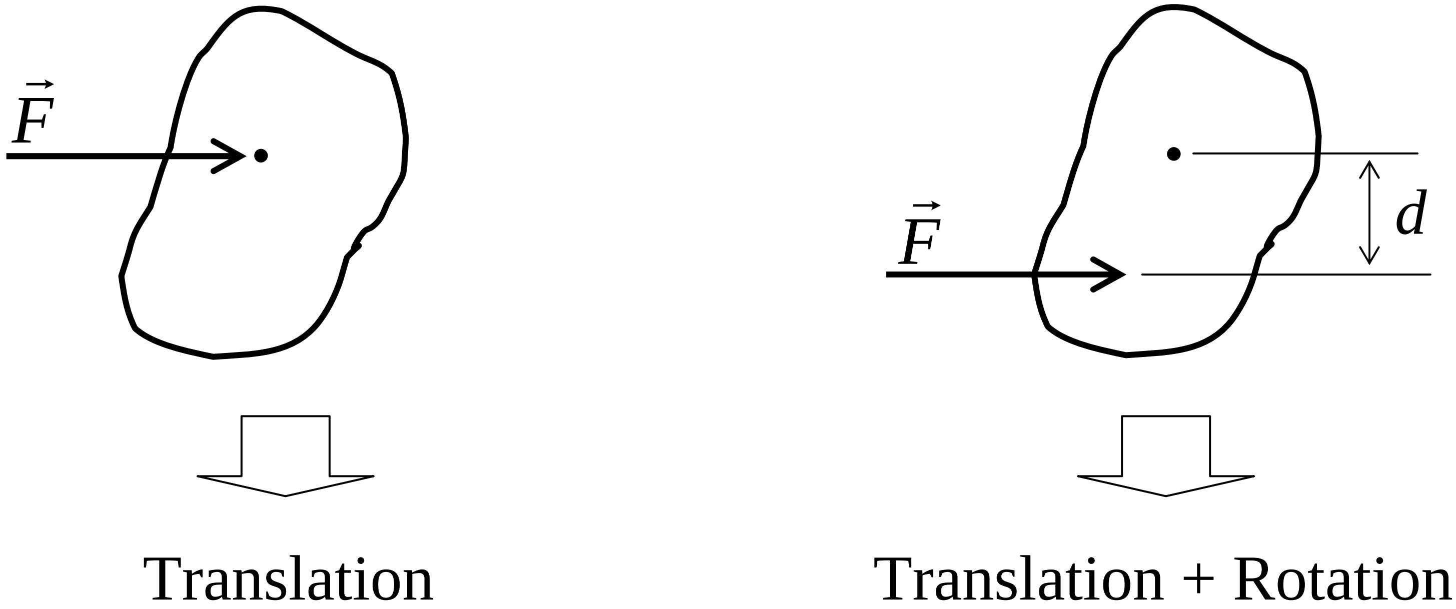
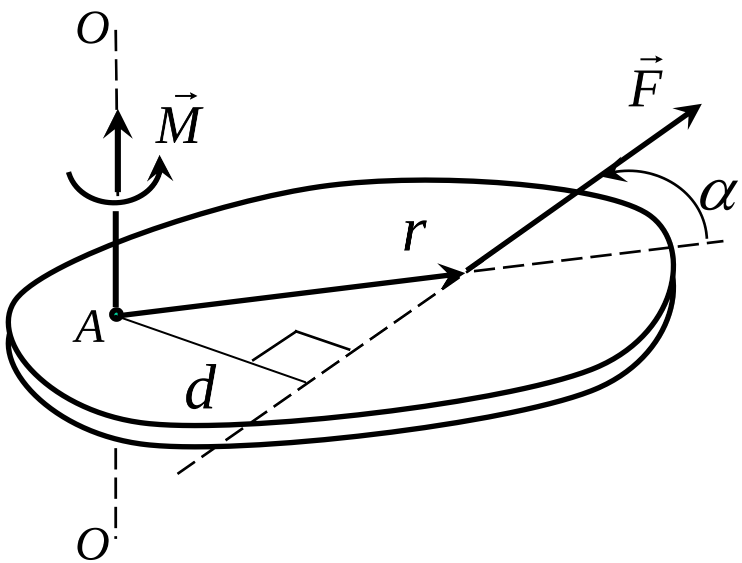
Moment



- This rotation tendency is known as *moment* $\vec{M}$ of force (*torque*)
- Axis of rotation may be any line which neither intersects nor parallel to the line of action of force
- Magnitude of moment depends on magnitude of $\vec{F}$ and the length d

Mathematical definition



Moment about axis *O*-*O* is defined as

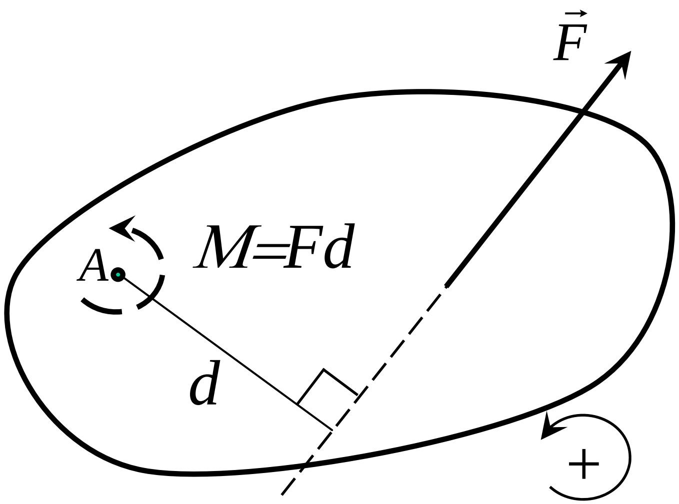
$$\boxed{M = Fd} \quad \text{(N.m)}$$

Moment is a *vector*

Direction, normal to *r*-*F* plane (right hand rule)

