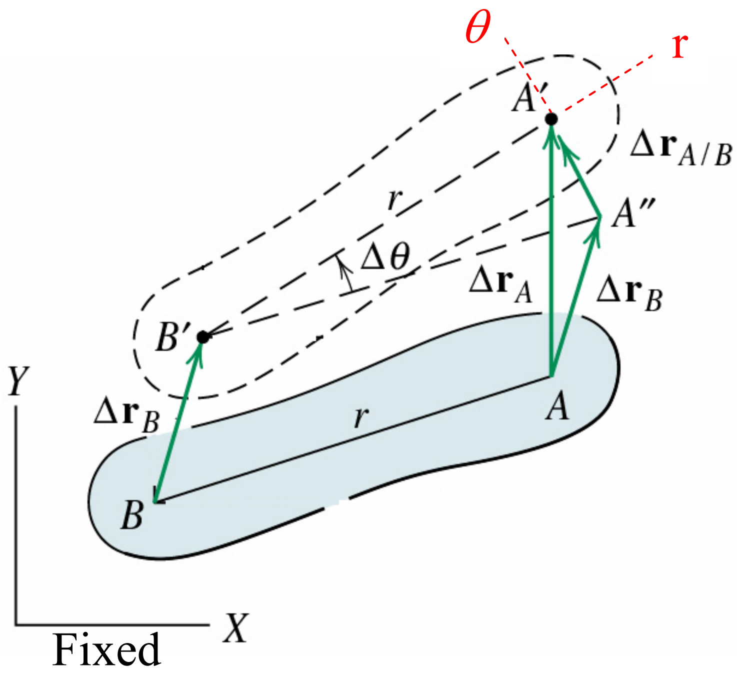
Relative velocity (1)



Observer in X-Y axes (no motion)

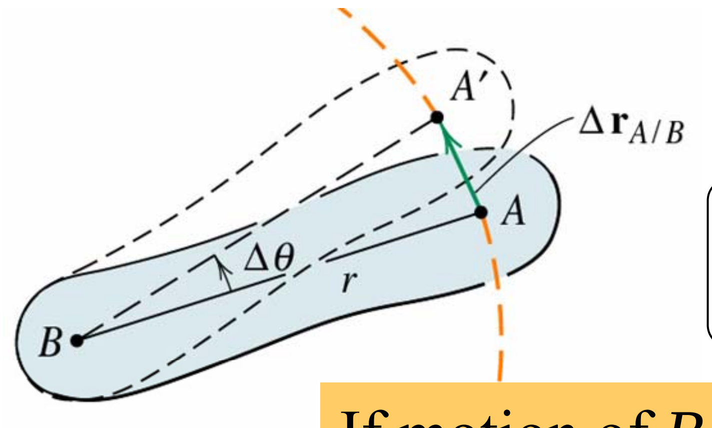
General plane motion
(translation + rotation)

Observers at *B* observe point *A* ?

(Consider in r-θ coordinate)

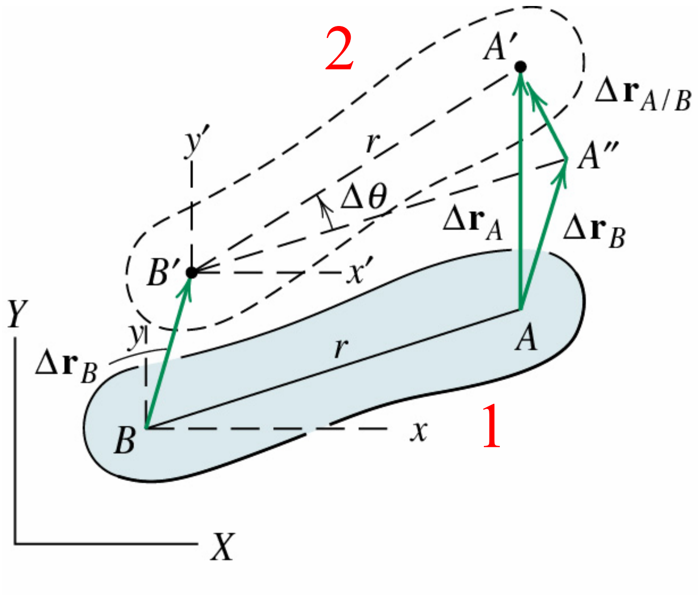
Distance between two points
on rigid body is constant $\dot{r} = 0$

Observers at *B* see *A* having no motion
or moving in circular motion around *B*



If motion of *B* (arbitrary) and relative motion are known,
motion of any point on a rigid body can be known

Relative velocity (2)



Consider the movement from 1 to 2 shown by line *BA*

Step 1: translation $\quad BA \Rightarrow B'A''$

$\Delta\vec{r}_B$ = Displacement vector (translation)

Step 2: rotation about point B'

