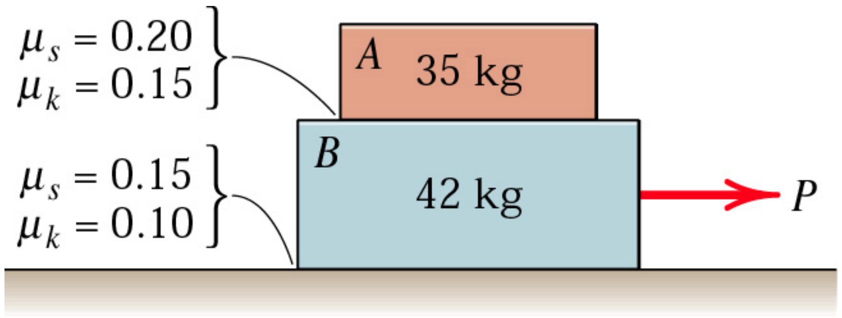
Sample 9 (3/43)

The sliders *A* and *B* are connected by a light rigid bar of length l = 0.5 m and move with negligible friction in the horizontal slots shown. For the position where x_A = 0.4 m, the velocity of *A* is v_A = 0.9 m/s to the right. Determine the acceleration of each slider and the force in the bar at this instant.



Sample 10 (3/46)

With the blocks initially at rest, the force P is increased slowly from zero to 260 N. Plot the accelerations of both masses as function of P.



Curvilinear motion

Rectangular coordinates

$$\sum \vec{F}_x = m\vec{a}_x \qquad \sum \vec{F}_y = m\vec{a}_y$$

where $a_x = \ddot{x}$ and $a_y = \ddot{y}$

n-t **coordinates**

$$\sum \vec{F}_n = m\vec{a}_n \qquad \sum \vec{F}_t = m\vec{a}_t$$

where $a_n = \rho\dot{\beta}^2 = v^2/\rho = v\dot{\rho}$, $a_t = \dot{v}$ and $v = \rho\dot{\beta}$

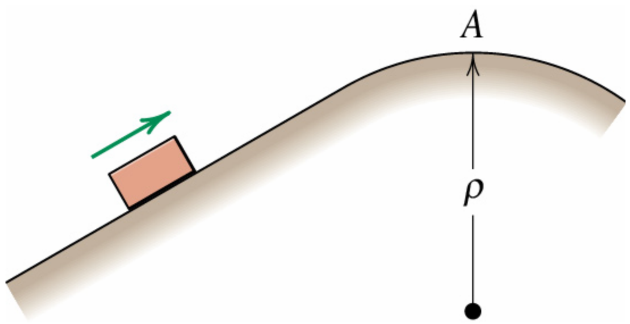
r-θ **coordinates (Polar)**

$$\sum \vec{F}_r = m\vec{a}_r \qquad \sum \vec{F}_\theta = m\vec{a}_\theta$$

where $a_r = \ddot{r} - r\dot{\theta}^2$ and $a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$

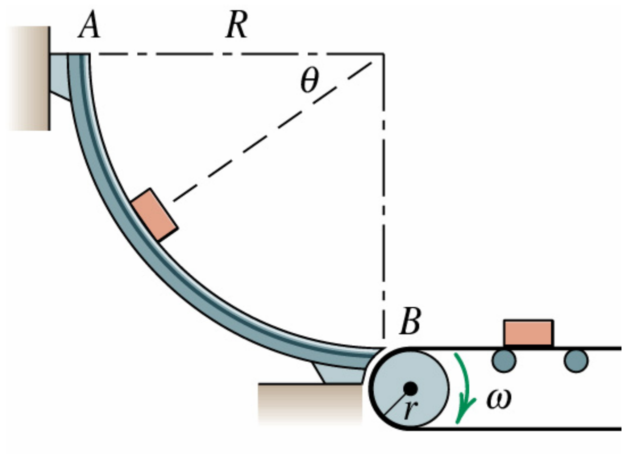
Sample problem 3/6

Determine the maximum speed *v* which the sliding block may have as it passes point *A* without losing contact with the surface.



