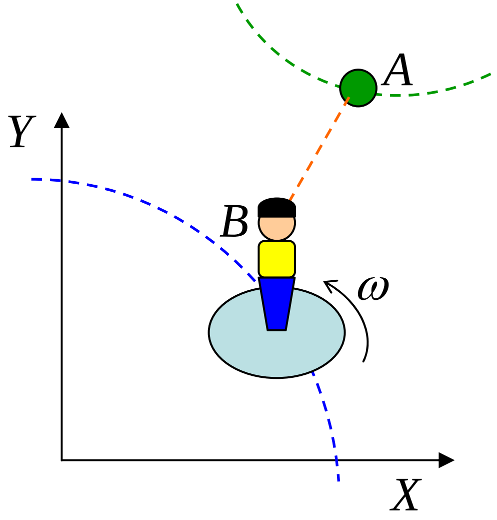
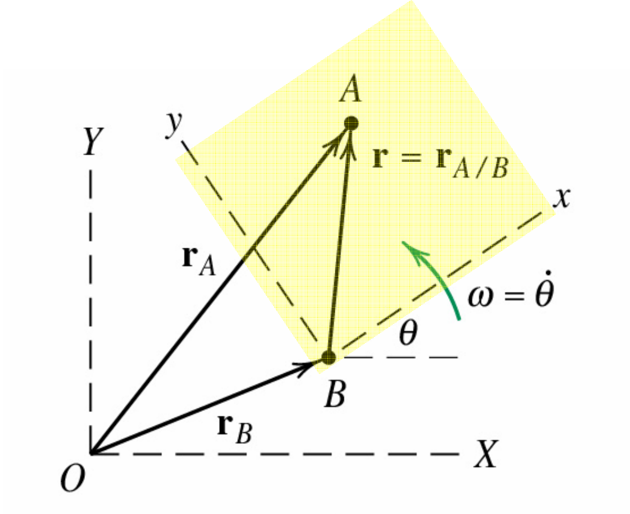
Motion relative to rotating axes





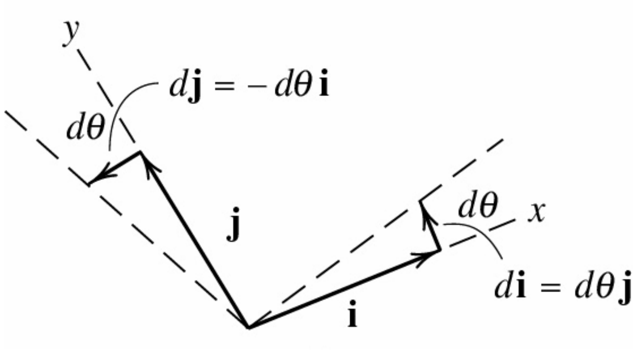
measured
from *x*-*y* axes

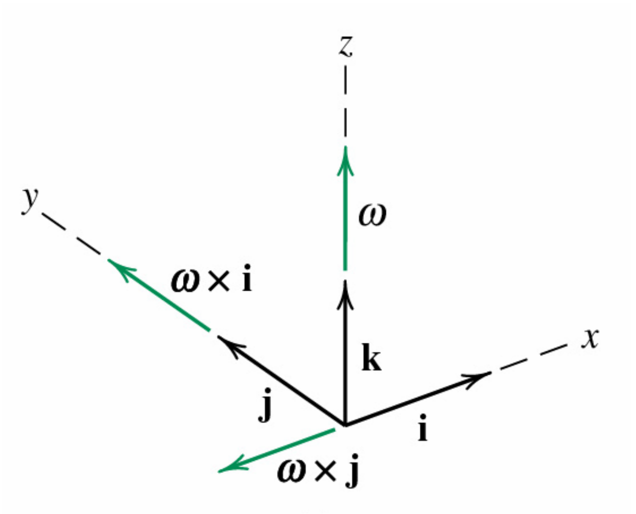
$$\vec{r}_A = \vec{r}_B + \vec{r} = \vec{r}_B + (x\hat{i} + y\hat{j})$$

velocity ⇨ $$\boxed{\dot{\vec{r}}_A = \dot{\vec{r}}_B + \frac{d}{dt}(x\hat{i} + y\hat{j})}$$

Rotating axes ⇨ Directions of unit vectors are also change.

Time derivatives of units vectors





$$\boxed{\dot{\hat{i}} = \omega\hat{j} \quad \text{and} \quad \dot{\hat{j}} = -\omega\hat{i}}$$

By using the cross product

$$\boxed{\dot{\hat{i}} = \vec{\omega}\times\hat{i} \quad \text{and} \quad \dot{\hat{j}} = \vec{\omega}\times\hat{j}}$$

Relative velocity

From $\dot{\hat{i}} = \omega \times \hat{i}$ and $\dot{\hat{j}} = \omega \times \hat{j}$

$$\dot{\vec{r}}_A = \dot{\vec{r}}_B + \frac{d}{dt}(x\hat{i} + y\hat{j})$$

$$\dot{\vec{r}}_A = \dot{\vec{r}}_B + (x\dot{\hat{i}} + y\dot{\hat{j}}) + (\dot{x}\hat{i} + \dot{y}\hat{j})$$