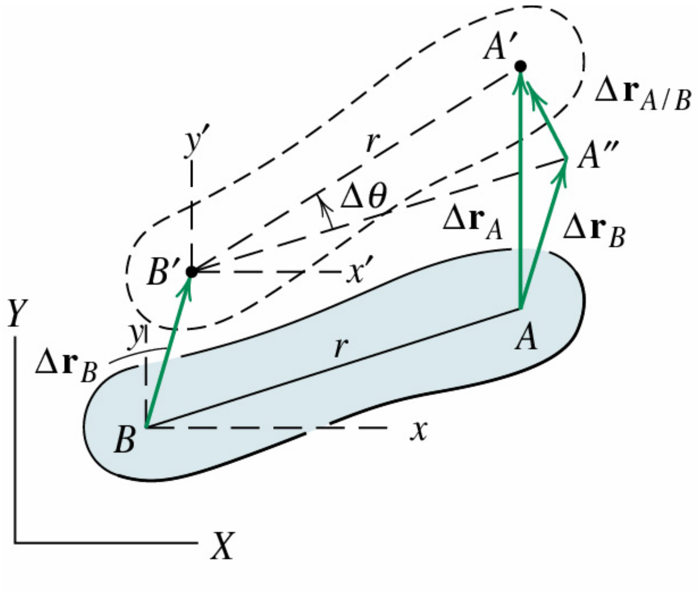
$B'A'' \Rightarrow B'A'$

$\Delta\vec{r}_{A/B}$ = change in position of *A* in this step
= change in position of *A* (observed by observer at B)

$\Delta\theta$ = change in the angle of line *BA*

Total change in position of *A* $\Rightarrow$ $$\Delta\vec{r}_A = \Delta\vec{r}_B + \Delta\vec{r}_{A/B}$$

Relative velocity (3)



$$\Delta\vec{r}_A = \Delta\vec{r}_B + \Delta\vec{r}_{A/B}$$

Divide by Δt and take limit $\Delta t \to 0$

$$\boxed{\vec{v}_A = \vec{v}_B + \vec{v}_{A/B}}$$

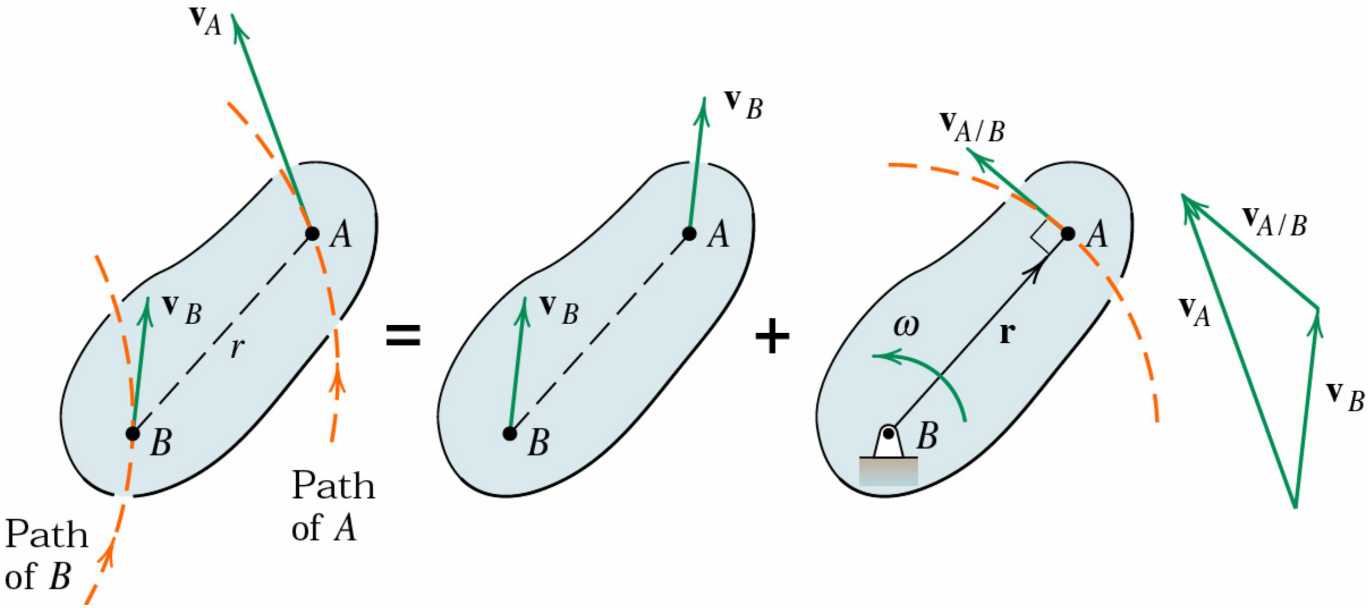
Similarly, from $\Delta\vec{r}_{A/B} = r\Delta\theta$

$$\boxed{v_{A/B} = r\omega}$$

In vector form $\boxed{\vec{v}_{A/B} = \vec{\omega}\times\vec{r}}$

Observers at *B* see point *A* moving in a circle around point *B* with the angular velocity of the body ω

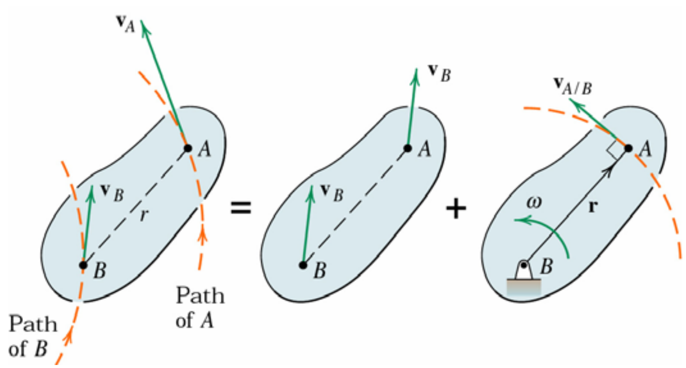
Relative velocity (4)



Observers at B see point A moving in a circle around point B

$\vec{v}_{A/B}$ is always perpendicular to the line AB

Solving the problems



<u>Concept</u>:

- Observer at *B* see point *A* moving in a circle around *B*
- Point *A* moving in circular motion