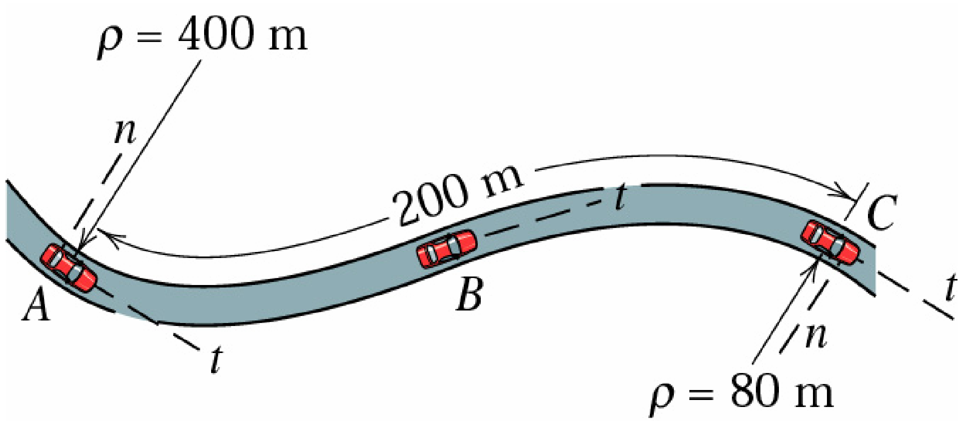
Sample problem 3/7

Small objects are released from rest at *A* and slide down the smooth circular surface of radius *R* to a conveyor *B*. Determine the expression for the normal contact force *N* between the guide and each object in terms of θ and specify the correct angular velocity ω of the conveyor pulley of radius *r* to prevent any sliding on the belt as the objects transfer to the conveyor.



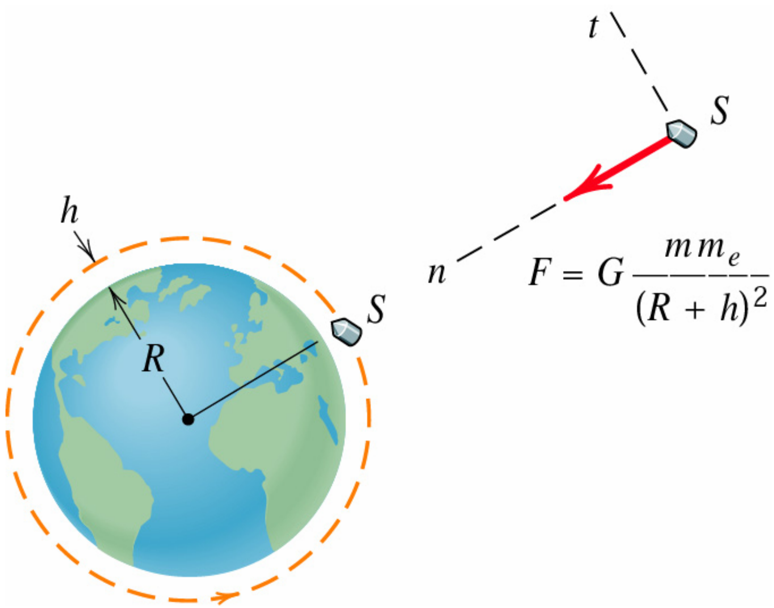
Sample problem 3/8

A 1500-kg car enters a section of curved road in the horizontal plane and slow down at a uniform rate from a speed of 100 km/h at *A* to a speed of 50 km/h as it passes *C*. The radius of curvature ρ of the road at *A* is 400 m and at *C* is 80 m. Determine the total horizontal force exerted by the road on the tires at position *A*, *B*, and *C*. Point *B* is the inflection point where the curvature changes direction.

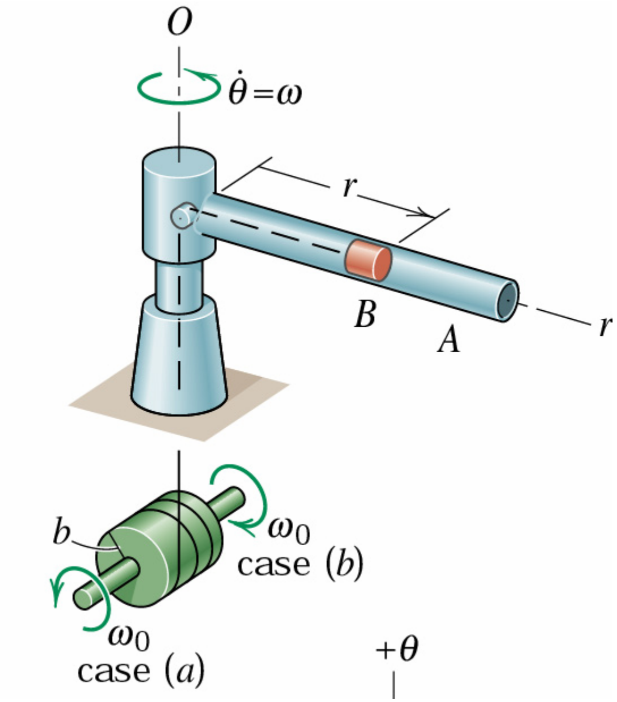


Sample problem 3/9

Compute the magnitude v of the velocity required for the spacecraft S to maintain a circular orbit of altitude 320 km above the surface of the earth.