Axis *O*-*O* is called *moment axis*

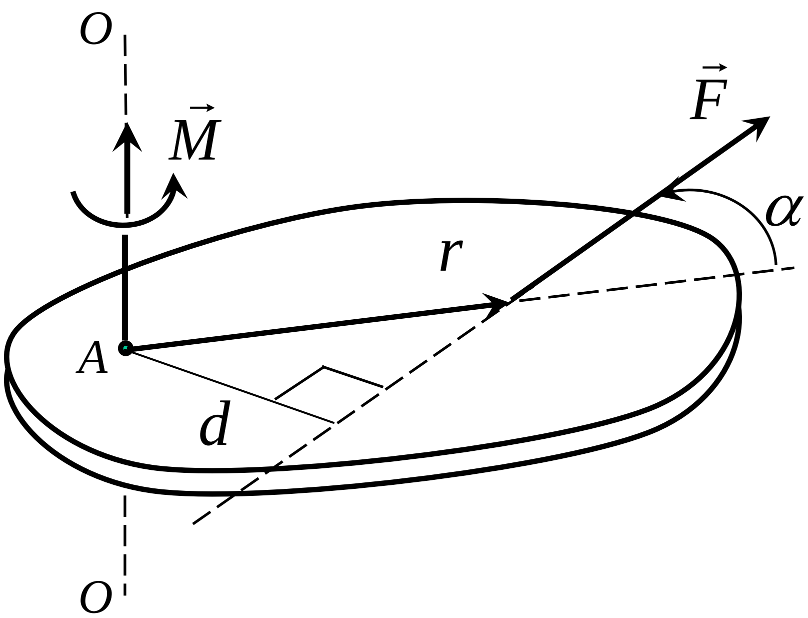
Moment is a *sliding vector*



2-D

- Axis becomes point
- Use sign convention to express direction (+ for CCW, – for CW)

The cross product



The moment of $\vec{F}$ about point A =

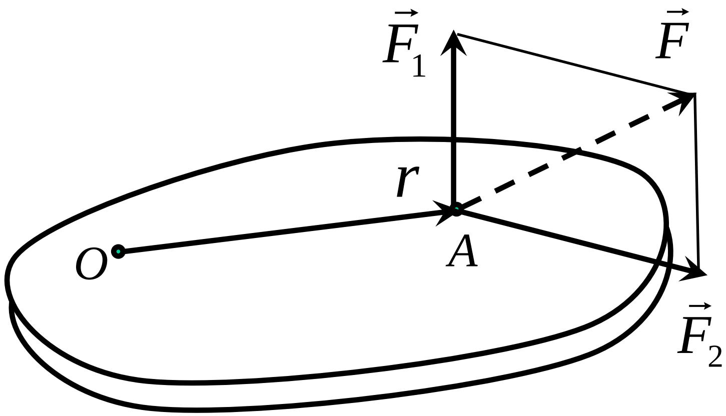
$$\vec{M} = \vec{r} \times \vec{F}$$

Magnitude $M = Fr\sin(\alpha) = Fd$

- Direction: normal to the $r - F$ plane, right hand rule
- *xyz* axis have to satisfy the right hand rule; $\hat{i} \times \hat{j} = \hat{k}$
- Sequence of ***r*** and ***F*** is important; $\vec{r} \times \vec{F} \neq \vec{F} \times \vec{r}$

Varignon's theorem

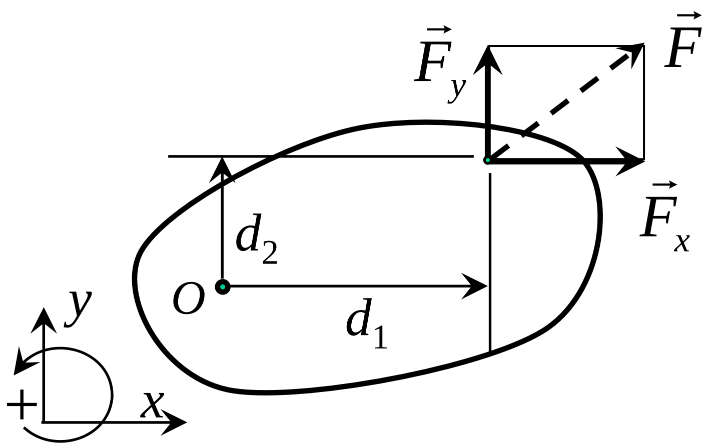
The moment of a force about any point = The moment of the <u>components of the force</u> about the same point