$$= (\omega \times x\hat{i} + \omega \times y\hat{j})$$

$$= \omega \times (x\hat{i} + y\hat{j})$$

$$= \vec{\omega} \times \vec{r}$$

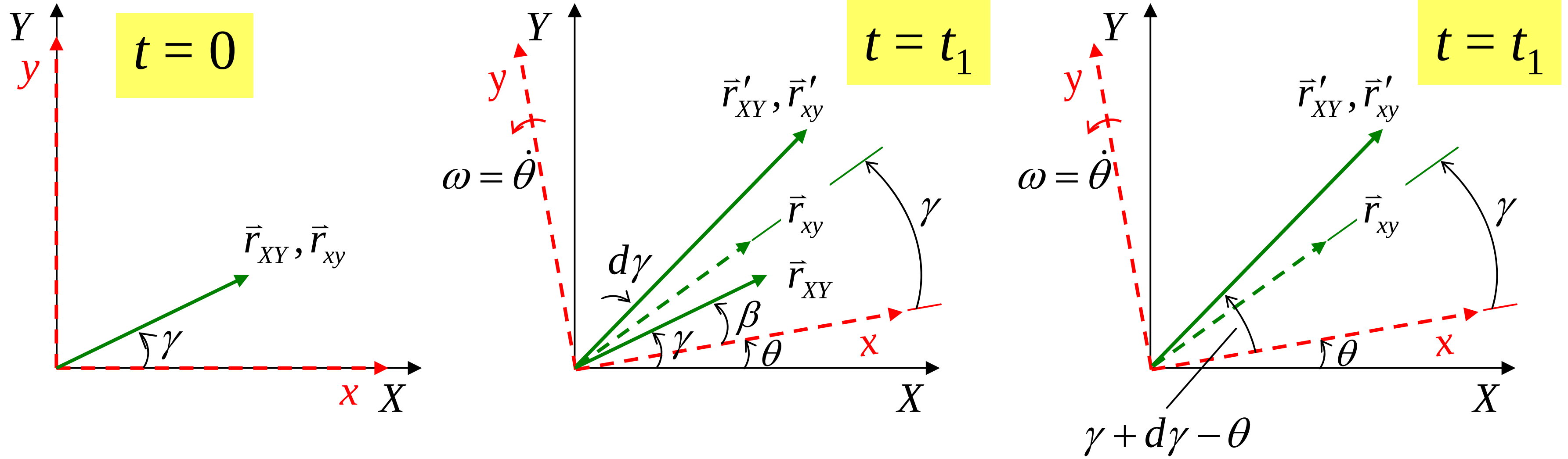
$= v_{rel}$

The velocity of *A* as measured relative to the plate.

$$\vec{v}_A = \vec{v}_B + \underline{\vec{\omega} \times \vec{r}} + \vec{v}_{rel}$$

Added from non-rotating axes

Understanding the effect of rotation



$\vec r$ = position vector

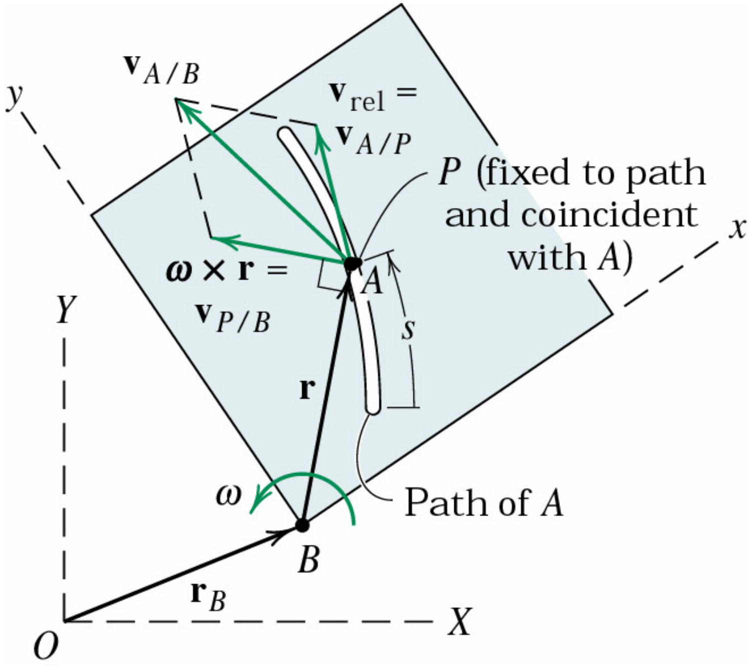
$$\left(\frac{d\vec r}{dt}\right)_{XY} = (\dot r_x \hat i + \dot r_y \hat j) + (r_x \dot{\hat i} + r_y \dot{\hat j})$$

$$\left(\frac{d\vec r}{dt}\right)_{XY} = \left(\frac{d\vec r}{dt}\right)_{xy} + \vec\omega \times \vec r$$

The effect of ω cannot be sensed by the rotating observer, therefore term $\vec\omega \times \vec M$ must be added for correction

This relation can be applied for any vector quantity

Understanding velocity components



Measured from rotating axes

$$\vec{v}_A = \vec{v}_B + \vec{\omega} \times \vec{r} + \vec{v}_{rel}$$

correction

The observer sees point A moving along path with speed v_{rel}

Measured from a nonrotating position

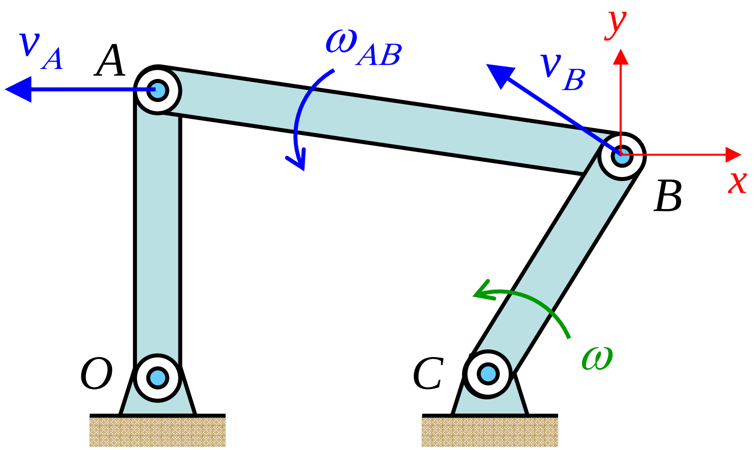
$$\vec{v}_A = \vec{v}_B + \vec{v}_{P/B} + \vec{v}_{A/P}$$