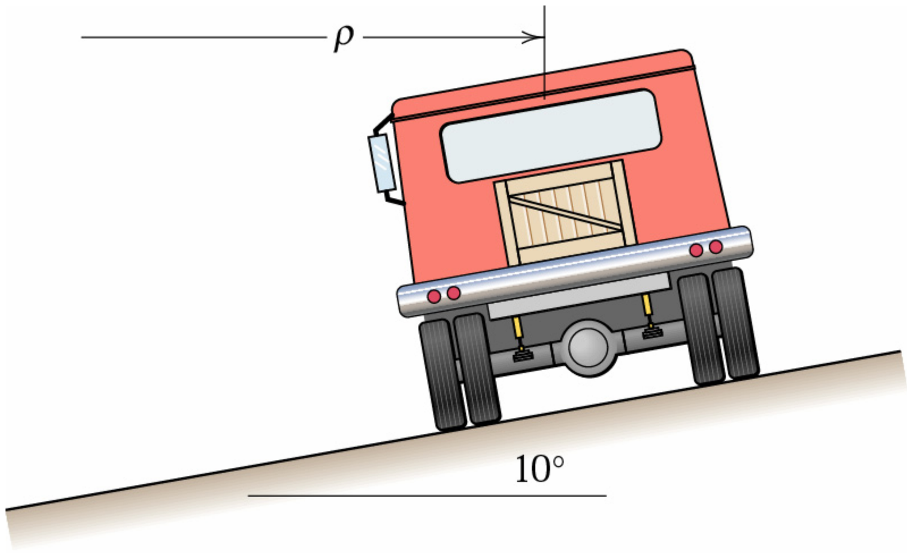
Sample problem 3/10



Tube *A* rotates about the vertical *O*-axis with a constant angular rate $\dot{\theta} = \omega$ and contains a small cylindrical plug *B* of mass *m* whose radial position is controlled by the cord which passes freely through the tube and shaft and is wound around the drum of radius *b*. Determine the tension *T* in the cord and the horizontal component F_θ of force exerted by the tube on the plug if the constant angular rate of rotation of the drum is ω_0 first in the direction for case (*a*) and second in the direction for case (*b*). Neglect friction.

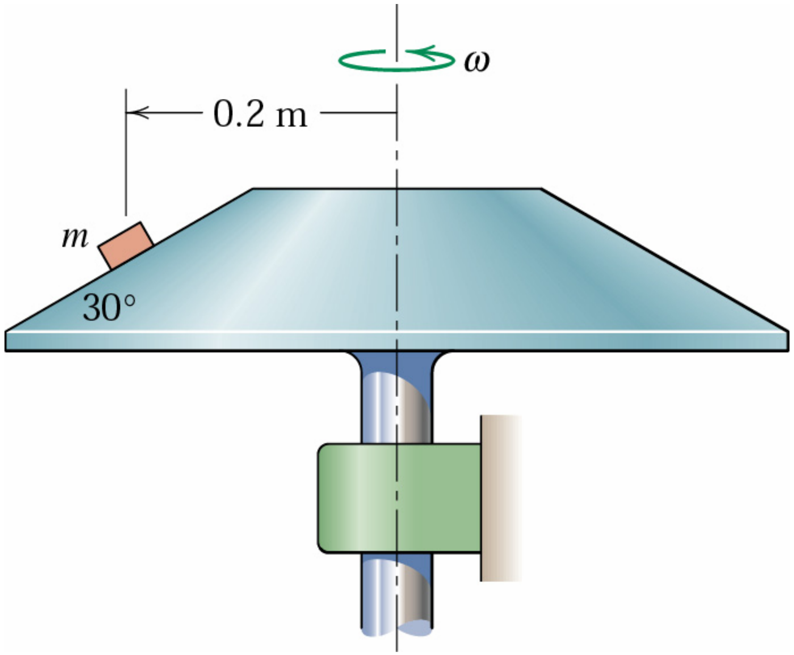
Sample 16 (3/69)

A flatbed truck going 100 km/h rounds a horizontal curve of 300-m radius inwardly banked at 10°. The coefficient of static friction between the truck bed and the 200-kg crate it carries is 0.7. Calculate the friction force *F* acting on the crate.



