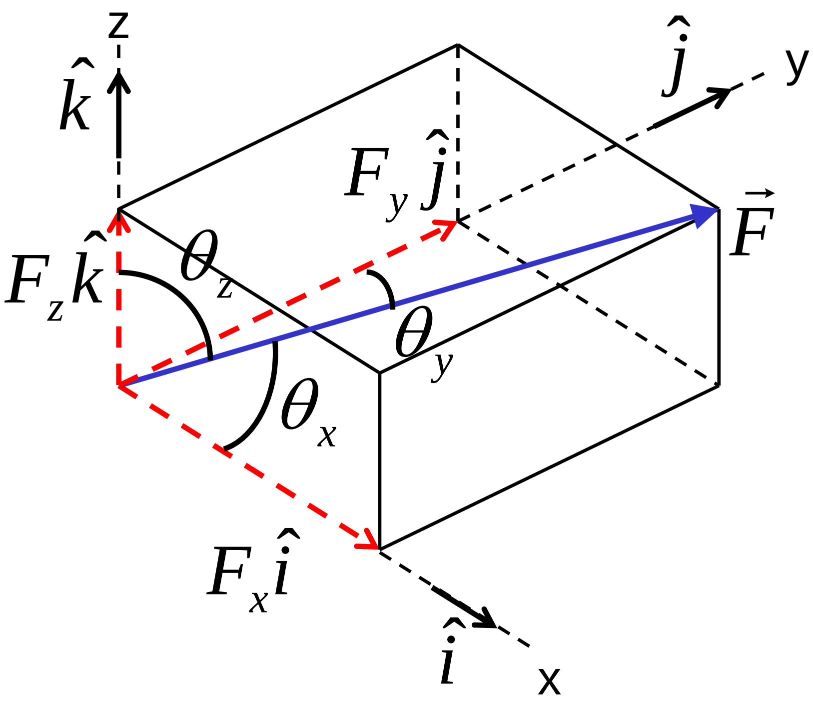
# Rectangular components




$$F_x = F\cos(\theta_x)$$
$$F_y = F\cos(\theta_y)$$
$$F_z = F\cos(\theta_z)$$
$$F = \sqrt{F_x^2 + F_y^2 + F_z^2}$$

$$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$$
$$\vec{F} = F(\cos\theta_x\hat{i} + \cos\theta_y\hat{j} + \cos\theta_z\hat{k})$$

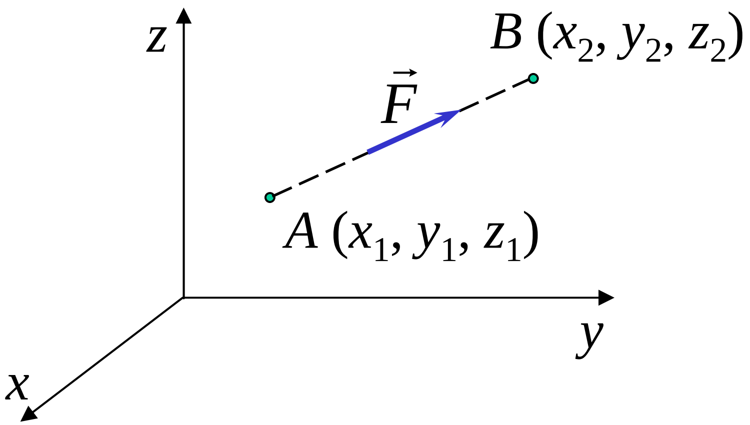
$$\vec{F} = F(l\hat{i} + m\hat{j} + n\hat{k})$$
$$\vec{F} = F\hat{n}_F$$

*Direction cosine*

$$l = \cos\theta_x \quad m = \cos\theta_y \quad n = \cos\theta_z$$
$$l^2 + m^2 + n^2 = 1$$