$$\vec{F} = \vec{F}_1 + \vec{F}_2$$

$$\vec{M}_o = \vec{r} \times \vec{F} = \vec{r} \times (\vec{F}_1 + \vec{F}_2)$$

$$\boxed{\vec{M}_o = \vec{r} \times \vec{F}_1 + \vec{r} \times \vec{F}_2}$$

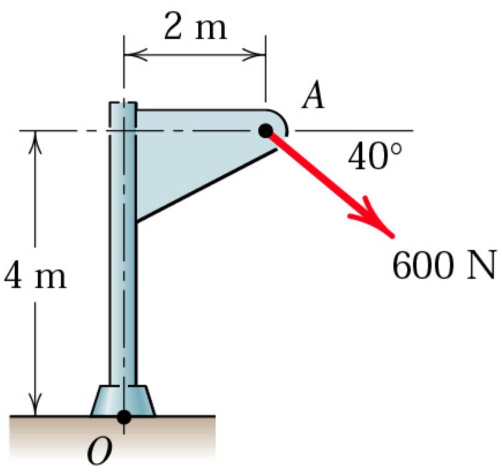


Useful with rectangular components

$$M_o = F_x d_2 - F_y d_1$$

Can use with more than 2 components

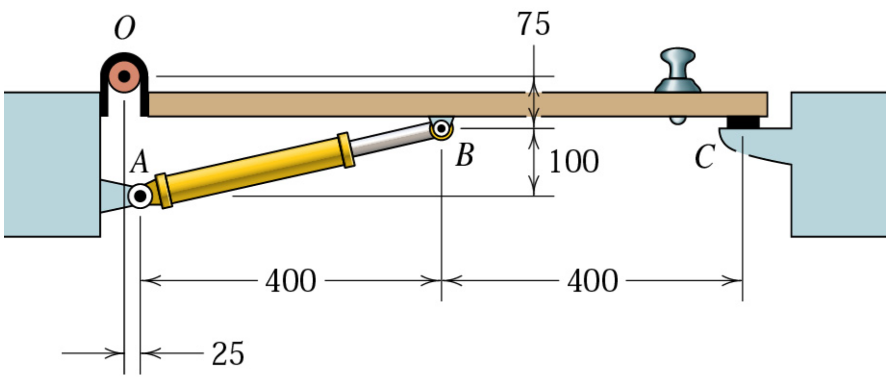
Sample (1)



Calculate the magnitude of the moment about the base point *O* of the 600-N force.

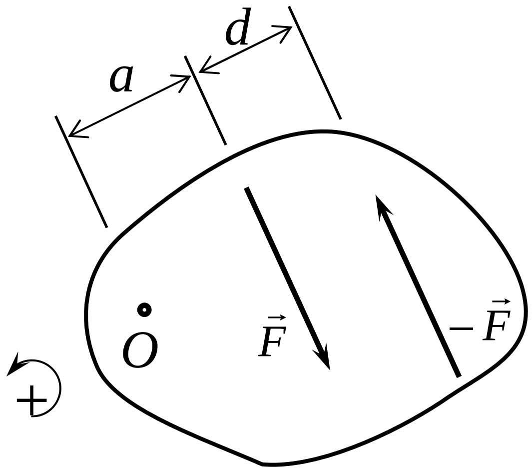
Sample (2)

The force exerted by the plunger of cylinder *AB* on the door is 40 N directed along the line *AB*, and this force tends to keep the door closed. Compute the moment of this force about the hinge *O*. What force F_c normal to the plane of the door must the door stop at *C* exert on the door so that the combined moment about *O* of the two forces is zero?



Dimensions in millimeters

Couple (1)



Couple is a *moment* produced by two equal, opposite, and noncollinear forces.

$$M = F(a+d) - Fa = Fd$$

The moment of a couple has the same value for all moment centers

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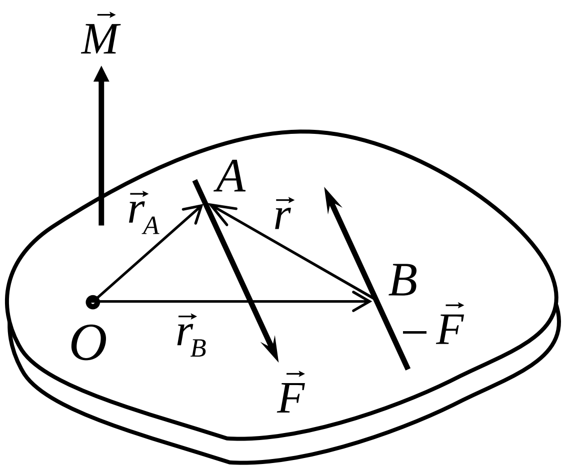
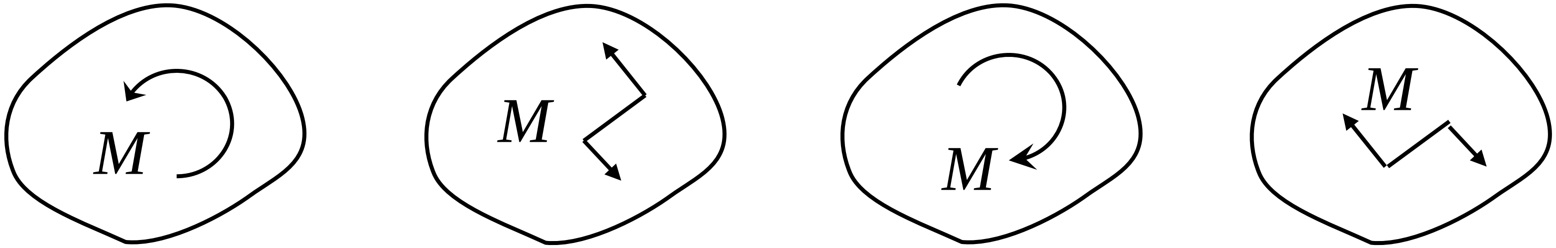


