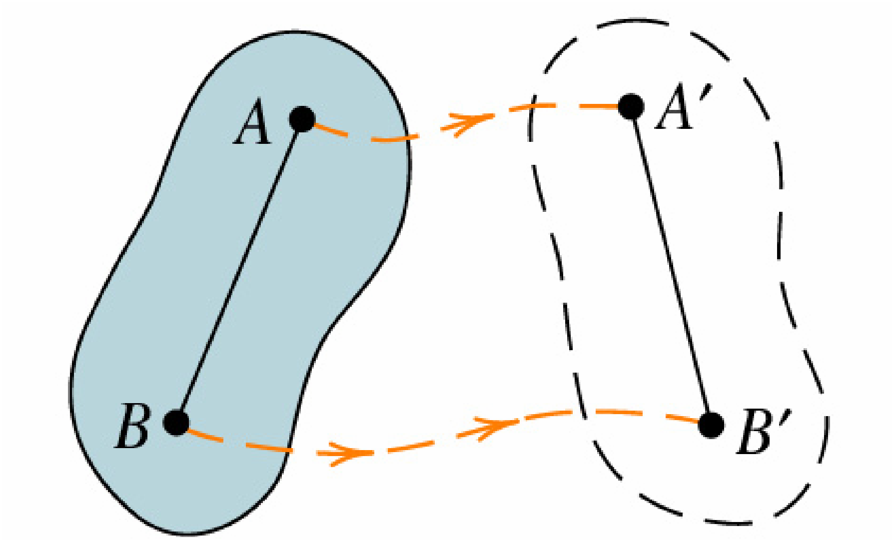
Ch.5 Plane Kinematics of Rigid Bodies

Rigid body: Distances between the particles remain unchanged,
Changes in shape are very small compared with the body movement

Kinematics of particle: Only the positions of particles are interested

Kinematics of rigid body: Movement of every part of rigid body is concerned (include rotational motion)

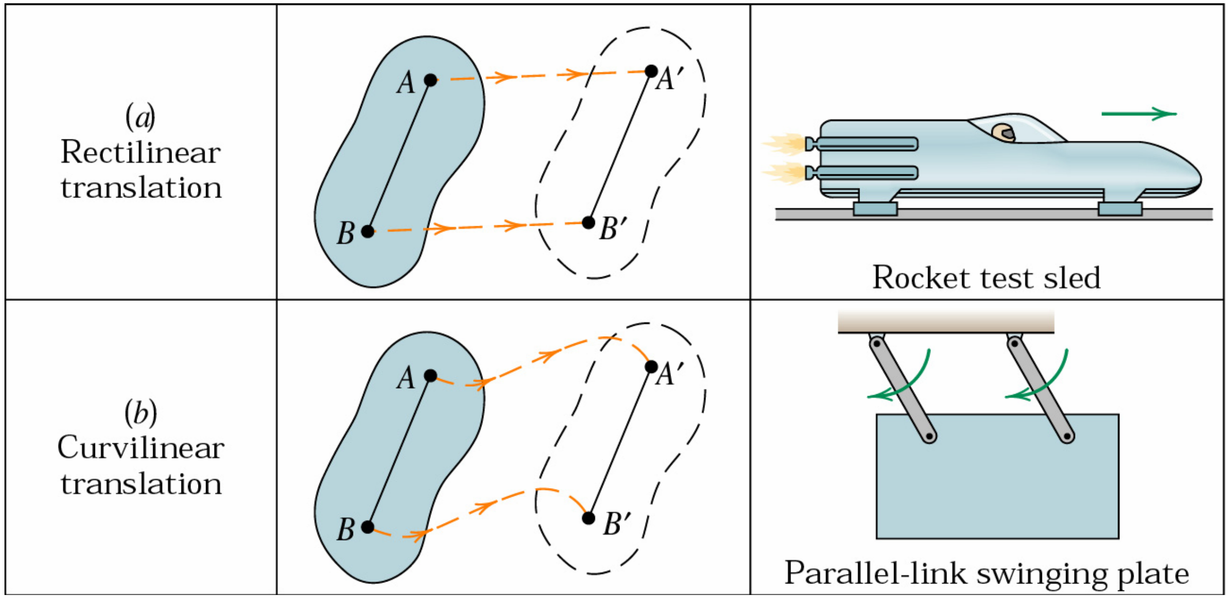


Plane motion (1)

Plane motion: all parts of the body move in parallel planes
(2-D problems)