# Writing a vector in 3D (1)

**1 Specification by two points on the line of action**




$$\vec{F} = F\hat{n}_F = F\frac{\vec{r}_{AB}}{|\vec{r}_{AB}|}$$

$$\vec{r}_{AB} = \vec{r}_B - \vec{r}_A$$

$$\vec{r}_A = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$$

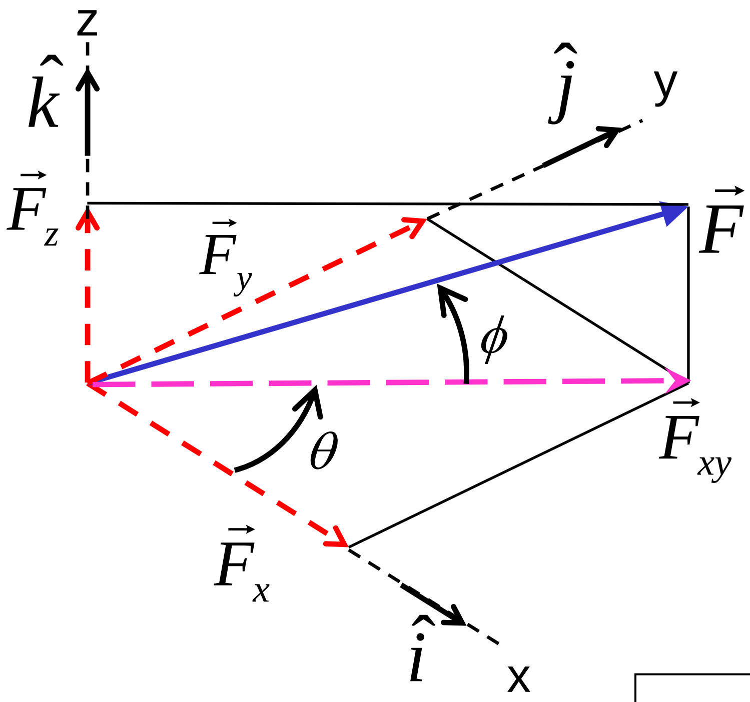
$$\vec{r}_B = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$$

$$\vec{r}_{AB} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$$

$$\vec{F} = F\frac{(x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}}$$

# Writing a vector in 3D (2)

**2 Specification by two angles which orient the line of action**



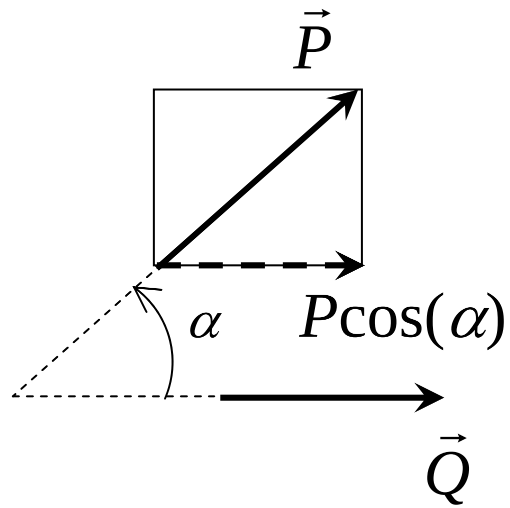

$$F_z = F\sin(\phi)$$

$$F_{xy} = F\cos(\phi)$$

$$F_x = F_{xy}\cos(\theta) = F\cos(\phi)\cos(\theta)$$

$$F_y = F_{xy}\sin(\theta) = F\cos(\phi)\sin(\theta)$$

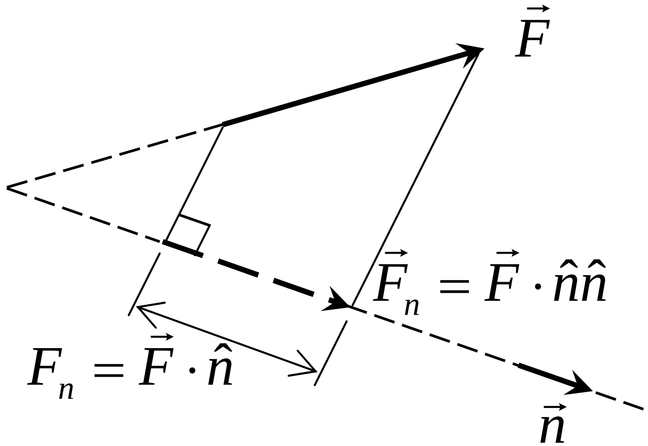
# Dot product




$$\vec{P}\cdot\vec{Q} = PQ\cos(\alpha)$$ (scalar)

$$\begin{pmatrix}\text{Projection of } \vec{P} \text{ in} \\ \text{the direction of } \vec{Q}\end{pmatrix} \bullet \left[Q\right]$$

If $\vec{Q}$ is a unit vector$(\hat{n})$ , dot product expresses the projection of vector in the unit vector direction




$$F_n = \vec{F}\cdot\hat{n} = F(l\hat{i} + m\hat{j} + n\hat{k})\cdot(\alpha\hat{i} + \beta\hat{j} + \gamma\hat{k})$$
$$= F(l\alpha + m\beta + n\gamma)$$

$$\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1$$

$$\hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{i} = \hat{i}\cdot\hat{k} = \hat{k}\cdot\hat{i} = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{j} = 0$$