$$\vec{M} = \vec{r}_A \times \vec{F} + \vec{r}_B \times (-\vec{F}) = (\vec{r}_A - \vec{r}_B) \times \vec{F}$$

$$\vec{M} = \vec{r} \times \vec{F}$$

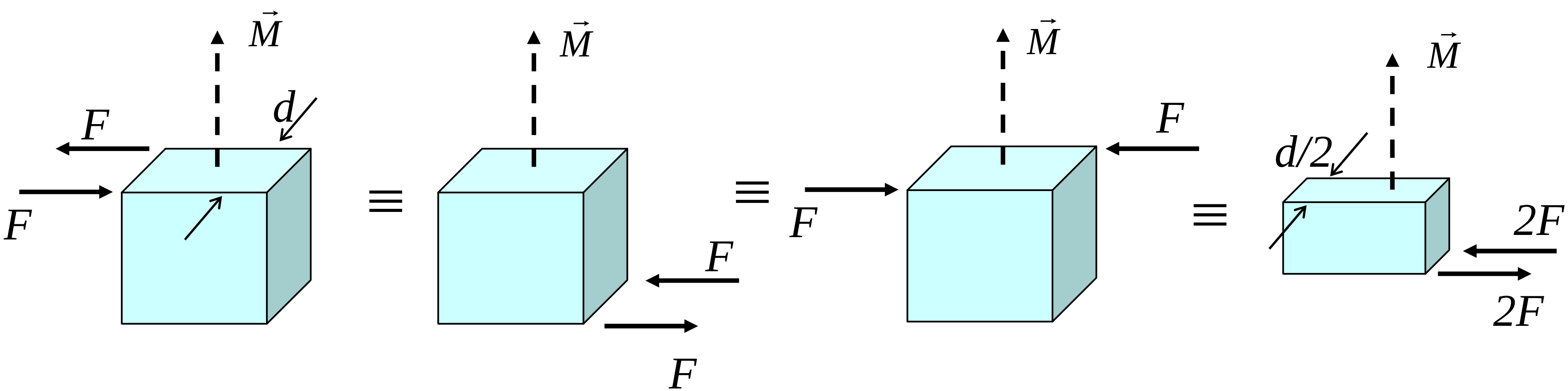
- Couple may be represented as a *free vector*
- Direction of couple is normal to the plane of two force

Couple (2)