Magnitude $v_{A/B} = r\omega$, Direction $\perp \overline{AB}$

1. Write eq.; Check known, unknown quantities

$$\vec{v}_A = \vec{v}_B + \vec{v}_{A/B}$$

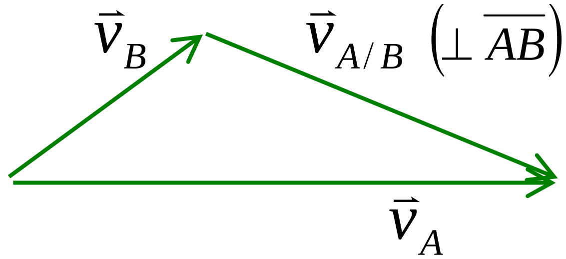
Mag.	×	○	$r\omega$
Dir.	×	○	$\perp \overline{AB}$

Which case?

Mag.	×	○	×
Dir.	○	○	$\perp \overline{AB}$

2. Calculate known quantities, angles

3. Draw vector diagram

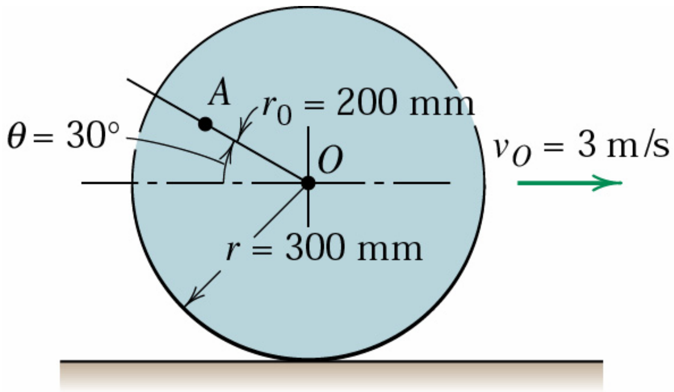


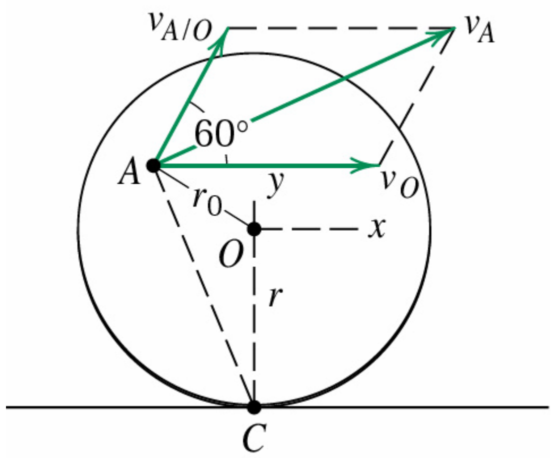
4. Calculate required quantities

<u>Ex</u>. Using sine law, cosine law

Sample problem 5/7

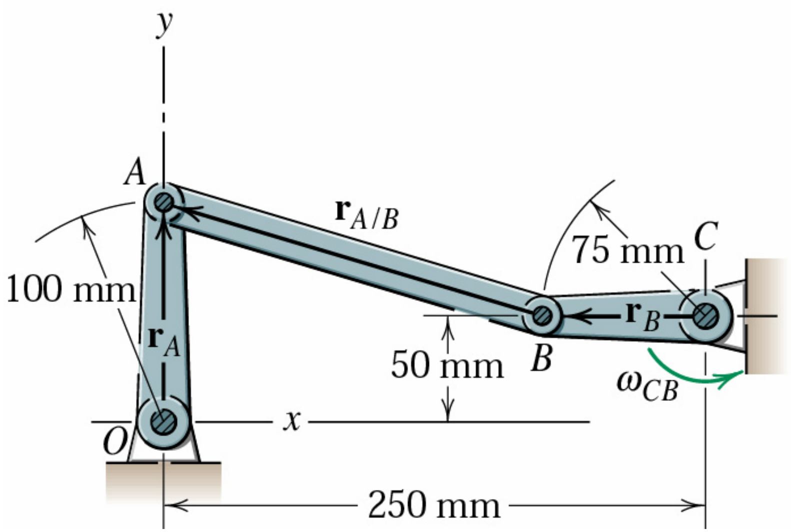
The wheel of radius $r = 300$ mm rolls to the right without slipping and has a velocity $v_O = 3$ m/s of its center O. Calculate the velocity of point A on the wheel for the instant represented.





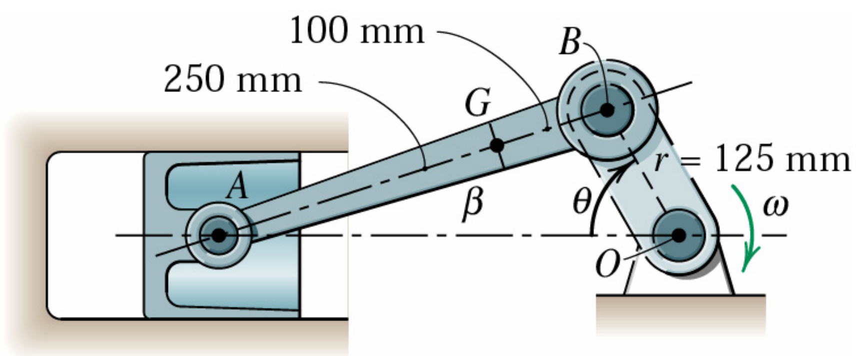
Sample problem 5/8

Crank *CB* oscillates about *C* through a limited arc, causing crank *OA* to oscillate about *O*. When the linkage passes the position shown with *CB* horizontal and *OB* vertical, the angular velocity of *CB* is 2 rad/s counterclockwise. For this instant, determine the angular velocities of *OA* and *AB*.