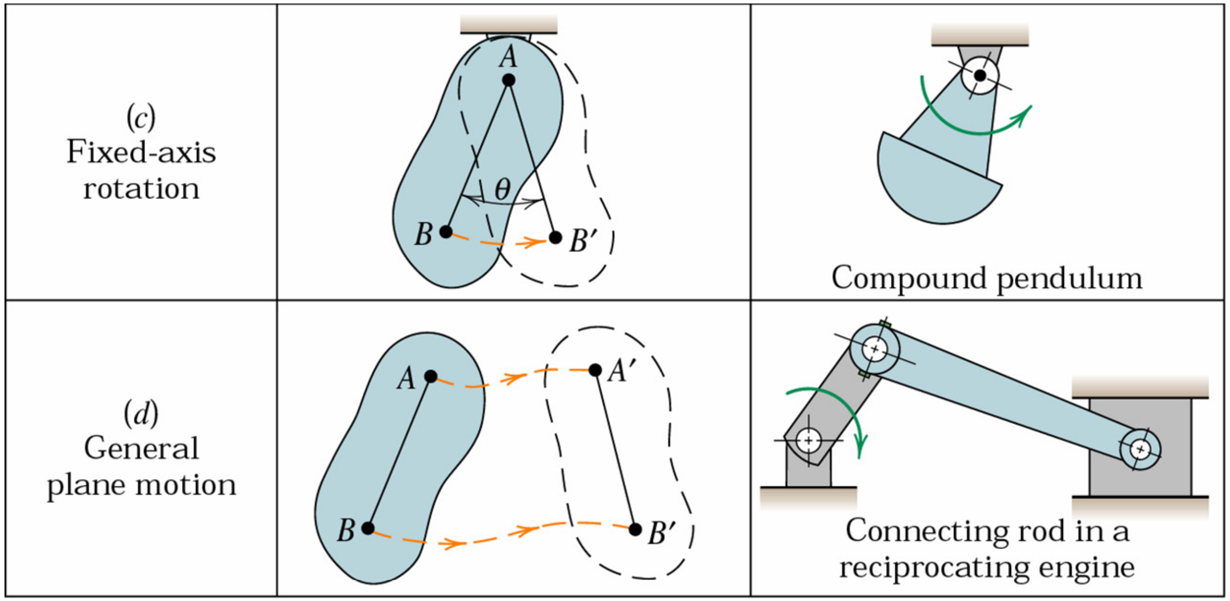
<u>Type of plane motion</u>

Translation

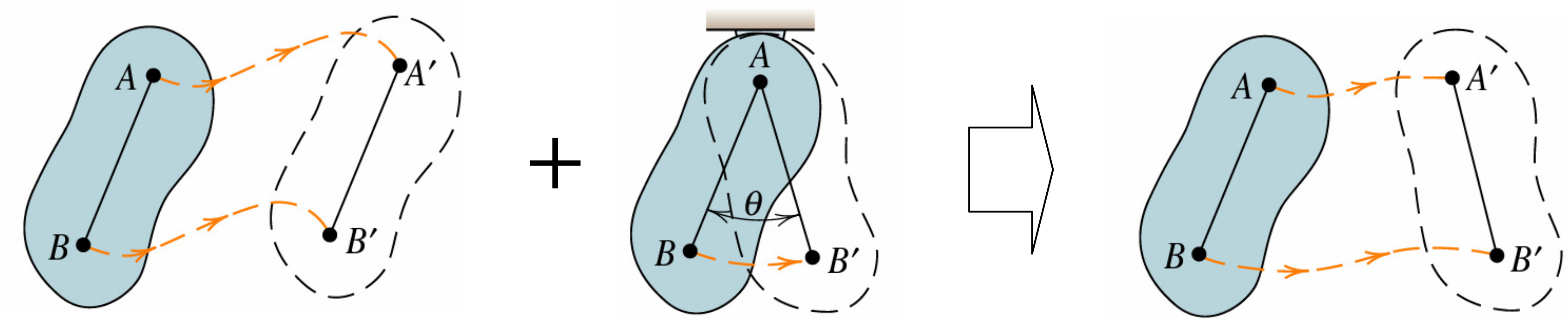


The motion of the body is completely specified by the motion of any point in the body ⟹ kinematics of particle

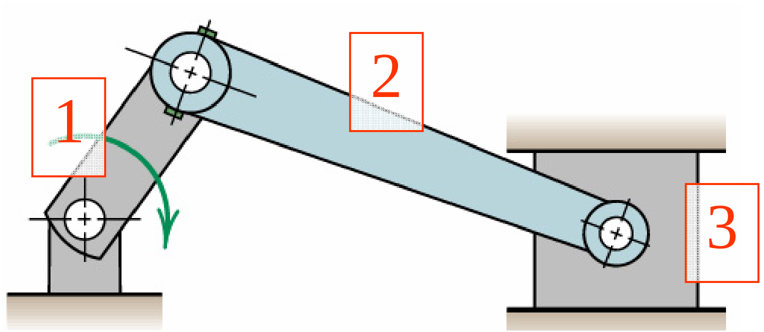
Plane motion (2)



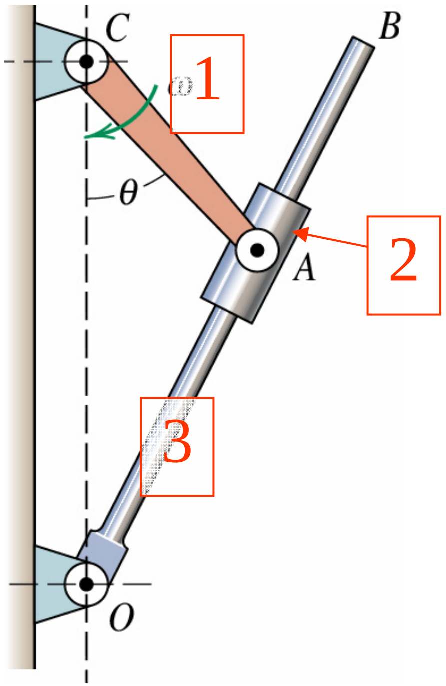
Translation + Rotation ⇨ General plane motion



Plane motion (3)

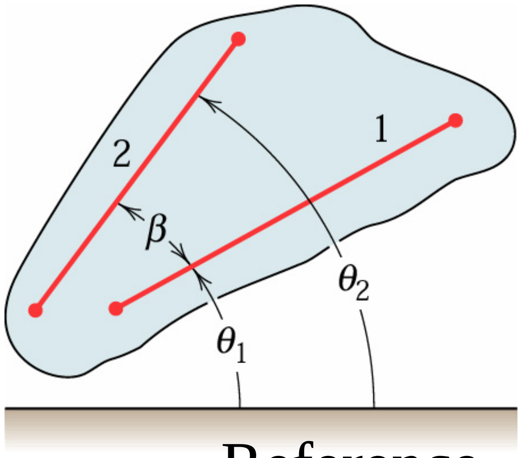


1. Rotation
2. General plane motion
3. Translation



1. Rotation
2. General plane motion
3. Rotation

Rotation



$$\theta_2 = \theta_1 + \beta$$

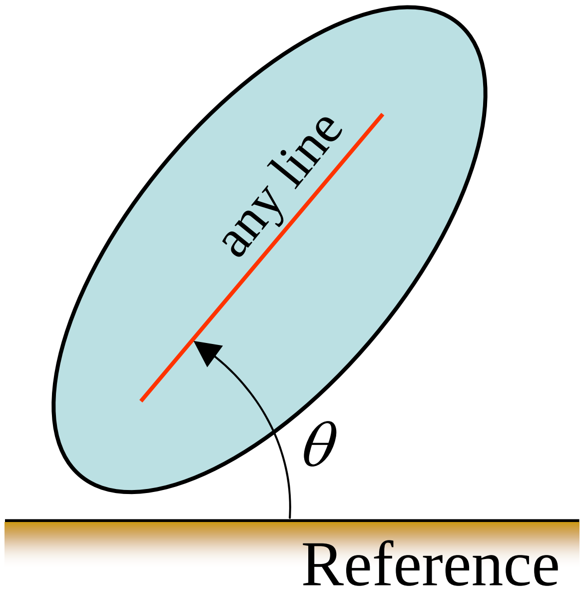
$$\dot{\beta} = 0 \text{ for rigid body}$$

$$\dot{\theta}_2 = \dot{\theta}_1, \quad \ddot{\theta}_2 = \ddot{\theta}_1, \quad \Delta\theta_2 = \Delta\theta_1$$

(at any reference)