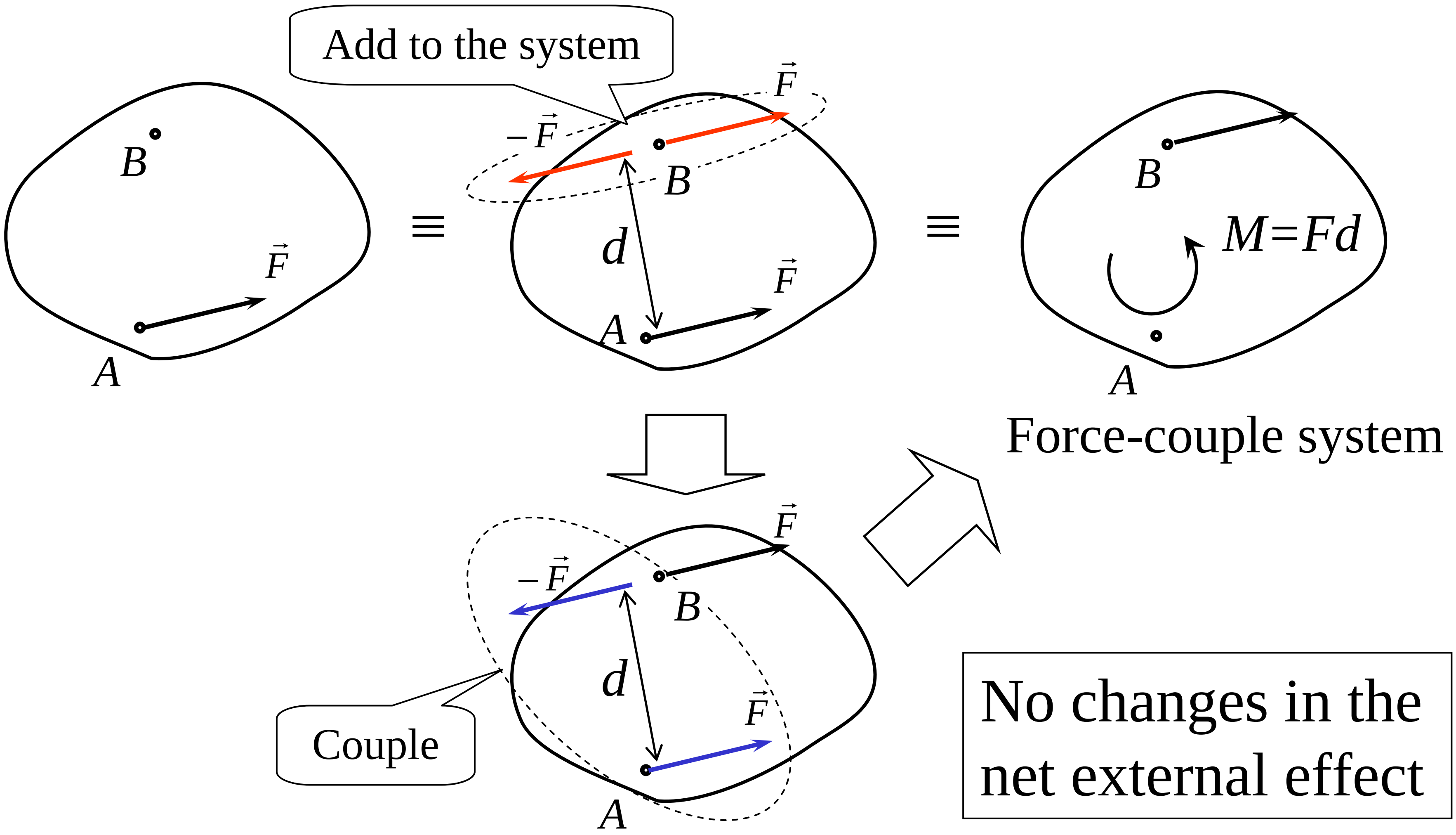
CCW couple CW couple

Since couple is a free vector, the followings are equivalent couples



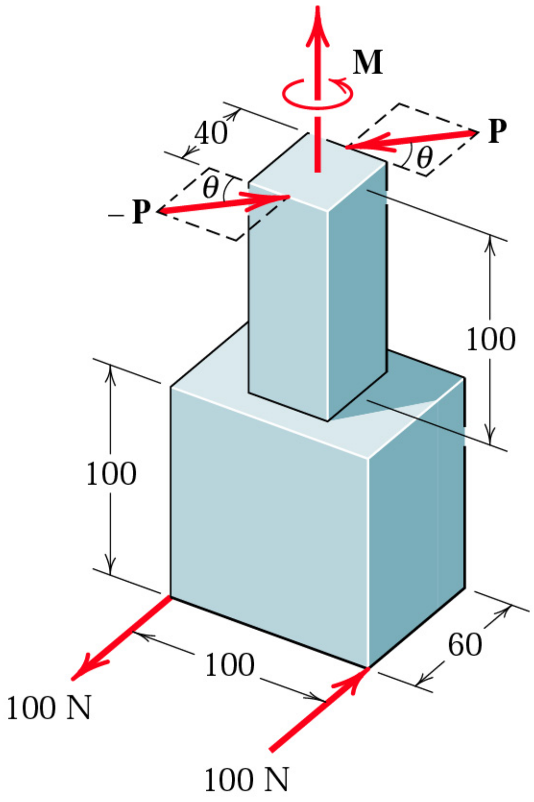
Force-couple systems

A given force can be replaced by an equal parallel force and a couple.



No changes in the net external effect

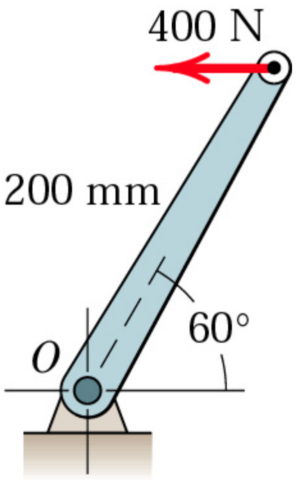
Sample (3)



Dimensions in millimeters

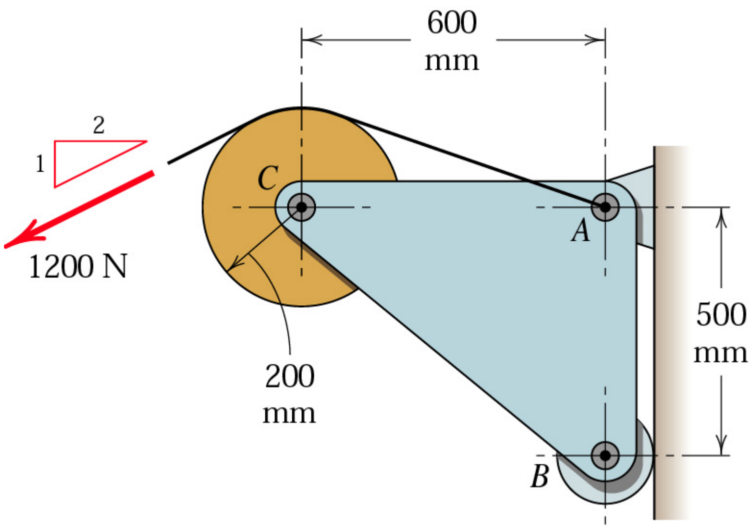
The rigid structural member is subjected to a couple consisting of the two 100-N forces. Replace this couple by an equivalent couple consisting of the two forces **P** and **–P**, each of which has a magnitude of 400 N. Determine the proper angle θ.

Sample (4)



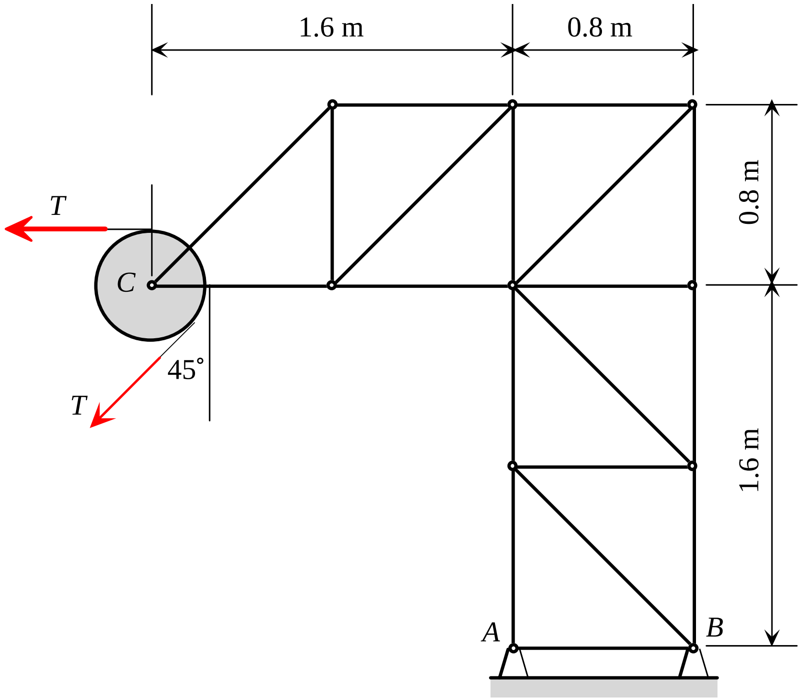
Replace the horizontal 400-N force acting on the lever by an equivalent system consisting of a force at *O* and a couple.