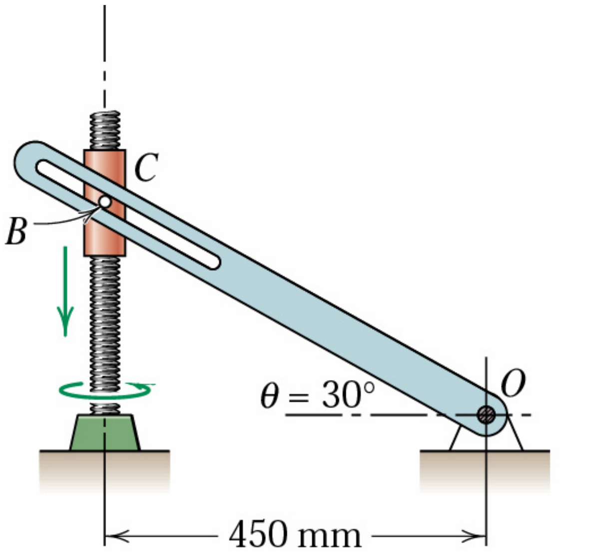
Sample problem 5/9

The common configuration of a reciprocating engine is that of the slider crank mechanism shown. If the crank *OB* has a clockwise rotational speed of 1500 rev/min, determine for the position where $\theta = 60°$ the velocity of the piston *A*, the velocity of point *G* on the connecting rod, and the angular velocity of the connecting rod.



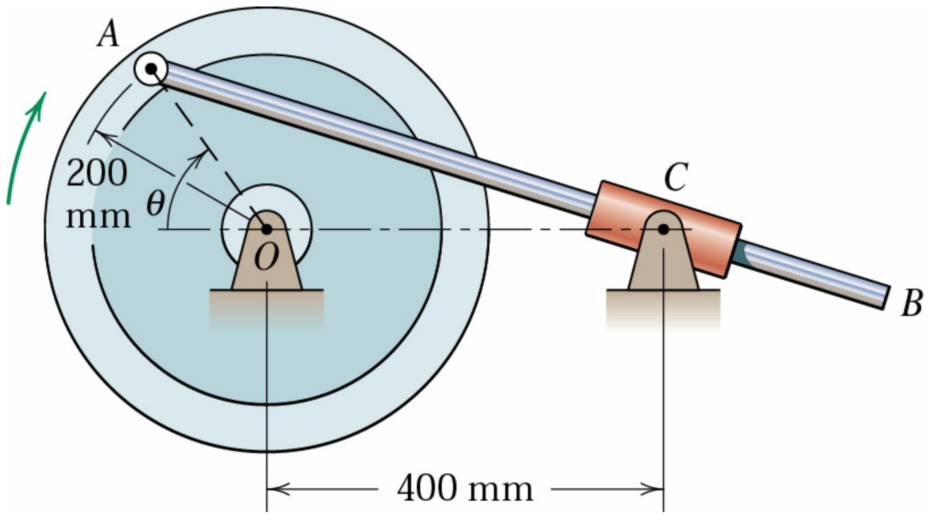
Sample problem 5/10

The power screw turns at a speed which gives the threaded collar C a velocity of 0.25 m/s vertically down. Determine the angular velocity of the slotted arm when $\theta = 30°$.



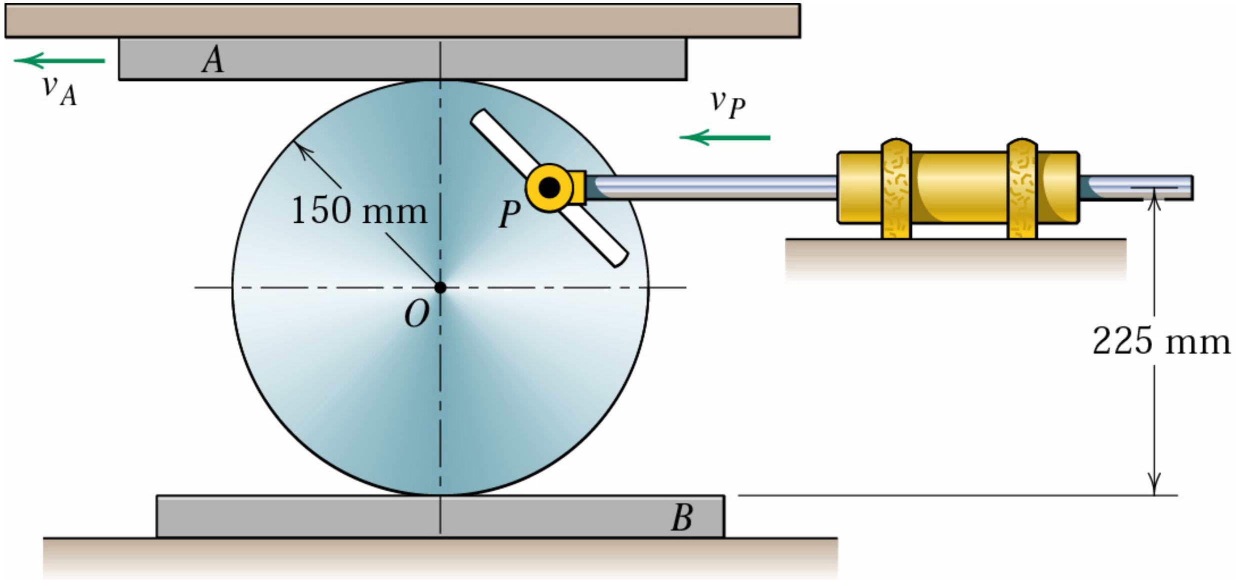
Sample 5 (5/77)

The flywheel turns clockwise with a constant speed of 600 rev/min, and the connecting rod *AB* slides through the pivoted collar at *C*. For the position $\theta = 45°$, determine the angular velocity ω_{AB} of *AB* by using the relative-velocity relations. (Suggestion: Choose a point *D* on *AB* coincident with *C* as a reference point whose direction of velocity is known.)