Velocity of P observed by B on the same frame = $\vec{\omega} \times \vec{r}$

An observer at P sees A moving along path

Rotating axes and non-rotating axes

Using non-rotating axes

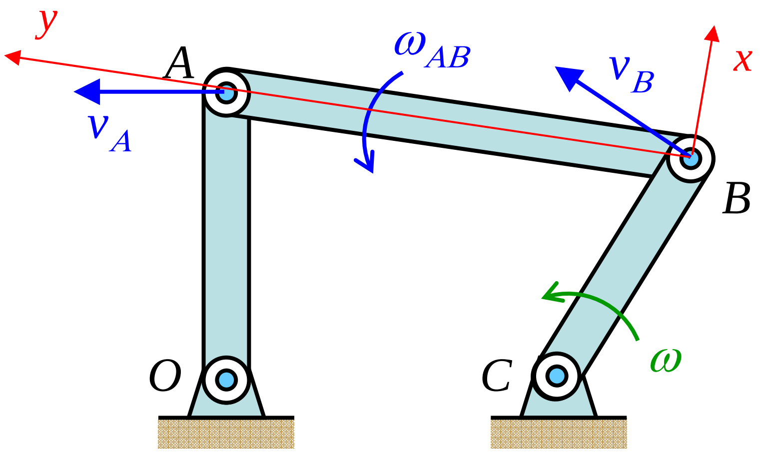


$$\vec{v}_A = \vec{v}_B + \vec{v}_{A/B}$$

$$\vec{\omega}_{AB} \times \vec{r}$$

An observer at *B* sees *A* moving in a circle around him

Using rotating axes



$$\vec{v}_A = \vec{v}_B + \vec{\omega} \times \vec{r} + \vec{v}_{rel}$$

$$= \vec{\omega}_{\text{rotating-frame}} \times \vec{r}$$

$$= \vec{\omega}_{AB} \times \vec{r}$$

$$= 0$$

A fixed with rot. frame, therefore *B* sees *A* having no motion

Relative acceleration

$$\vec{v}_A = \vec{v}_B + \vec{\omega}\times\vec{r} + \vec{v}_{rel}$$

$$\vec{a}_A = \vec{a}_B + \dot{\vec{\omega}}\times\vec{r} + \vec{\omega}\times\dot{\vec{r}} + \dot{\vec{v}}_{rel}$$

$$\dot{\vec{r}} = \frac{d}{dt}(x\hat{i} + y\hat{j}) = (x\dot{\hat{i}} + y\dot{\hat{j}}) + (\dot{x}\hat{i} + \dot{y}\hat{j})$$

$$= \vec{\omega}\times\vec{r} + v_{rel}$$

- -

$$\vec{\omega}\times\dot{\vec{r}} = \vec{\omega}\times(\vec{\omega}\times\vec{r} + v_{rel})$$

$$= \vec{\omega}\times(\vec{\omega}\times\vec{r}) + \vec{\omega}\times v_{rel}$$

$$\dot{\vec{v}}_{rel} = \frac{d}{dt}(\dot{x}\hat{i} + \dot{y}\hat{j})$$

$$= (\dot{x}\dot{\hat{i}} + \dot{y}\dot{\hat{j}}) + (\ddot{x}\hat{i} + \ddot{y}\hat{j})$$

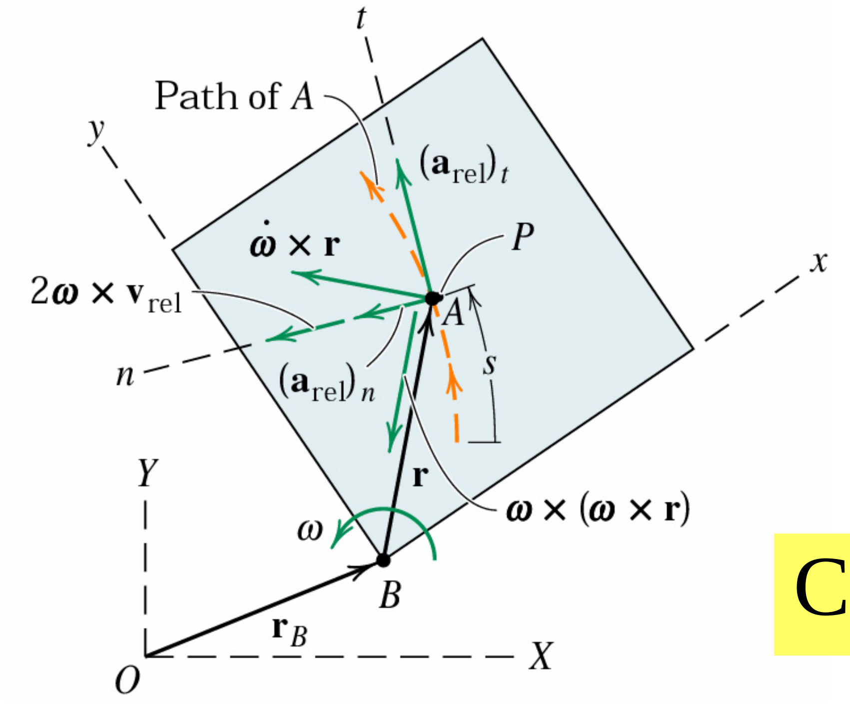
$$= \vec{\omega}\times(\dot{x}\hat{i} + \dot{y}\hat{j}) + (\ddot{x}\hat{i} + \ddot{y}\hat{j})$$