Sample (5)



Calculate the moment of the 1200-N force about pin *A* of the bracket. Begin by replacing the 1200-N force by a force-couple system at point *C*.

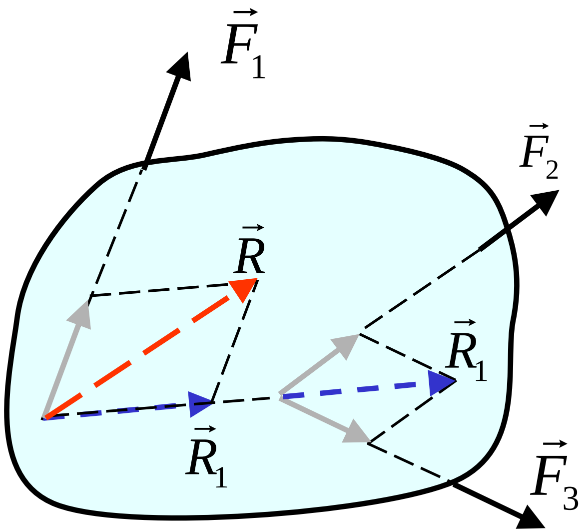
Sample (6)



Determine the combined moment M_A about point A due to the two equal tensions $T = 8$ kN in the cable acting on the pulley. Is it necessary to know the pulley diameter?

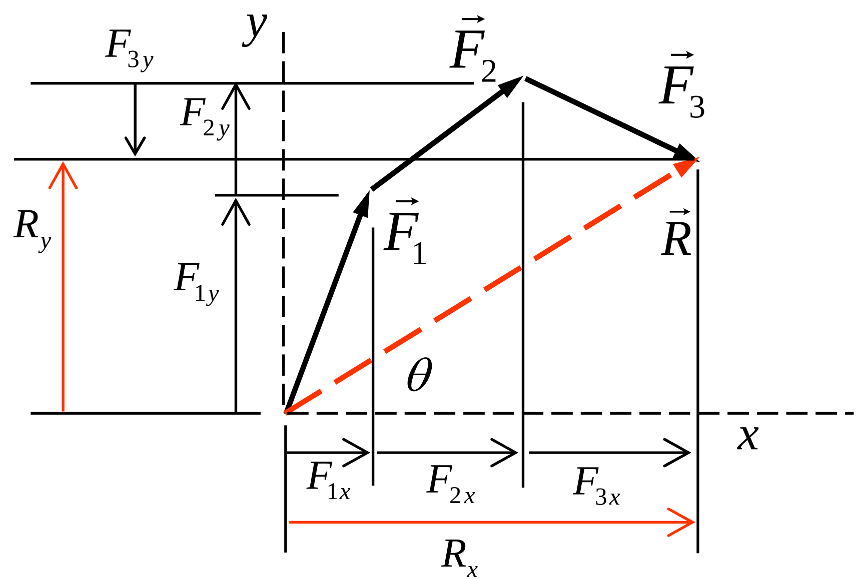
Resultants

The **Resultant** is the <u>simplest force combination</u> which can replace the original forces <u>without</u> altering the external effect on the body



(1) $\vec{F}_2 + \vec{F}_3 = \vec{R}_1$

(2) $\vec{R}_1 + \vec{F}_1 = \vec{R}$



$$\vec{R} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \ldots = \sum \vec{F}$$

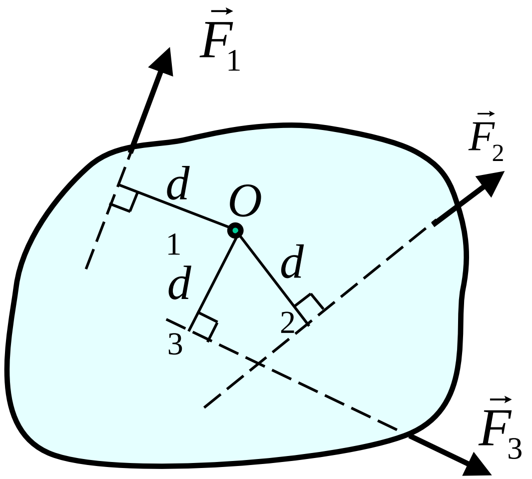
$$R_x = \sum F_x, \quad R_y = \sum F_y$$