All lines on a rigid body in its plane of motion have the same angular displacement, velocity, and acceleration

Angular motion relation



Similar to the linear motion:

Define θ = angular position

ω = angular velocity

α = angular acceleration

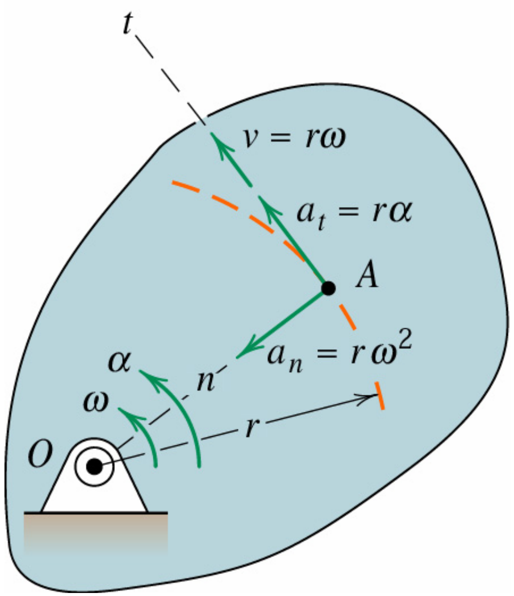
$$s \Leftrightarrow \theta$$
$$v \Leftrightarrow \omega$$
$$a \Leftrightarrow \alpha$$

$$\omega = \frac{d\theta}{dt} = \dot{\theta}$$

$$\alpha = \frac{d\omega}{dt} = \dot{\omega} = \frac{d^2\theta}{dt^2} = \ddot{\theta}$$

$$\omega d\omega = \alpha d\theta \quad or \quad \dot{\theta} d\dot{\theta} = \ddot{\theta} d\theta$$

Rotation about a fixed axis



Use n-t coordinate

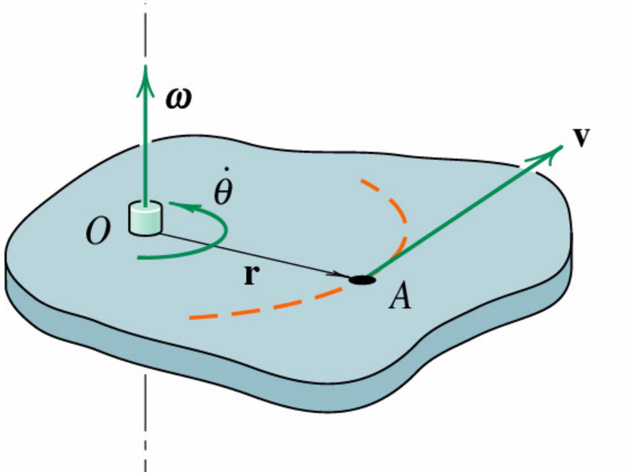
At point *A*

$$v = r\omega$$

$$a_n = r\omega^2 = v^2/r = v\omega$$

$$a_t = r\alpha$$

Rotation about a fixed axis (vector)



Direction of ω is known by the right hand rule

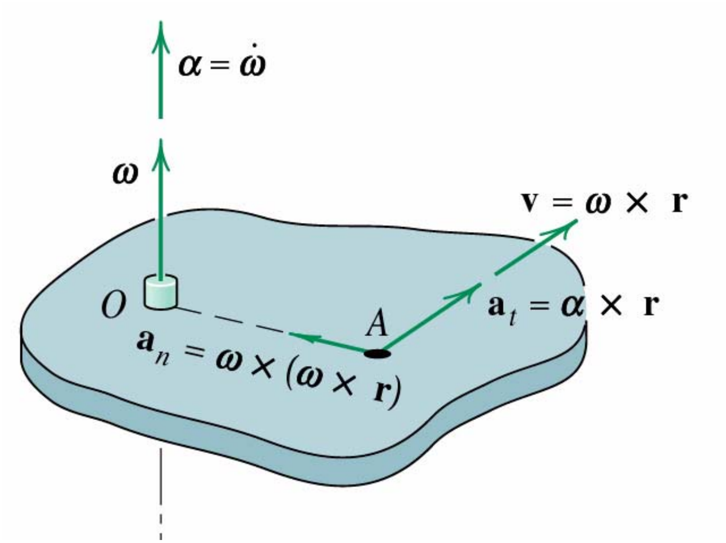
$$\vec{v} = \dot{\vec{r}} = \vec{\omega} \times \vec{r}$$

$$\vec{a} = \dot{\vec{v}} = \vec{\omega} \times \dot{\vec{r}} + \dot{\vec{\omega}} \times \vec{r}$$

$$= \vec{\omega} \times (\vec{\omega} \times \vec{r}) + \dot{\vec{\omega}} \times \vec{r}$$