# Angle between two vectors

From relation of dot product $\vec{P} \cdot \vec{Q} = PQ\cos(\theta)$

The angle between vectors $\vec{P}$ and $\vec{Q}$ is

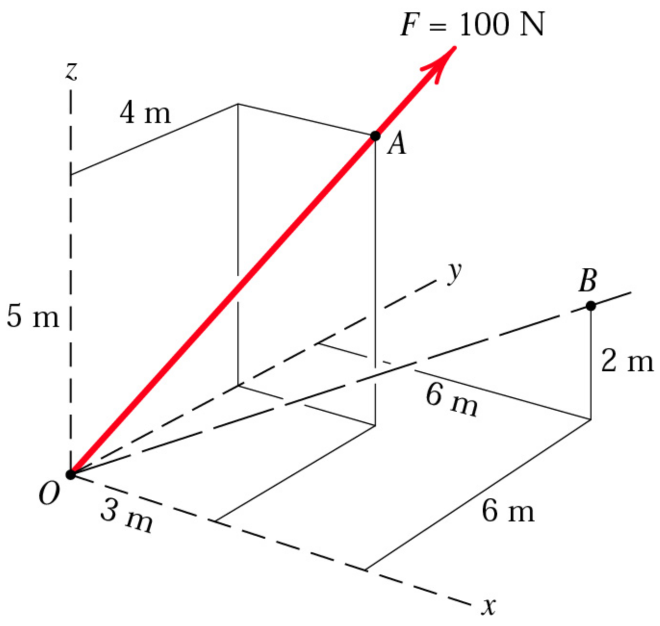
$$\theta = \cos^{-1} \frac{\vec{P} \cdot \vec{Q}}{PQ}$$

The angle between vectors $\vec{F}$ and $\hat{n}$ is

$$\theta = \cos^{-1} \frac{\vec{F} \cdot \hat{n}}{F}$$

$$\vec{F} \cdot \hat{n} = 0 \longrightarrow \vec{F} \perp \hat{n}$$

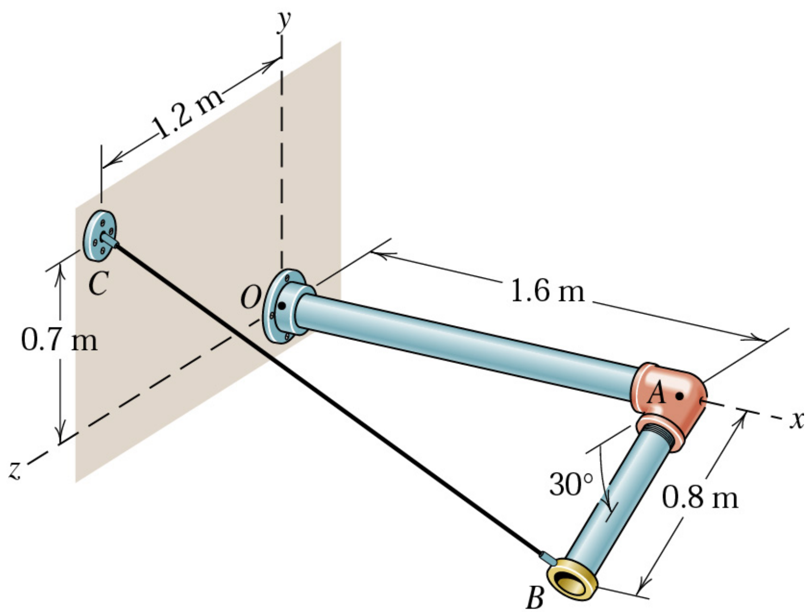
# Sample 1




A force **F** with a magnitude of 100 N is applied at the origin *O* of the axes *x*-*y*-*z* as shown. The line of action of **F** passes through a point *A* whose coordinates are 3m, 4m and 5m. Determine (a) the *x*, *y* and *z* scalar components of **F**, (b) the projection $F_{xy}$ of **F** on the *x*-*y* plane, and (c) the projection $F_{OB}$ of **F** along the line *OB*.

# Sample 2

The cable *BC* carries a tension of 750 N. Write this tension as a force **T** acting on point *B* in terms of the unit vector **i**, **j** and **k**. The elbow at *A* forms a right angle.



# Sample 3

The tension in supporting cable *BC* is 3200 N. Write the force which this cable exerts on the boom *OAB* as a vector **T**. Determine the angles $\theta_x$, $\theta_y$ and $\theta_z$ which the line of action of **T** forms with the positive *x*-, *y*- and *z*-axes.