$\longrightarrow$

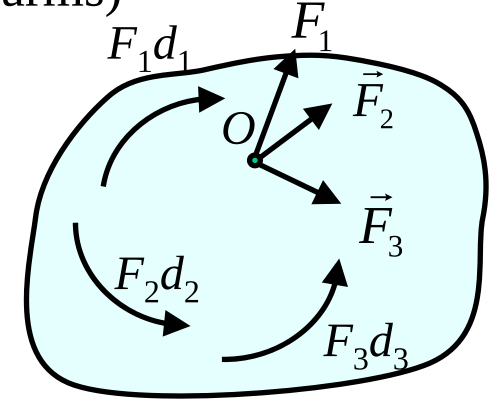
$$R = \sqrt{(\sum F_x)^2 + (\sum F_y)^2}$$

$$\theta = \tan^{-1}(R_y / R_x)$$

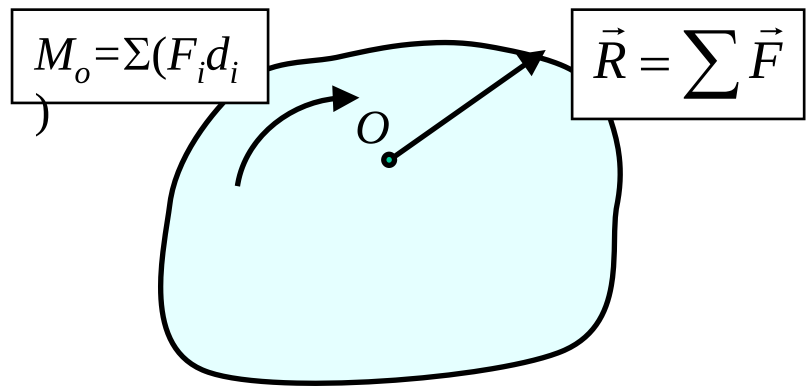
Method to get a resultant



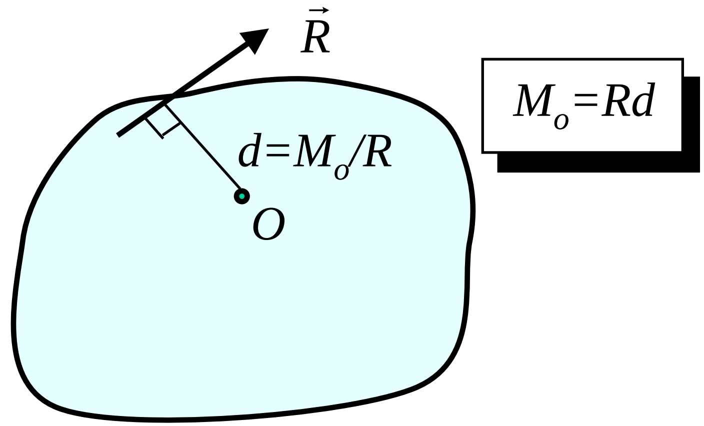
1) Pick a point (easy to find moment arms)