$$= \vec{\omega} \times \vec{v} + \vec{\alpha} \times \vec{r}$$

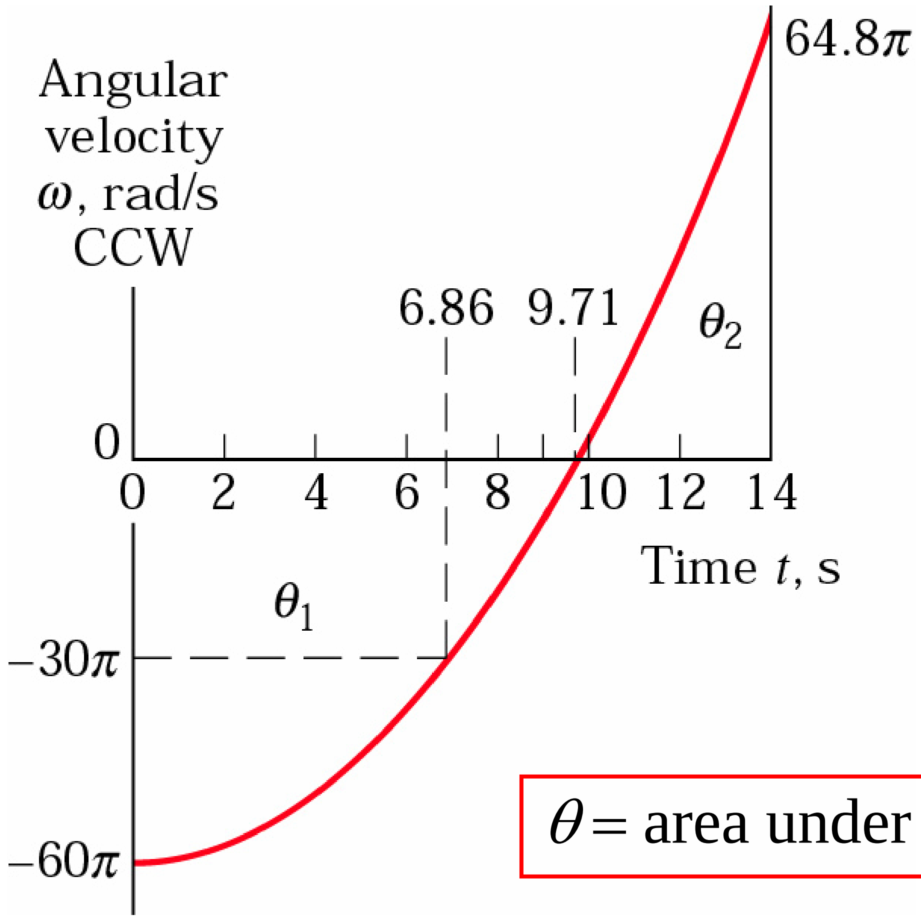
a_n a_t



Sample problem 5/1

A flywheel rotating freely at 1800 rev/min CW is subjected to a variable CCW torque which is first applied at time $t = 0$. The torque produces a CCW angular acceleration $\alpha = 4t$ rad/s^2, where t is the time in seconds during which the torque is applied. Determine (*a*) the time required for the flywheel to reduce its CW angular speed to 900 rev/min, (*b*) the time required for the flywheel to reverse its direction of rotation, and (c) the total number of revolutions, CW plus CCW, turned by the flywheel during the first 14 seconds of torque application.

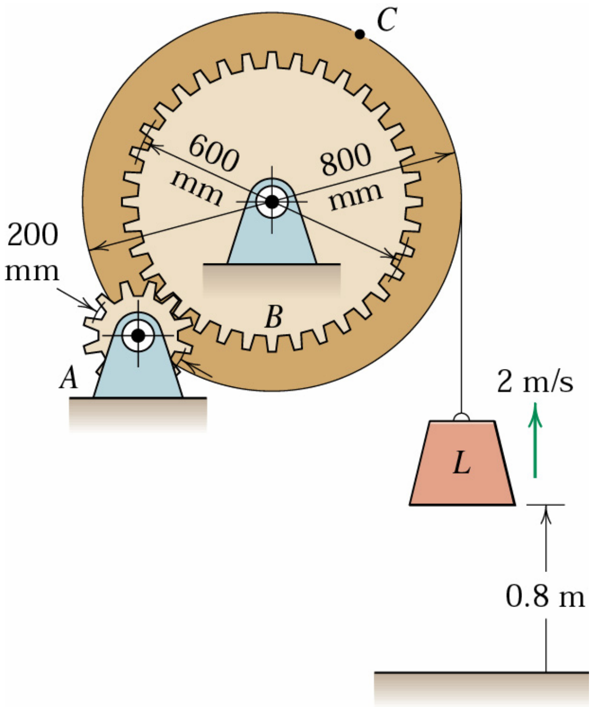
Sample problem 5/1



$\theta =$ area under graph ω-t

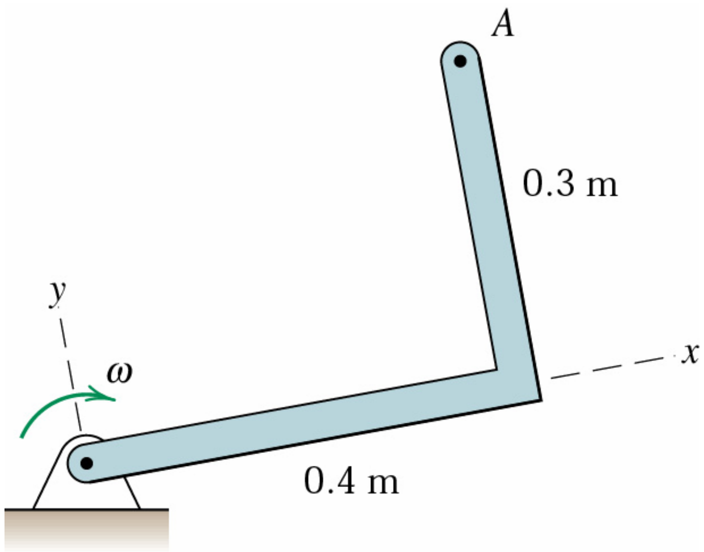
Sample problem 5/2

The pinion *A* of the hoist motor drives gear *B*, which is attached to the hoisting drum. The load *L* is lifted from its rest position and acquires an upward velocity 2 m/s in a vertical rise of 0.8 m with constant acceleration. As the load passes this position, compute (*a*) the acceleration of point *C* on the cable in contact with the drum and (*b*) the angular velocity and angular acceleration of the pinion *A*.