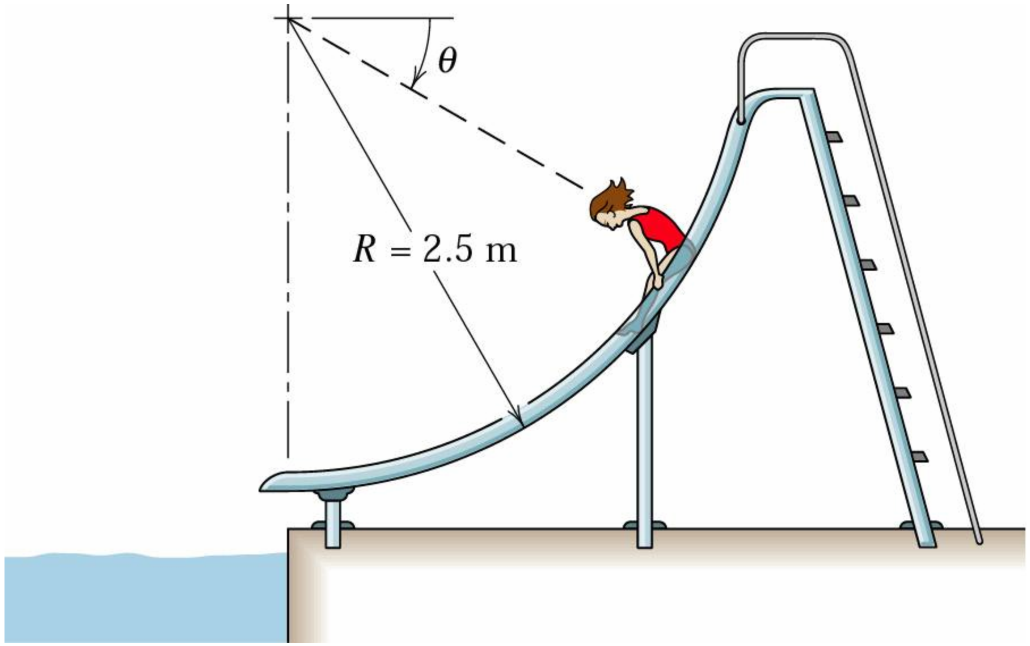
Sample 17 (3/73)

The small object of mass *m* is placed on the rotating conical surface at the radius shown. If the coefficient of static friction between the object and the rotating surface is 0.8, calculate the maximum angular velocity ω of the cone about the vertical axis for which the object will not slip. Assume very gradual angular velocity changes.

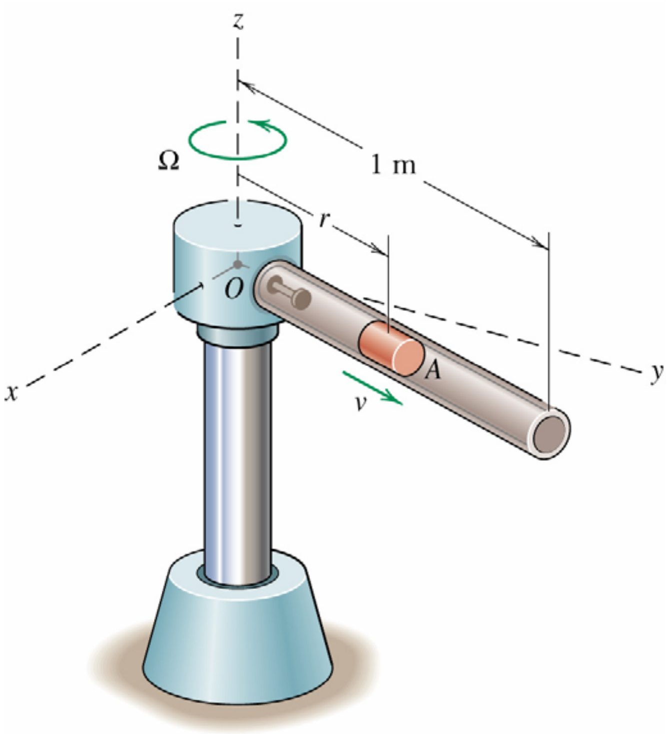


Sample 18 (3/75)

Beginning from rest when $\theta = 20°$, a 35-kg child slides with negligible friction down the sliding board which is in the shape of a 2.5-m circular arc. Determine the tangential acceleration and speed of the child, and the normal force exerted on her (*a*) when $\theta = 30°$ and (*b*) when $\theta = 90°$.