2) Replace each force with a force at point O + a couple

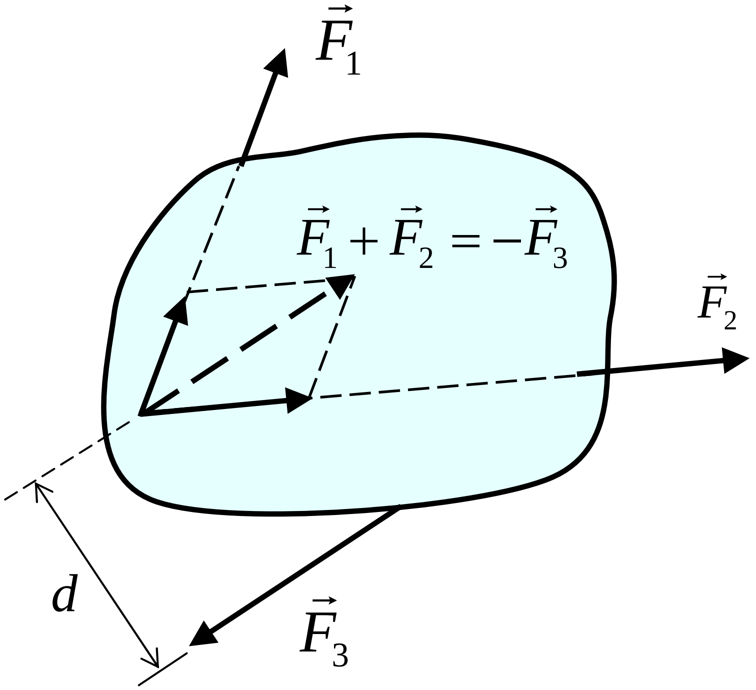


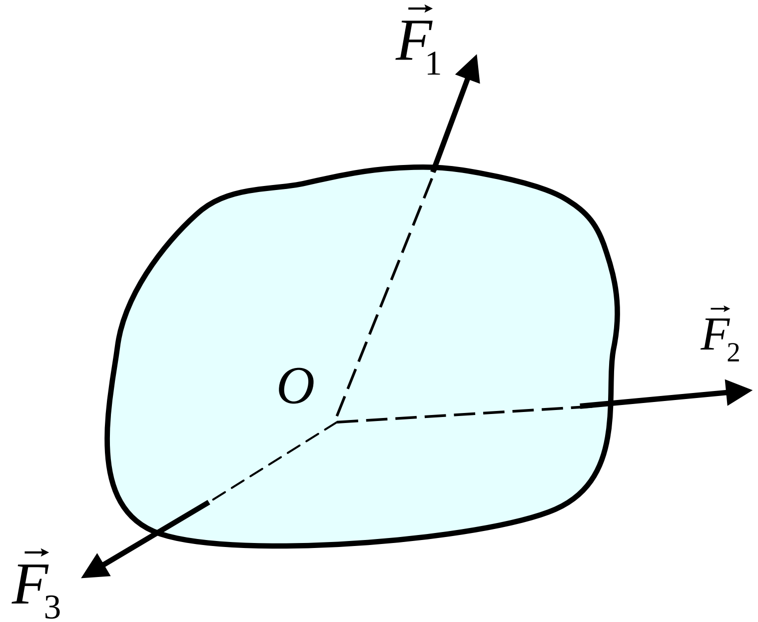
3) Add forces and moments



4) Replace force-couple system with a single force

Other cases





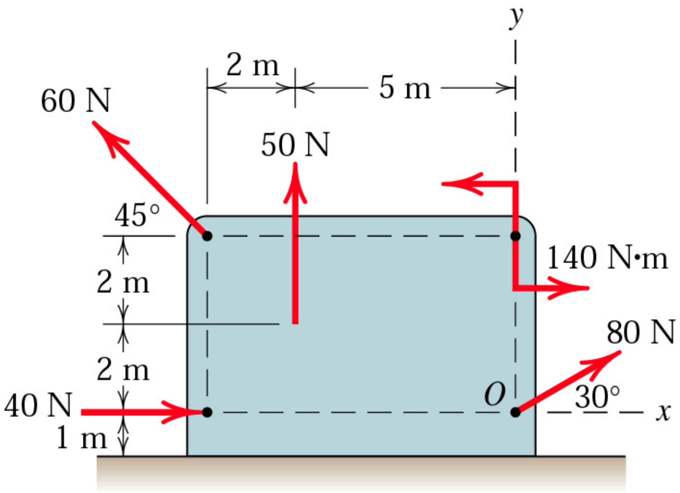
$$\sum \vec{F} = 0$$

$$R = M_O = F_3 \cdot d$$

$$\sum \vec{M}_O = 0$$

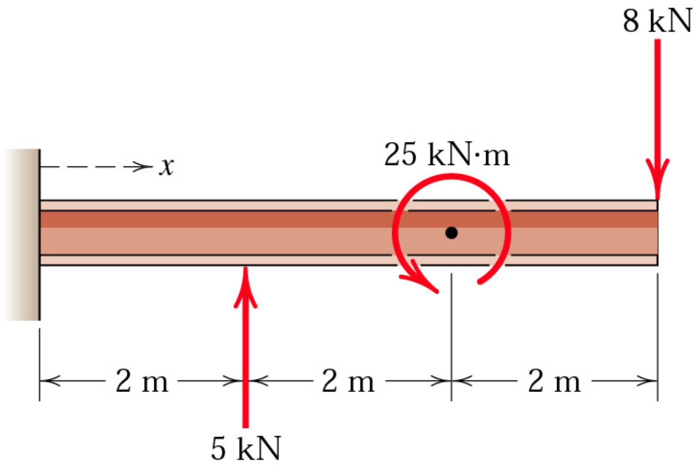
$$\vec{R} = \sum \vec{F}$$

Sample (7)



Determine the resultant of the four forces and one couple that act on the plate shown.

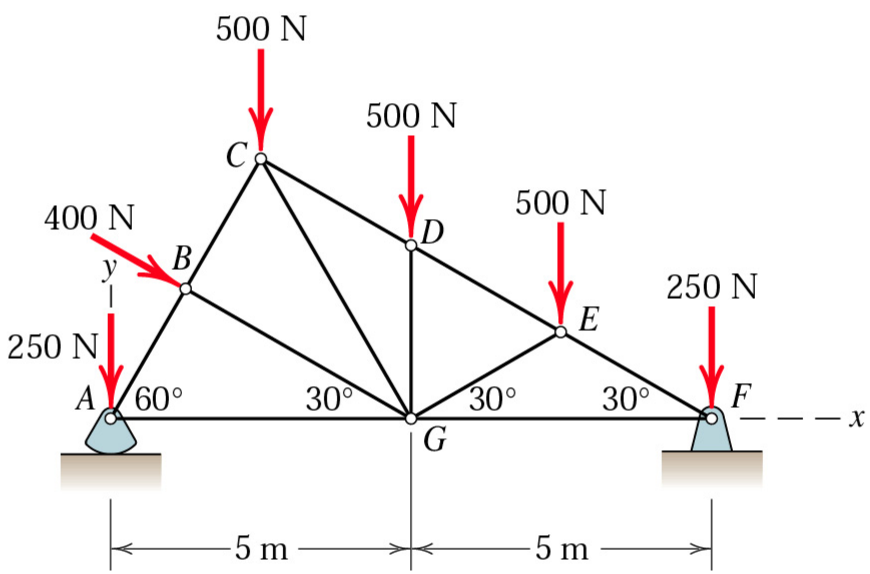
Sample (8)



Determine and locate the resultant **R** of the two force and one couple acting on the I-beam.

Sample (9)

The five vertical loads represent the effect of the weights of the truss and supported roofing materials. The 400-N load represents the effect of wind pressure. Determine the equivalent force-couple system at *A*. Also, compute the *x*-intercept of the line of action of the system resultant treated as a single force **R**.



Sample (10)

Replace the three forces acting on the bent pipe by a single equivalent force **R**. Specify the distance *x* from point *O* to the point on the *x*-axis through which the line of action of **R** passes.