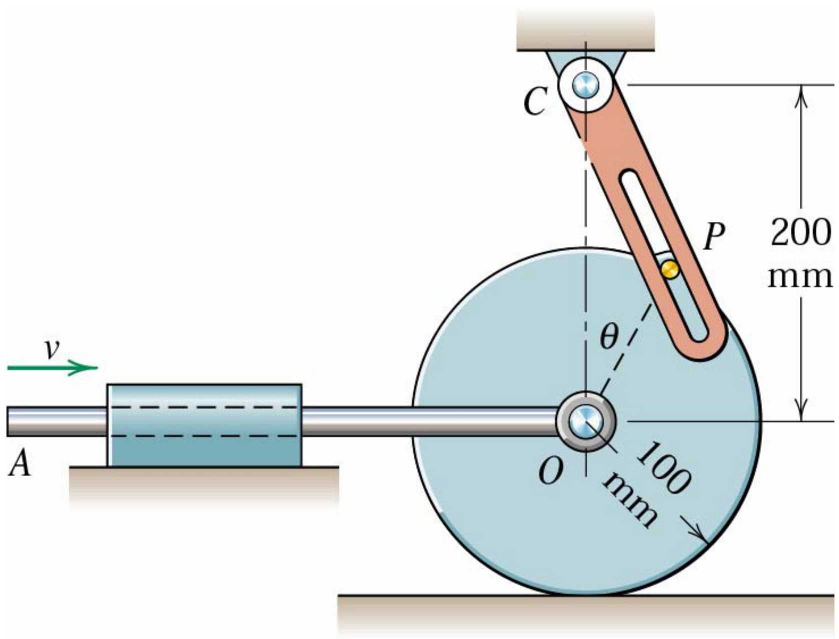
Sample 6 (5/87)

Pin *P* on the end of the horizontal rod slides freely in the slotted gear. The gear engages the moving rack *A* and the fixed rack *B* (teeth not shown) so it rolls without slipping. If *A* has a velocity of 120 mm/s to the left for the instant shown, determine the velocity v_P of the rod for this position.



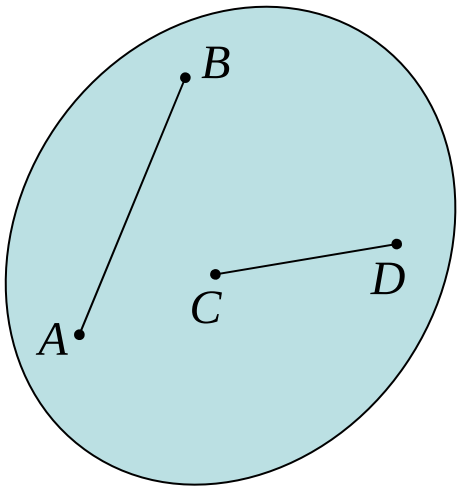
Sample 7 (5/89)

The wheel rolls without slipping. For the instant portrayed, when O is directly under point C, link OA has a velocity $v = 1.5$ m/s to the right and $\theta = 30°$. Determine the angular velocity ω of the slotted link.



Instantaneous Center of Zero Velocity

Rigid body : distances between any two points are constant



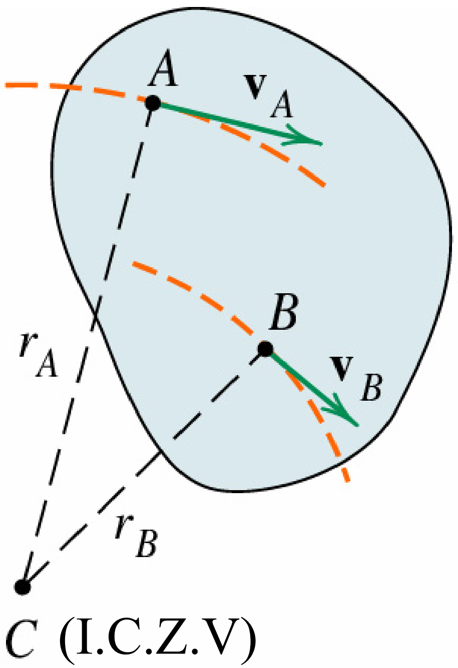
Consider ω of each line, if ω of lines are different

- Line AB and CD rotate at different ω
- Distances between two lines (points) are changed ⟹ Impossible

- All lines in the rigid body must rotate with the same ω
- There is a point that is the center of rotation of all points in the rigid body at that instant → $v = 0$
- If this point is known, v at any point can be calculated → $v = \omega r$

Instantaneous Center of Zero Velocity

- For a moving body at each instant of time, there is always a point on the body (or on the extended body) that can be though as the center of rotation.
- This point has zero velocity ,and be called as "Instantaneous center of zero velocity (I.C.Z.V)"