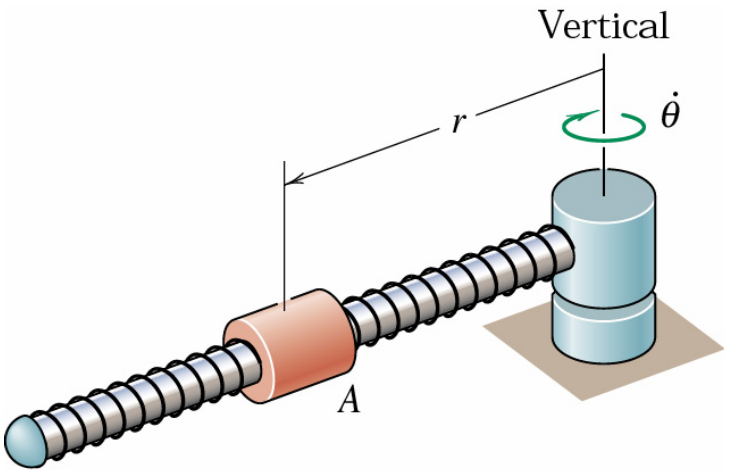
Sample 19 (3/83)



The small 180-g slider *A* moves without appreciable friction in the hollow tube, which rotates in a horizontal plane with a constant speed $\Omega = 7$ rad/s. The slider is launched with an initial speed $\dot{r}_0 = 20$ m/s relative to the tube at the inertial coordinate $x = 150$ mm and $y = 0$. Determine the magnitude *P* of the horizontal force exerted on the slider by the tube just before the slider exits the tube.

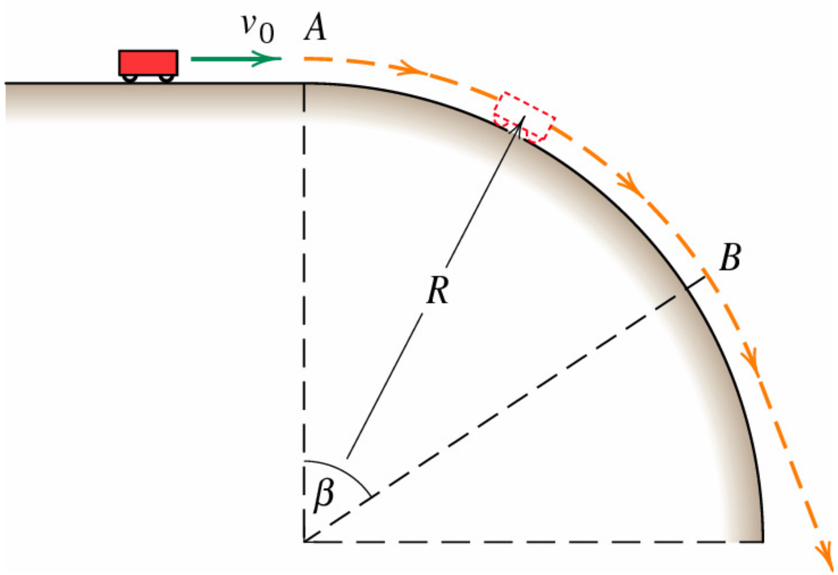
Sample 20 (3/85)

The spring-mounted 0.8-kg collar *A* oscillates along the horizontal rod, which is rotating at the constant angular rate $\dot{\theta}$ = 6 rad/s. At a certain instant, *r* is increasing at the rate of 800 mm/s. If the coefficient of kinetic friction between the collar and the rod is 0.40, calculate the friction force *F* exerted by the rod on the collar at this instant.



Sample 21 (3/87)

A small vehicle enters the top *A* of the circular path with a horizontal velocity v_0 and gathers speed as it moves down the path. Determine an expression for the angle β which locates the point where the vehicle leaves the path and becomes a projectile. Evaluate your expression for $v_0 = 0$. Neglect friction and treat the vehicle as a particle.



Sample 22 (3/98)

The small pendulum of mass m is suspended from a trolley which runs on a horizontal rail. The trolley and pendulum are initially at rest with $\theta = 0$. If the trolley is given a constant acceleration $a = g$, determine the maximum angle θ_{max} through which the pendulum swings. Also find the tension T in the cord in terms of θ.