$$= \vec{\omega}\times\vec{v}_{rel} + \vec{a}_{rel}$$

$$\vec{a}_A = \vec{a}_B + \dot{\vec{\omega}}\times\vec{r} + \vec{\omega}\times(\vec{\omega}\times\vec{r}) + 2\vec{\omega}\times\vec{v}_{rel} + \vec{a}_{rel}$$

$\vec{a}_{rel}$ ➡ The velocity of *A* as measured relative to the plate.

Understanding acc. components



To understand each acceleration component, equations written for rotating and nonrotating axes are compared together.

Coriolis acc.?

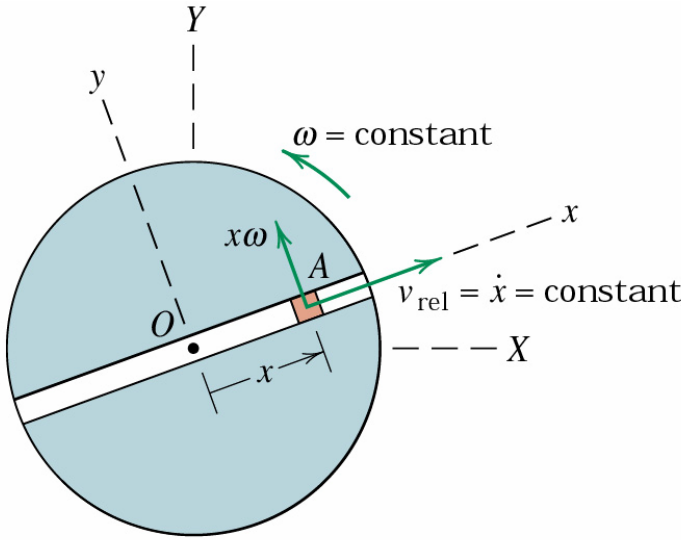
$$\boxed{\begin{aligned}(\vec{a}_{rel})_t &= \ddot{s}\\ (\vec{a}_{rel})_n &= v_{rel}^2/\rho\end{aligned}}$$

Rot. $$\vec{a}_A = \vec{a}_B + \underbrace{\dot{\vec{\omega}} \times \vec{r} + \vec{\omega} \times (\vec{\omega} \times \vec{r})} + \underbrace{2\vec{\omega} \times \vec{v}_{rel} + \vec{a}_{rel}}$$

Nonrot. $$\vec{a}_A = \vec{a}_B + \vec{a}_{P/B} + \vec{a}_{A/P}$$

Coriolis acceleration (1)

Simple example for understanding coriolis acc.



velocity

$$\vec v_A = \cancelto{0}{\vec v_O} + \vec\omega\times\vec r + \vec v_{rel}$$

$$\vec v_A = \omega x\hat j + \dot x\hat i$$

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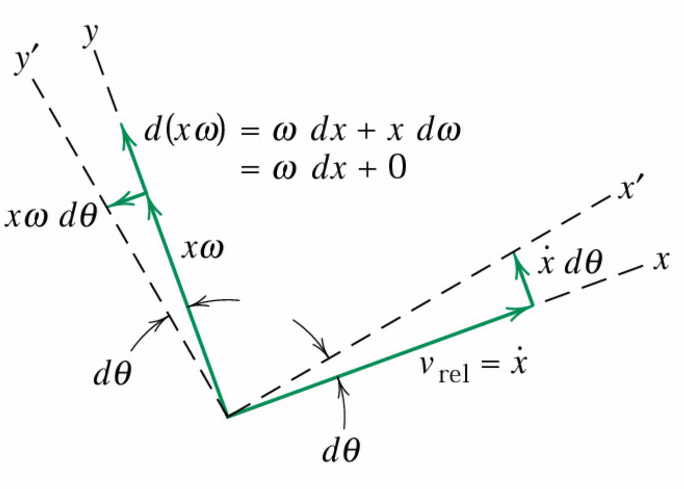
acceleration

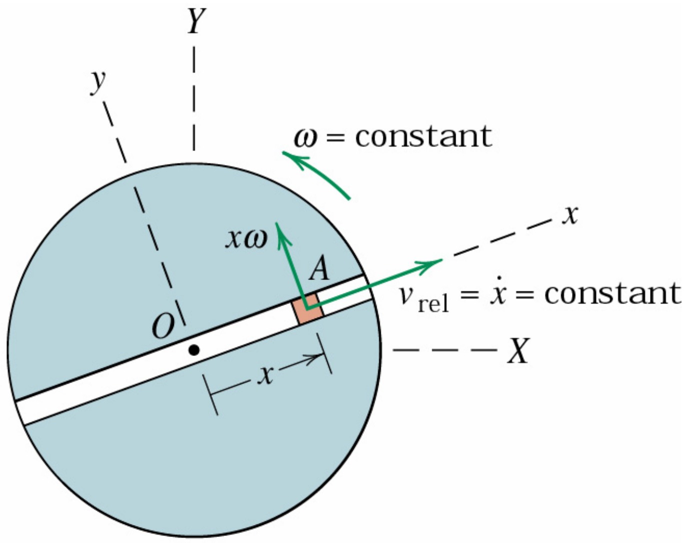
$$\vec a_A = \cancelto{0}{\vec a_B} + \cancelto{0}{\dot{\vec\omega}\times\vec r} + \vec\omega\times(\vec\omega\times\vec r) + 2\vec\omega\times\vec v_{rel} + \cancelto{0}{\vec a_{rel}}$$