C (I.C.Z.V)
Observer

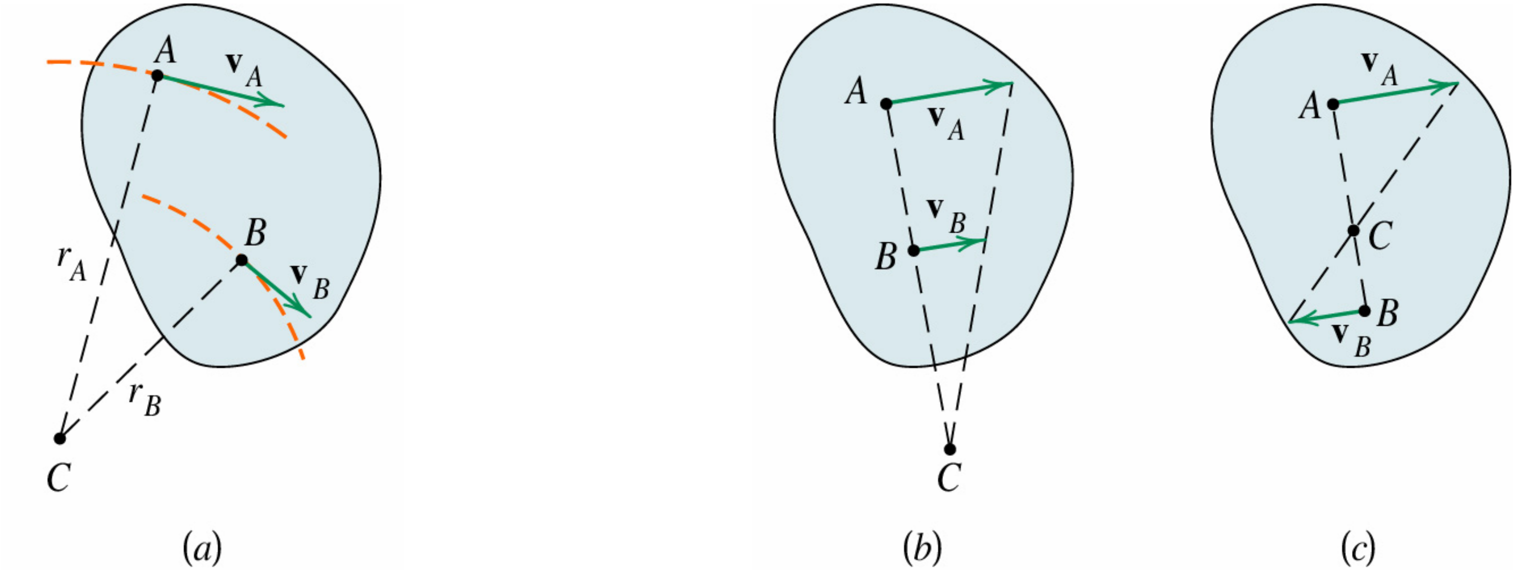
$$\vec{v}_A = \cancelto{0}{\vec{v}_C} + \vec{v}_{A/C}$$

Absolute velocity = relative velocity
(Observing from an I.C.Z.V)

$$\boxed{\vec{v}_A = \vec{\omega} \times \vec{r}}$$

$\vec{v}_A$ must be perpendicular to CA

Locating the instantaneous center



$\vec{v}_A$ perpendicular to CA $\qquad v_A = \omega(CA)$

$\vec{v}_B$ perpendicular to CB $\qquad v_B = \omega(CB)$

Although $v_C = 0$, a_C usually $\neq 0$ (see sample)

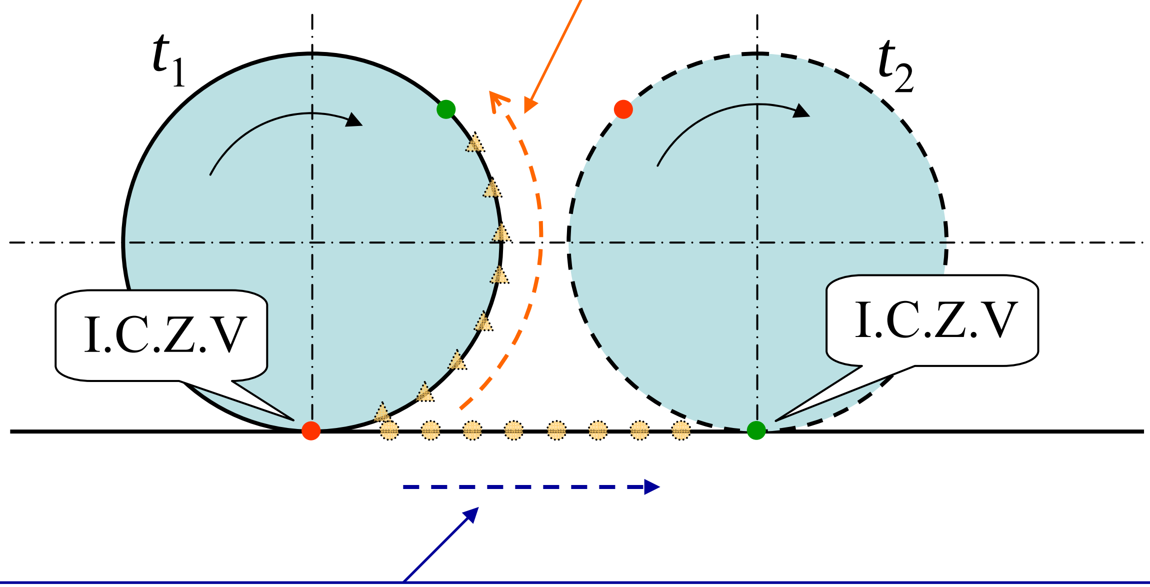
Point C (I.C.Z.V) can be used for calculating **velocity only**.