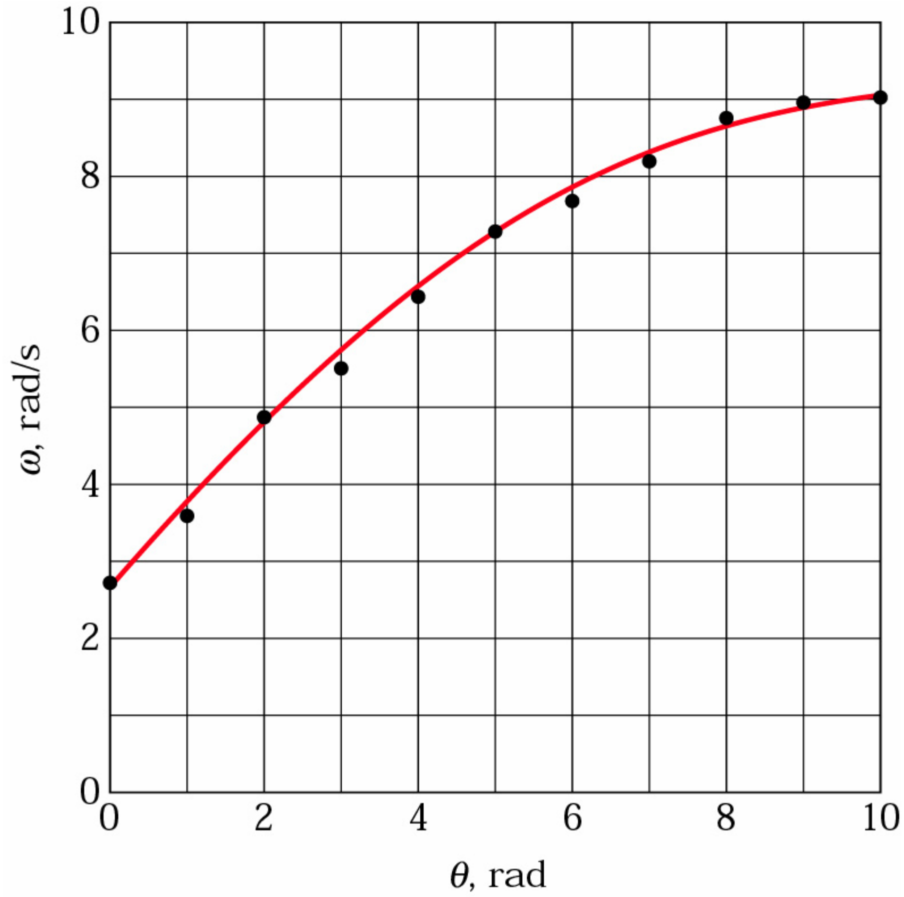
Sample problem 5/3

The right-angle bar rotate clockwise with an angular velocity which is decreasing at the rate of 4 rad/s^2. Write the vector expression for the velocity and acceleration of point *A* when $\omega = 2$ rad/s.



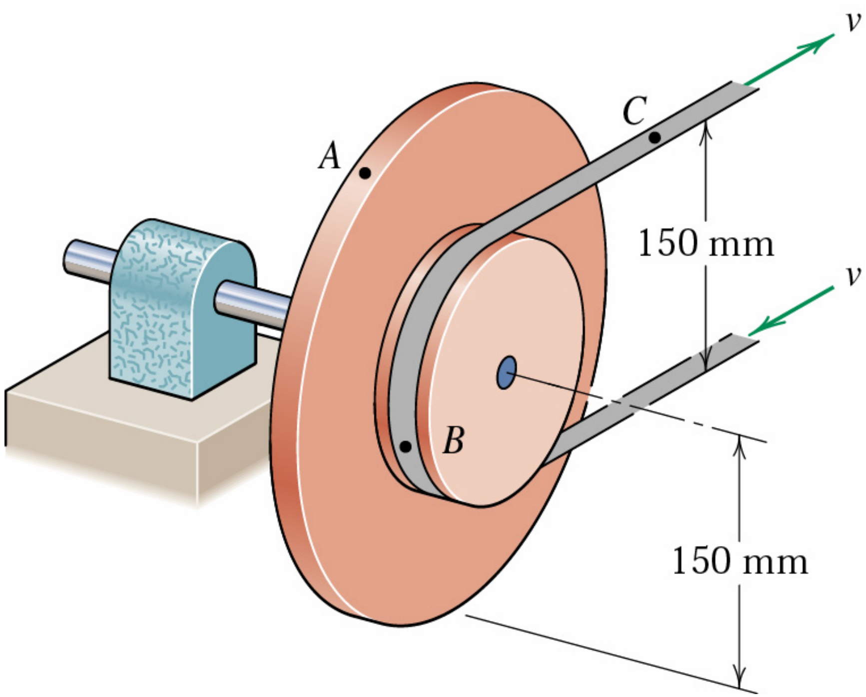
Sample 4 (5/11)

Experimental data for a rotating control element reveal the plotted relation between angular velocity and the angular coordinate θ as shown. Approximate the angular acceleration α of the element when $\theta = 6$ rad.



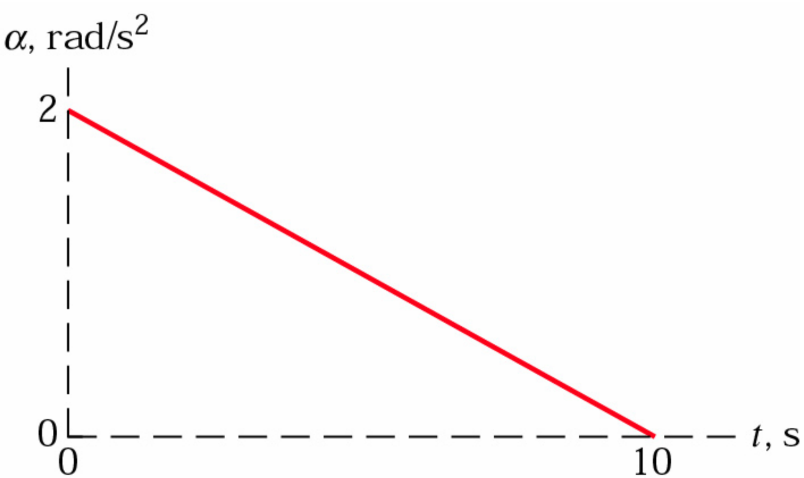
Sample 5 (5/15)

The belt-driven pulley and attached disk are rotating with increasing angular velocity. At a certain instant the speed v of the belt is 1.5 m/s, and the total acceleration of point A is 75 m/s^2. For this instant determine (a) the angular acceleration α of the pulley and disk, (b) the total acceleration of point B, and (c) the acceleration of point C on the belt.



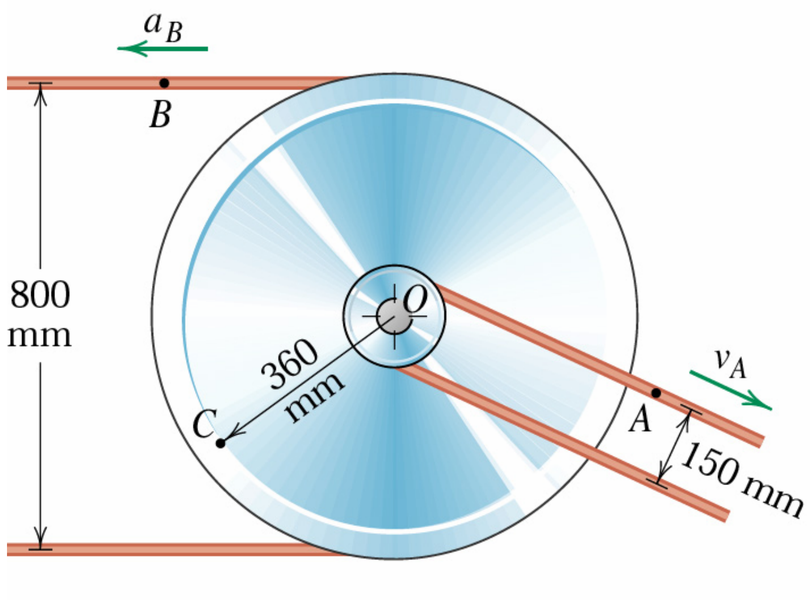
Sample 6 (5/19)

A gear rotating at a clockwise speed of 200 rev/min is subjected to a torque which gives it a clockwise angular acceleration α which varies with the time as shown. Find the speed N of the gear when $t = 5$ s.