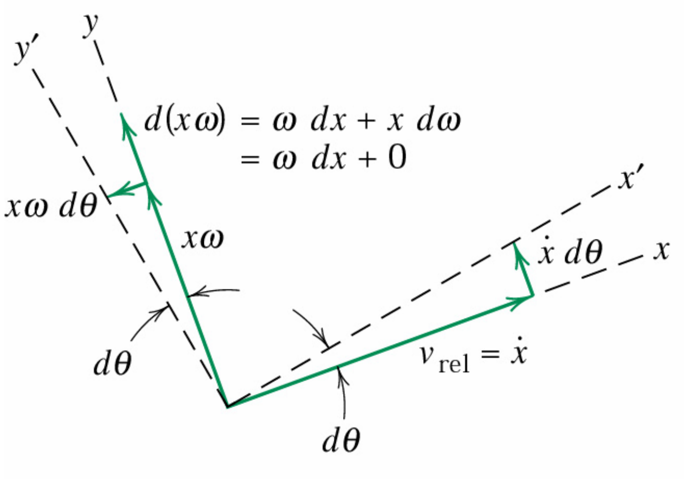
$$\boxed{a_A = 2\vec\omega\times\vec v_{rel} + \vec\omega\times(\vec\omega\times\vec r)}$$

$$\underbrace{\qquad\qquad}_{\text{Coriolis acc.}}$$



Coriolis acceleration (2)





acceleration

From the figure, velocity change:

- Change in magnitude of v_{rel} = 0
- Change in direction of $v_{rel} = \dot{x} d\theta$
- Change in magnitude of $x\omega = \omega\, dx$
- Change in direction of $x\omega = x\omega\, d\theta$

$$dv_A = (\dot{x} d\theta + \omega dx)\hat{j} - (x\omega\, d\theta)\hat{i}$$

$$a_A = (\dot{x}\omega + \omega\dot{x})\hat{j} - (x\omega^2)\hat{i}$$

$$\boxed{a_A = 2\vec{\omega}\times\vec{v}_{rel} + \vec{\omega}\times(\vec{\omega}\times\vec{r})}$$

Coriolis acc.

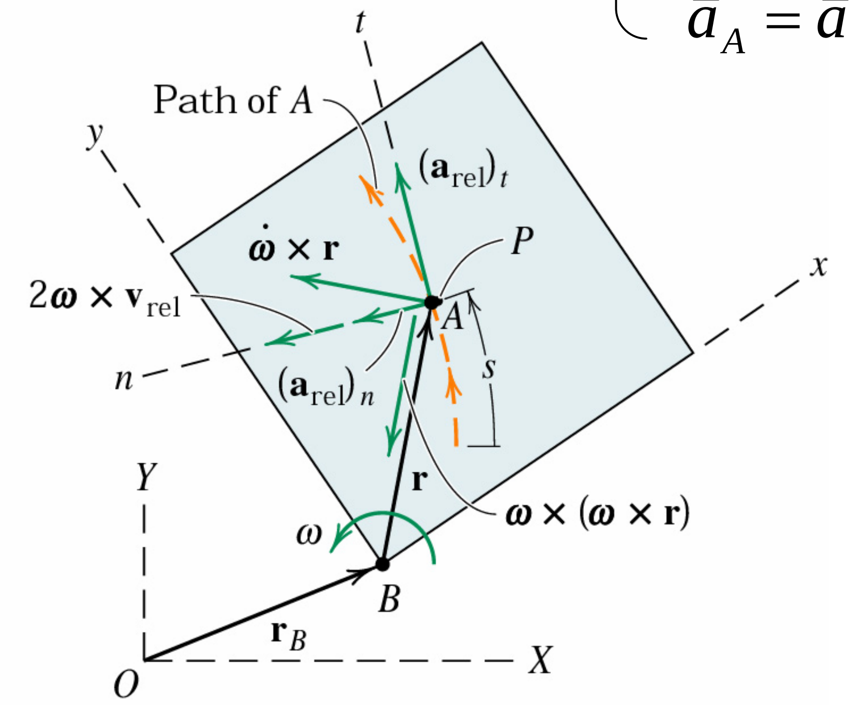
Rotating vs. nonrotating systems

Rot.

$$\vec{a}_A = \vec{a}_B + \underbrace{\dot{\vec{\omega}}\times\vec{r} + \vec{\omega}\times(\vec{\omega}\times\vec{r})} + \underbrace{2\vec{\omega}\times\vec{v}_{rel} + \vec{a}_{rel}}$$

Nonrot.

$$\left\{\begin{aligned} \vec{a}_A &= \underbrace{\vec{a}_B + \vec{a}_{P/B}} + \vec{a}_{A/P} \\ \vec{a}_A &= \underbrace{\vec{a}_P + \vec{a}_{A/P}} \\ \vec{a}_A &= \vec{a}_B + \vec{a}_{A/B} \end{aligned}\right.$$