Motion of the instantaneous center

As the body changes its position, the instantaneous center also changes it position.

Consider in body: I.C.Z.V move along this locus

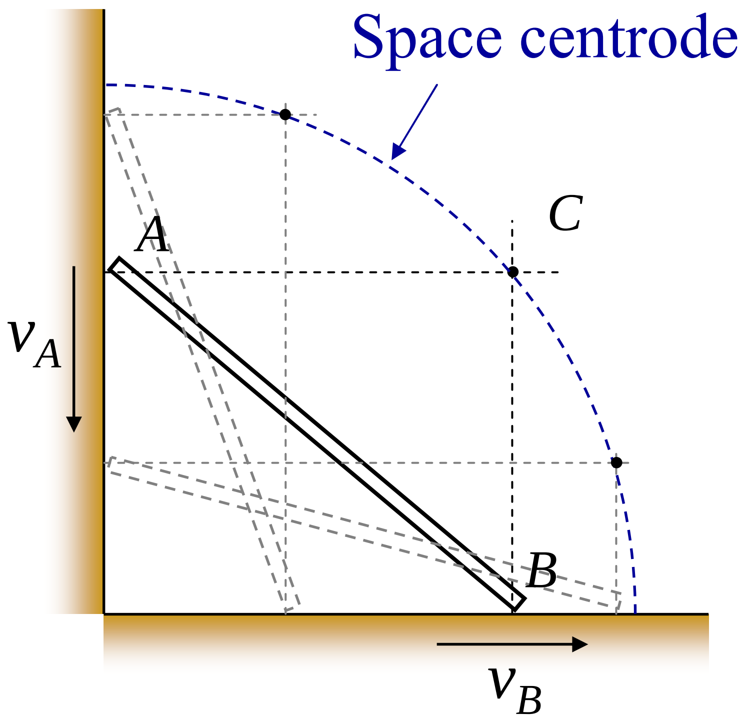
⟹ Body centrode



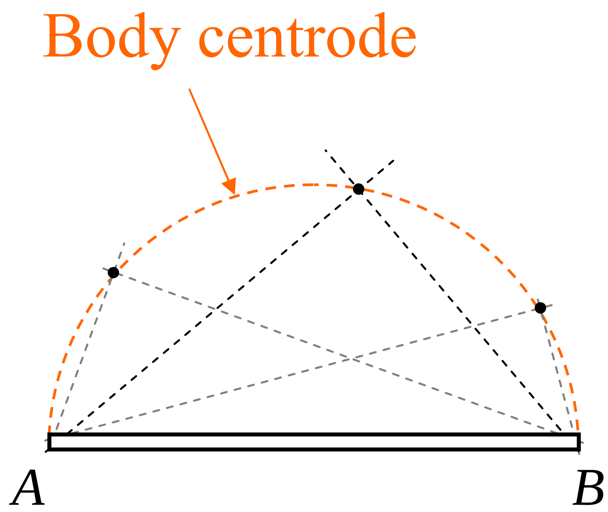
Consider in space: I.C.Z.V move along this locus

⟹ Space centrode

Example I.C.Z.V



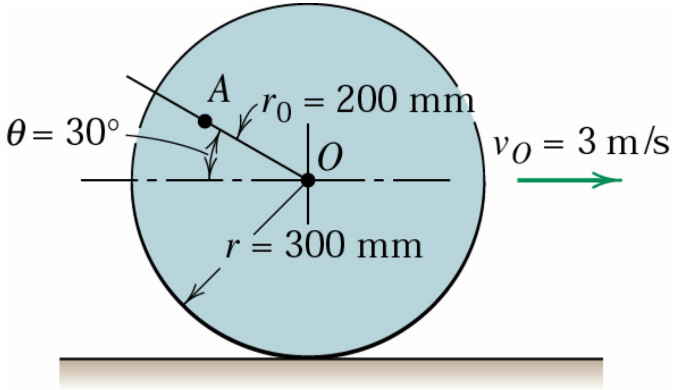
The change of the position of I.C.Z.V in the space