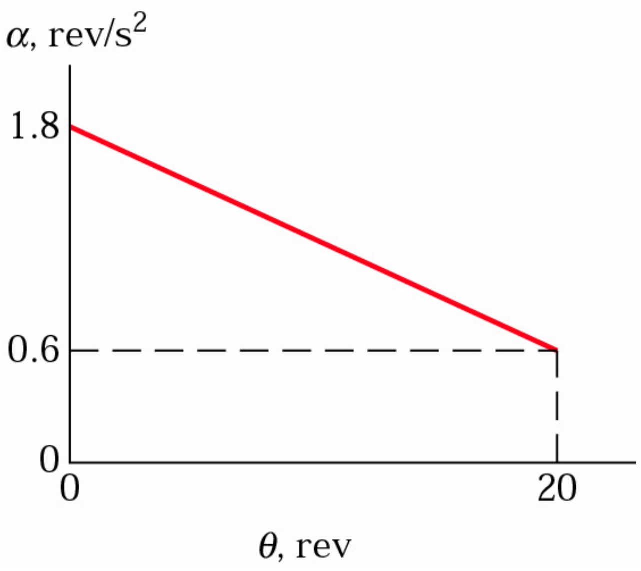
Sample 7 (5/21)

The two V-belt pulleys form an integral unit and rotate about the fixed axis at *O*. At a certain instant, point *A* on the belt of the smaller pulley has a velocity v_A = 1.5 m/s, and point *B* on the belt of the larger pulley has an acceleration a_B = 45 m/s^2 as shown. For this instant determine the magnitude of the acceleration a_C of point *C* and sketch the vector in your solution.



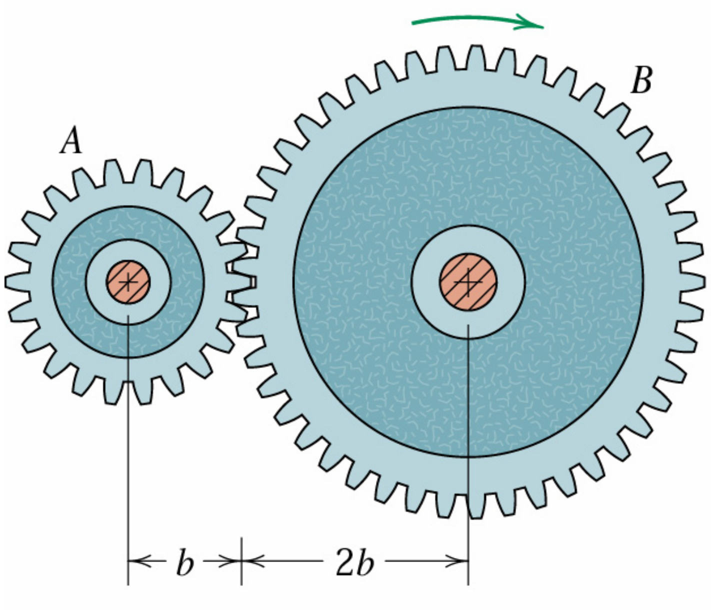
Sample 8 (5/22)

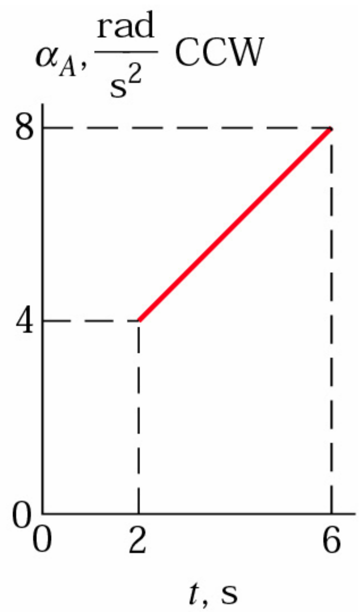
A clockwise variable torque is applied to a flywheel at time $t = 0$ causing its clockwise angular acceleration to decrease linearly with angular displacement θ during 20 revolutions of the wheel as shown. If the clockwise speed of the flywheel was 300 rev/min at $t = 0$, determine its speed N after turning the 20 revolutions. (*Suggestion*: Use units of revolution instead of radians.)



Sample 9 (5/23)

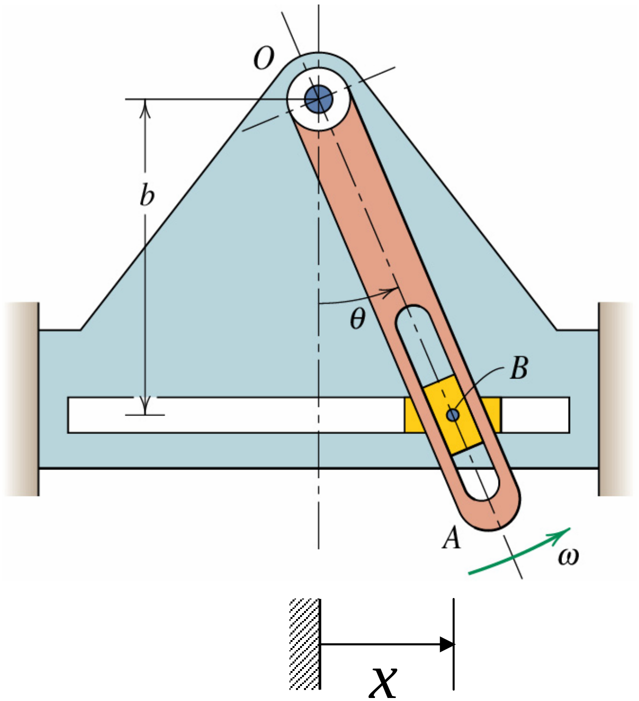
The design characteristics of a gear-reduction unit are under review. Gear B is rotating clockwise with a speed of 300 rev/min when a torque is applied to gear A at time $t = 2$ s to give gear A a counterclockwise acceleration α with varies with time for a duration of 4 seconds as shown. Determine the speed N_B of gear B when $t = 6$ s.