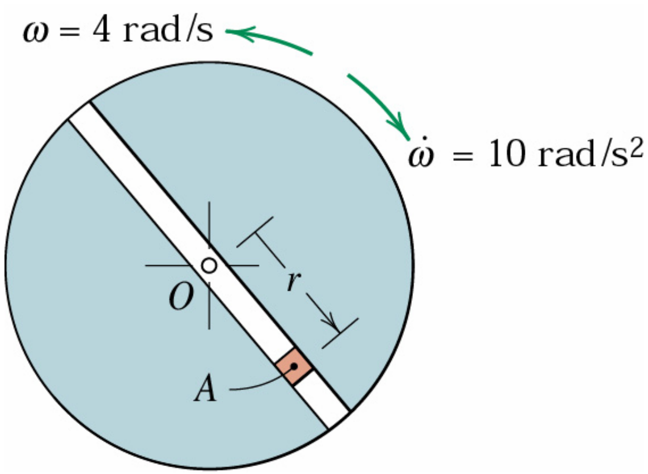
Another expression

$$\boxed{\vec{a}_A = \vec{a}_P + 2\vec{\omega}\times\vec{v}_{rel} + \vec{a}_{rel}}$$

Point *P* is attached to the rotating reference and is coincident with *A*

Sample problem 5/16

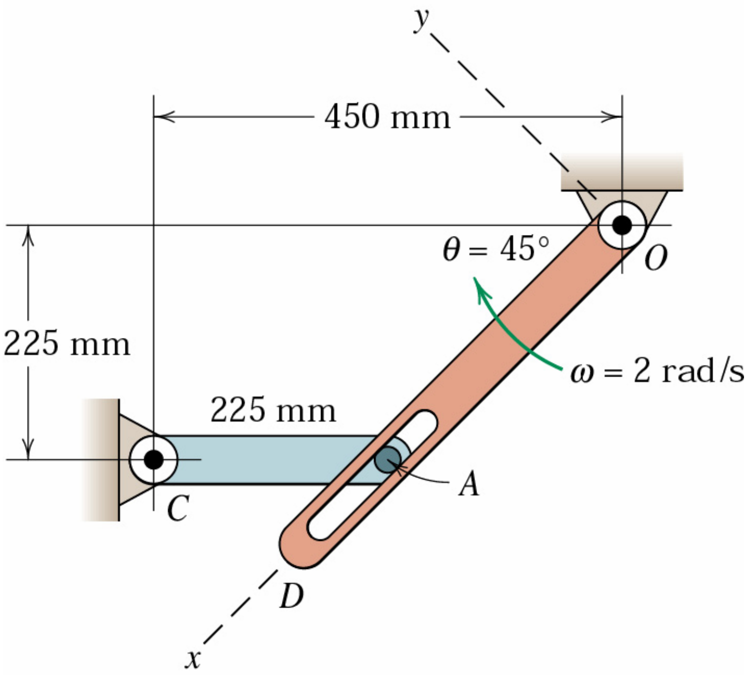
At the instant represented, the disk with the radial slot is rotating about *O* with a CCW angular velocity of 4 rad/s which is decreasing at the rate of 10 rad/s^2. The motion of slider *A* is separately controlled, and at this instant, r = 150 mm. $\dot{r}$ = 125 mm/s, and $\ddot{r}$ = 2025 mm/s^2. Determine the absolute velocity and acceleration of *A* for this position.



Sample problem 5/17

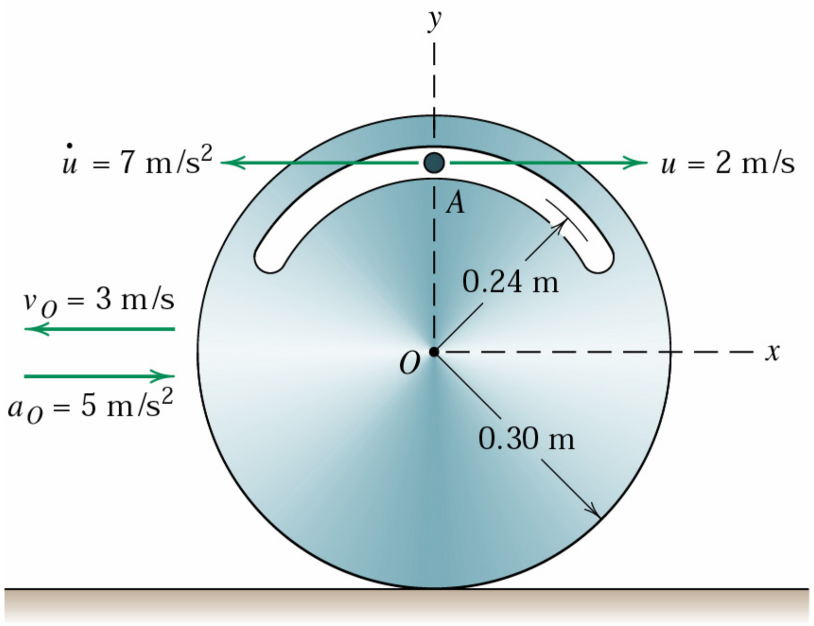
The pin *A* of the hinged link *AC* is confined to move in the rotating slot of link *OD*. The angular velocity of *OD* is $\omega = 2$ rad/s CW and is constant for the interval of motion concerned. For the position where $\theta = 45°$ with *AC* horizontal, determine the velocity of pin *A* and the velocity of *A* relative to the rotating slot in *OD*.

For this condition, determine the angular acceleration of *AC* and the acceleration of *A* relative to the rotating slot in arm *OD*.