The change of the position of I.C.Z.V relative to bar *AB*

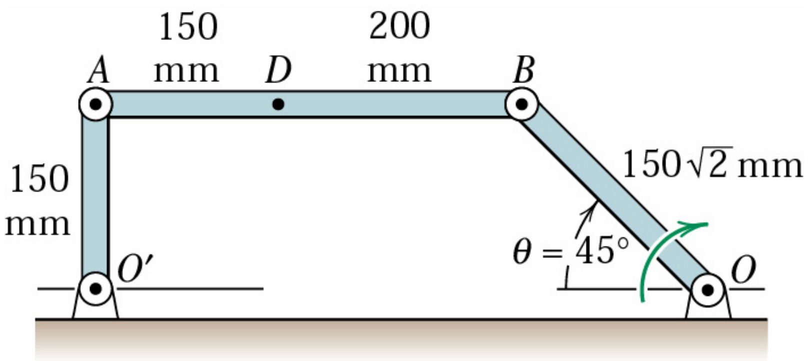
Sample problem 5/11

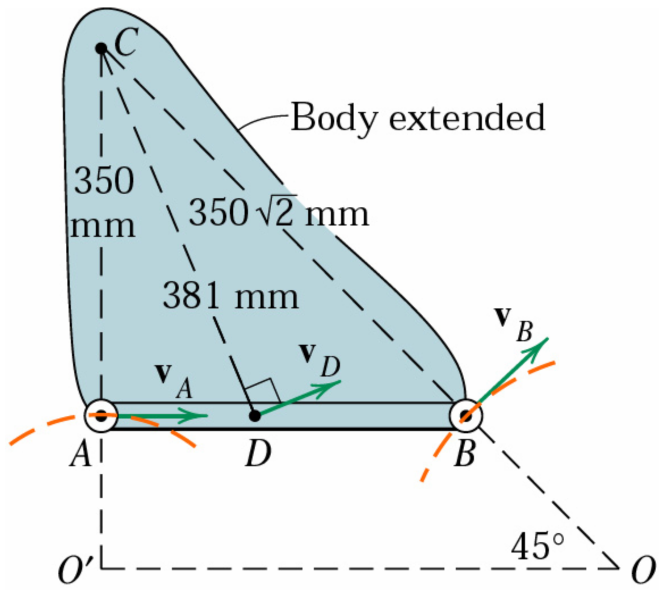
The wheel rolls to the right without slipping, with its center *O* having a velocity $v_O = 3$ m/s. Locate the instantaneous center of zero velocity and use it to find the velocity of point A for the position indicated.



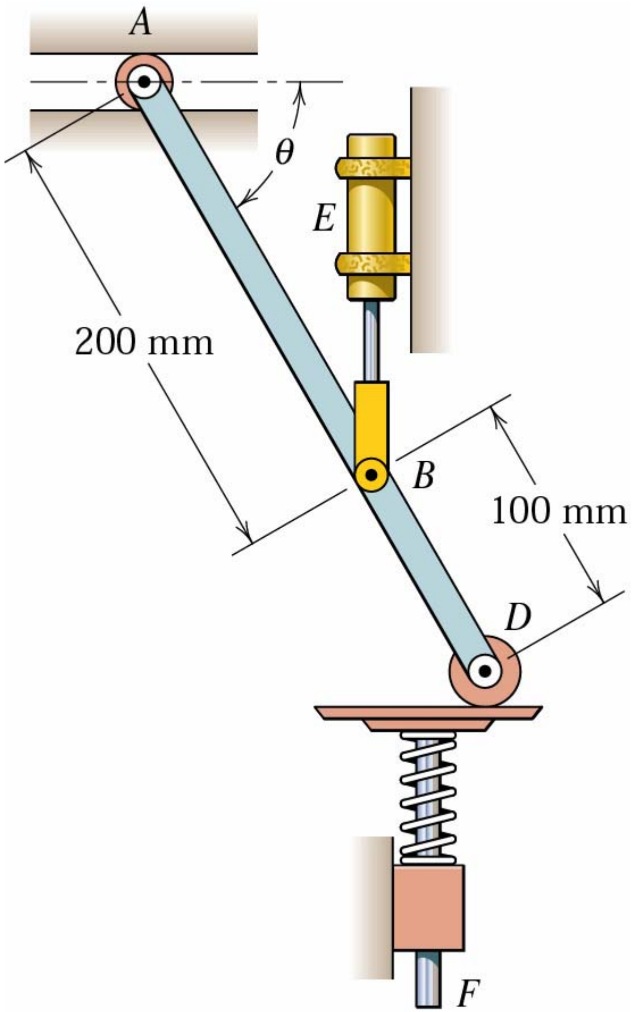
Sample problem 5/12

Arm *OB* of the linkage has a clockwise angular velocity of 10 rad/s in the position shown where $\theta = 45°$. Determine the velocity of *A*, the velocity of *D*, and the angular velocity of link *AB* for the position shown.





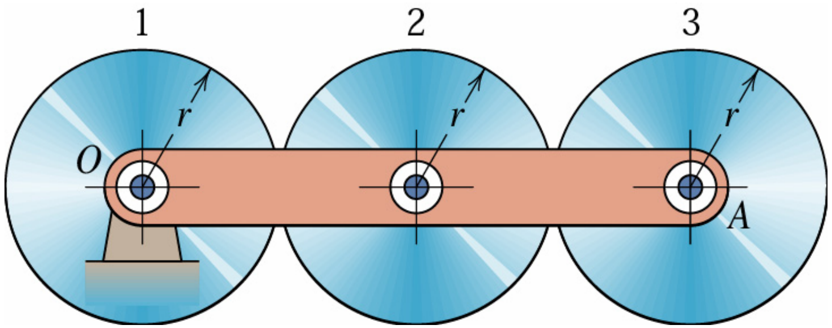
Sample 8 (5/113)



Vertical oscillation of the spring-loaded plunger *F* is controlled by a periodic change in pressure in the vertical hydraulic cylinder *E*. For the position $\theta = 60°$, determine the angular velocity of *AD* and the velocity of the roller *A* in its horizontal guide if the plunger *F* has a downward velocity of 2 m/s.

Sample 9 (5/115)

The three gears 1, 2, and 3 of equal radii are mounted on the rotating arm as shown. (Gear teeth are omitted from the drawing.) Arm *OA* rotates clockwise about *O* at the angular rate of 4 rad/s, while gear 1 rotates independently at the counterclockwise rate of 8 rad/s. Determine the angular velocity of gear 3.



Sample 10 (5/117)

The shaft at *O* drives the arm *OA* at a clockwise speed of 90 rev/min about the fixed bearing *O*. Use the method of the instantaneous center of zero velocity to determine the rotational speed of gear *B* (gear teeth not shown) if (*a*) ring gear *D* is fixed and (*b*) ring gear *D* rotates counterclockwise about *O* with a speed of 80 rev/min.