Absolute motion

- Write equation showing the geometric relation of the rigid bodies.
- Find time derivatives to obtain velocities and acceleration



ω = constant

Find v_B, a_B

$$x = b\tan\theta$$

$$v_B = \dot{x} = b\dot{\theta}\sec^2\theta$$

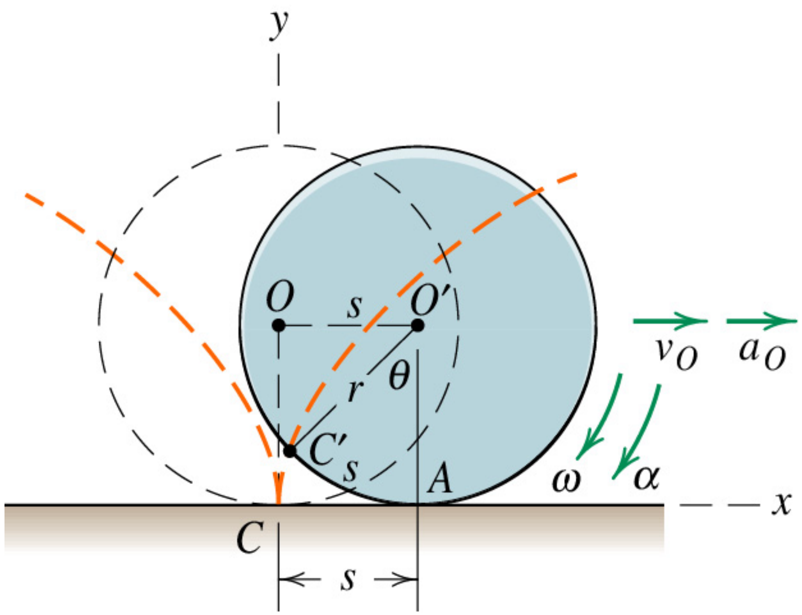
$$a_B = \ddot{x} = b\ddot{\theta}\sec^2\theta + 2b\dot{\theta}^2\sec^2\theta\tan\theta$$

($\ddot{\theta} \rightarrow 0$)

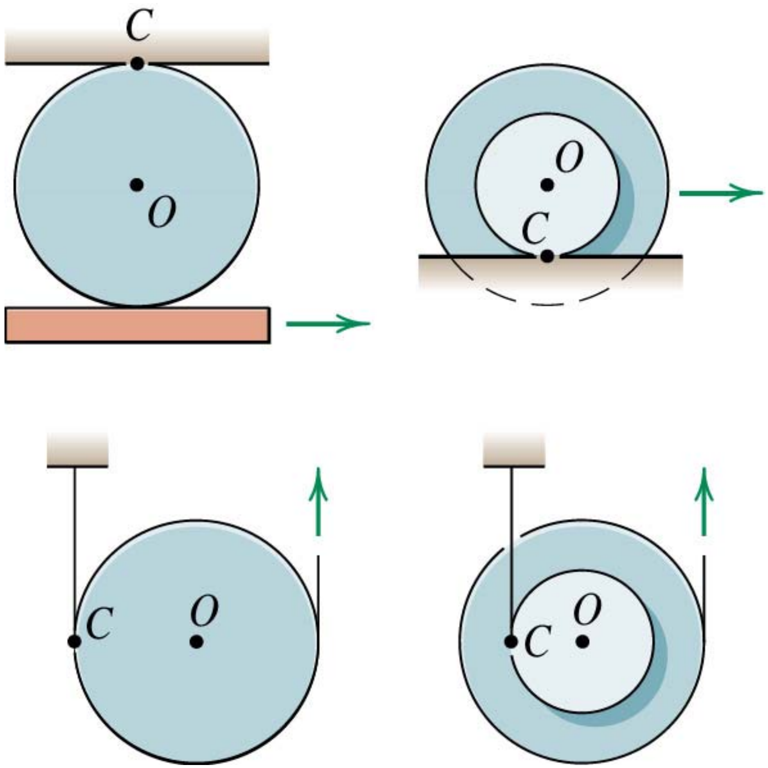
$$= 2b\dot{\theta}^2\sec^2\theta\tan\theta$$

Sample problem 5/4

A wheel of radius *r* rolls on a flat surface without slipping. Determine the angular motion of the wheel in terms of the linear motion of its center *O*. Also determine the acceleration of a point on the rim of the wheel as the point comes into contact with the surface on which the wheel rolls.



Sample problem 5/4



Sample problem 5/5

From the figure, each cable is wrapped securely around its respective pulley so it does not slip. The two pulley to which *L* is attached are fastened together to form a single rigid body. Calculate the velocity and acceleration of the load *L* and the corresponding angular velocity ω and angular acceleration α of the double pulley under the following conditions:

Case (a)

$\omega_1 = \alpha_1 = 0$ (at rest)

$\omega_2 = 2$ rad/s,

$\alpha_2 = -3$ rad/s^2

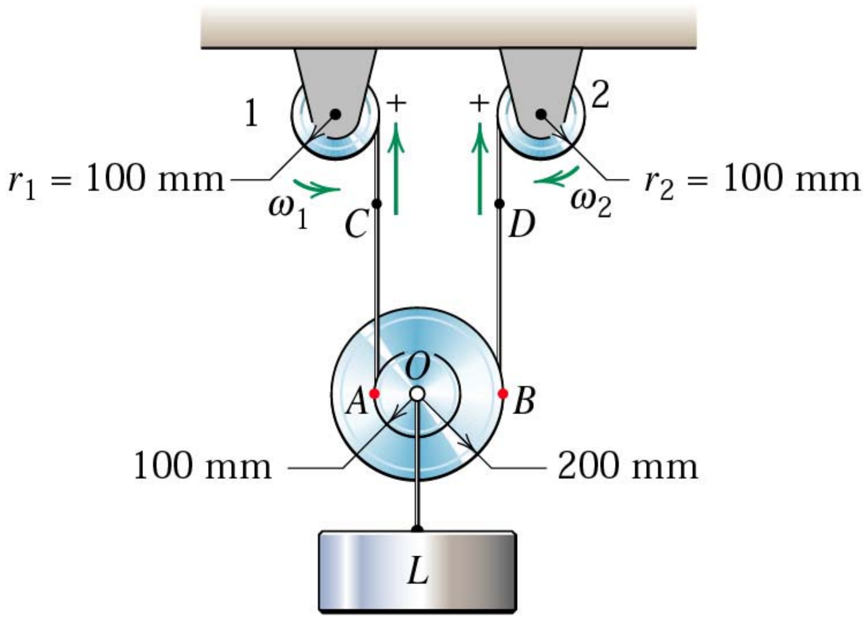
Case (b)