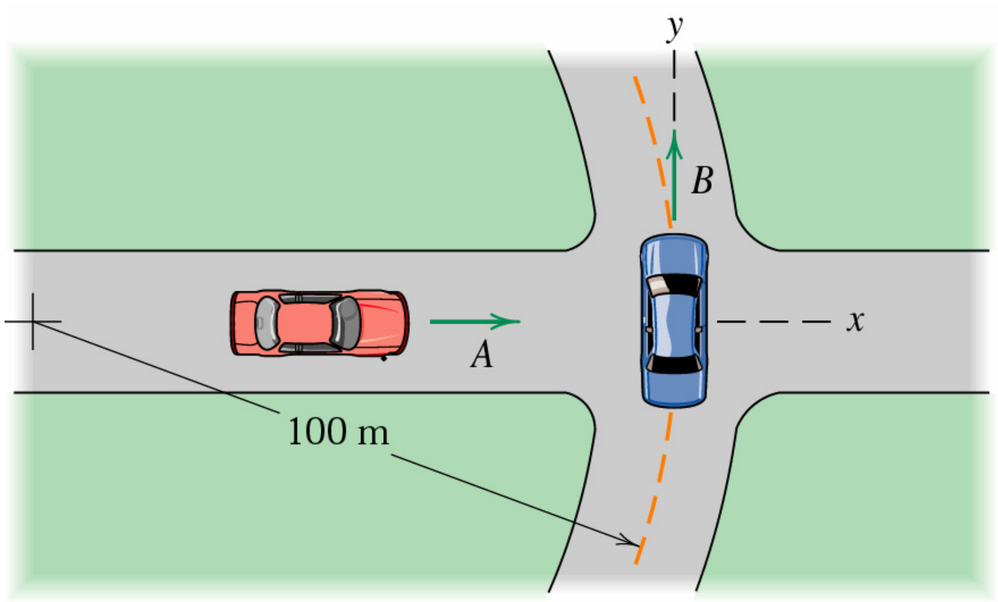
Sample 3 (5/153)

The disk rolls without slipping on the horizontal surface, and at the instant represented, the center *O* has the velocity and acceleration shown in the figure. For this instant, the particle *A* has the indicated speed u and time-rate-of-change of speed $\dot{u}$, both relative to the disk. Determine the absolute velocity and acceleration of particle *A*.



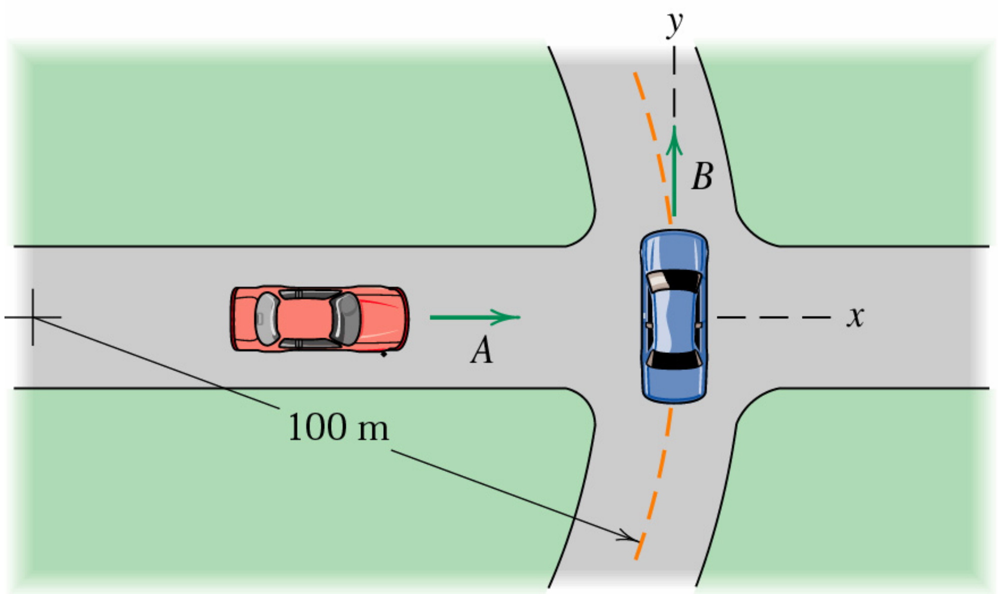
Sample 4 (5/156)

Car *B* is rounding the curve with a constant speed of 54 km/h, and car *A* is approaching car *B* in the intersection with a constant speed of 75 km/h. Determine the velocity which car *A* appears to have to an observer riding in and turning with car *B*. The *x*-*y* axes are attached to car *B*. Is this apparent velocity the negative of the velocity which *B* appears to have to a nonrotating observer in car *A*? The distance separating the two cars at the instant depicted is 40 m.