$\omega_1 = 1$ rad/s,

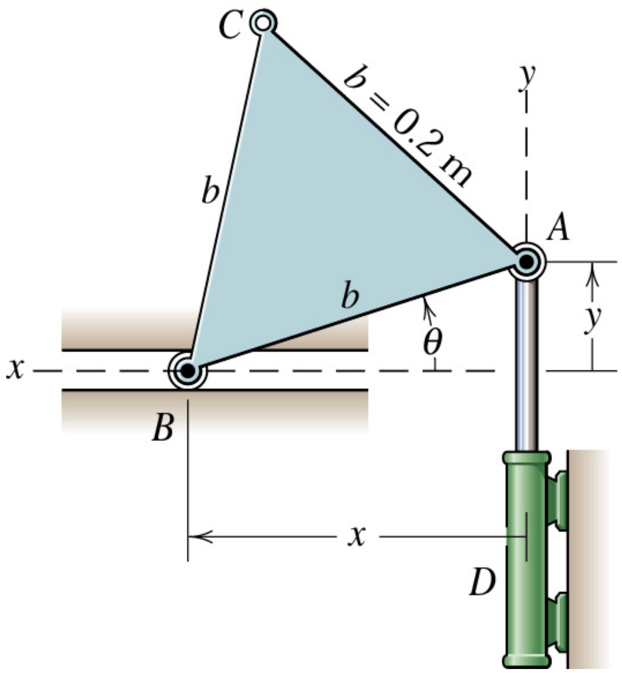
$\alpha_1 = 4$ rad/s

$\omega_2 = 2$ rad/s,

$\alpha_2 = -2$ rad/s^2



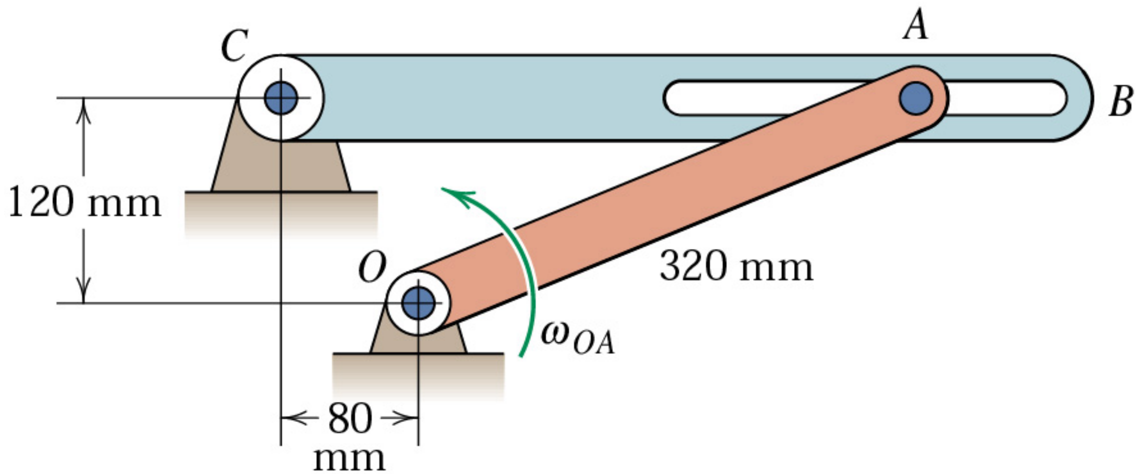
Sample problem 5/6



Motion of the equilateral triangular plate *ABC* in its plane is controlled by the hydraulic cylinder *D*. If the piston rod in the cylinder is moving upward at the constant rate of 0.3 m/s during an interval of its motion, calculate for the instant when $\theta = 30°$ the velocity and acceleration of the center of the roller *B* in the horizontal guide and the angular velocity and angular acceleration of edge *CB*.

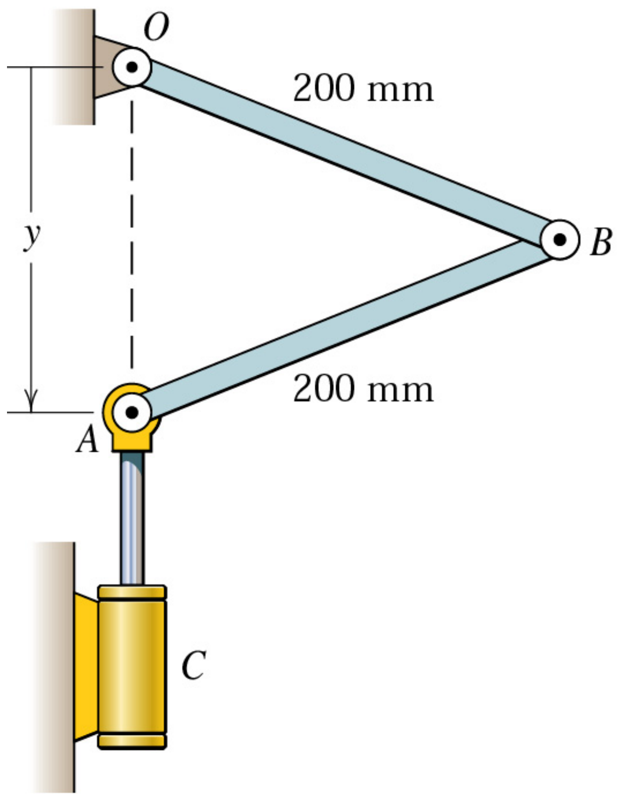
Sample 13 (5/41)

Link *OA* has an angular velocity ω_{OA} = 8 rad/s as it passes the position shown. Determine the corresponding angular velocity ω_{CB} of the slotted link *CB*. Solve by considering the relation between the infinitesimal displacements involved.



Sample 14 (5/51)

For the instant when $y = 200$ mm, the piston rod of the hydraulic cylinder *C* imparts a vertical motion to the pin *A* of $\dot{y} = 400$ mm/s and $\ddot{y} = -100$ mm/s^2. For this instant determine the angular velocity ω and the angular acceleration α of link *AB*.



Sample 15 (5/53)

The Geneva wheel is a mechanism for producing intermittent rotation. Pin *P* in the integral unit of wheel *A* and plate *B* engages the slots in wheel *C* thus turning wheel *C* one-forth of a revolution for each revolution of the pin. At the position shown, $\theta = 45°$. For a constant CW angular velocity ω_1 = 2 rad/s of wheel *A*, determine the corresponding CCW angular velocity ω_2 of wheel *C* for $\theta =$ 20° .