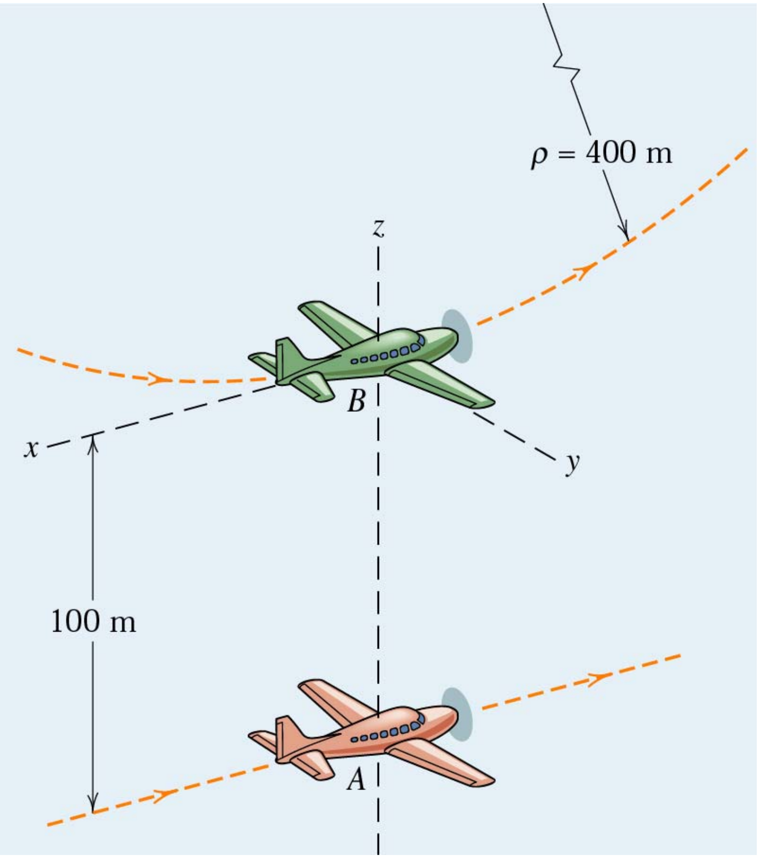


Sample 5 (5/157)

For the car of Prob. 5/156 traveling with constant speed, determine the acceleration with car *A* appears to have to an observer riding in and turning with car *B*.



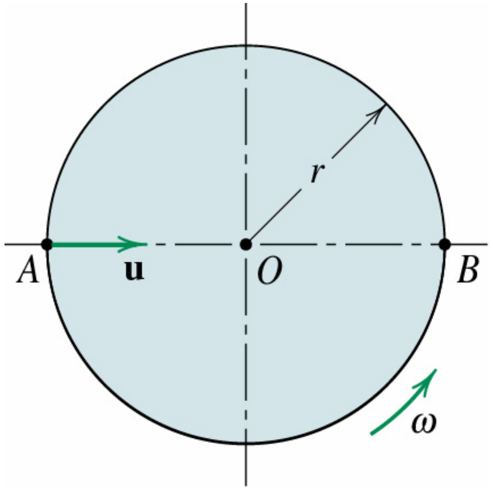
Sample 6 (5/159)



Aircraft *B* has a constant speed of 540 km/h at the bottom of a circular loop of 400-m radius. Aircraft *A* flying horizontally in the plane of the loop passes 100 m directly under *B* at a constant speed of 360 km/h. With coordinate axes attached to *B* as shown, determine the acceleration which *A* appears to have to the pilot of *B* for this instant.

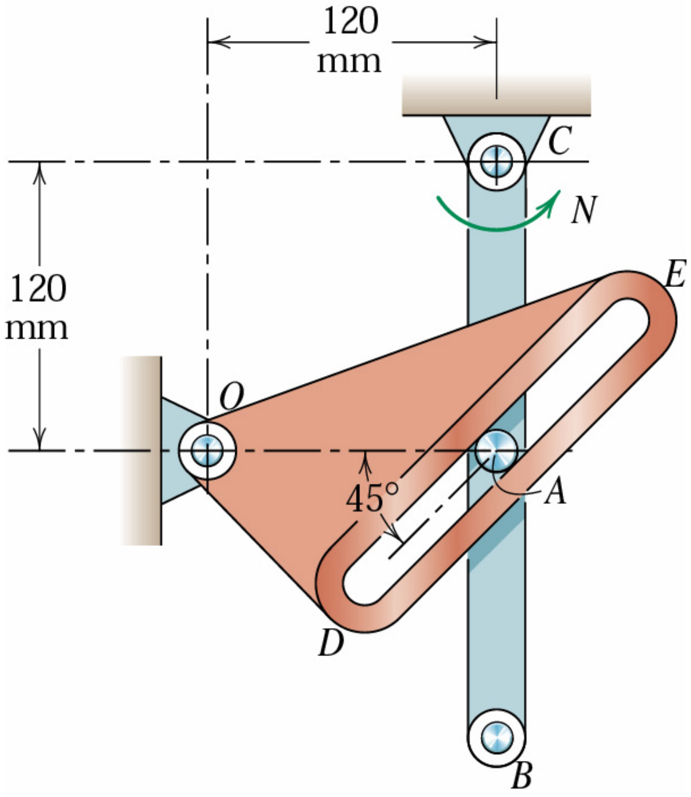
Sample 7 (5/161)

Two boys *A* and *B* are sitting on opposite sides of a horizontal turntable which rotates at a constant CCW angular velocity ω as seen from above. Boy *A* throws a ball toward *B* by giving it a horizontal velocity **u** relative to the turntable toward *B*. Assume that the ball has no horizontal acceleration once released and write an expression for the acceleration $\mathbf{a}_{\text{rel}}$ which *B* would observe the ball to have in the plane of the turntable just after it is thrown. Sketch the path of the ball on the turntable as observed by *B*.



Sample 8 (5/168)

For the instant represented, link *CB* is rotating CCW at a constant rate $N = 4$ rad/s, and its pin *A* causes a CW rotation of the slotted member *ODE*. Determine the angular velocity ω and angular acceleration α of *ODE* for this instant.



Sample 9 (5/173)

Link *OA* has a constant CW angular velocity of 3 rad/s for a brief interval of its rotation. Determine the angular acceleration α_{BC} of *BC* for the instant when $\theta = 60°$. First use a rotating-frame analysis, and then verify your result with an absolute-motion approach.