Equilibrium conditions

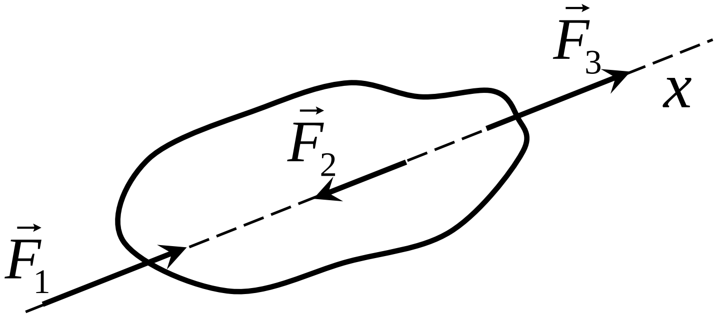
In 2D, equilibrium equation can be written in scalar form as

$$\Sigma F_x = 0 \qquad \Sigma F_y = 0 \qquad \Sigma M_o = 0$$

- The moment sum can be calculated about any point *O* on or off the body.
- A body moving with constant velocity (zero acceleration) can be treated as in a state of equilibrium.
- Equilibrium conditions (i.e. horizontal force, vertical force or moment) are independent, and one may hold without another.

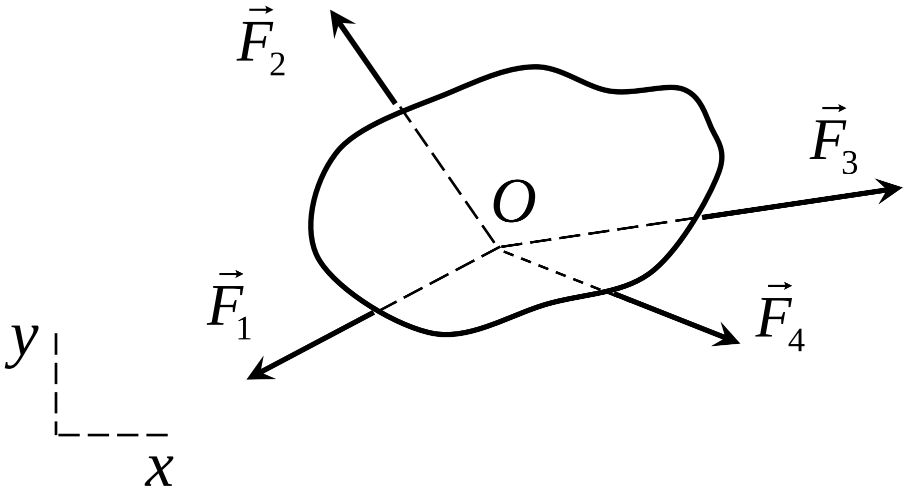
Categories of equilibrium(1)

1. Collinear



$\Rightarrow$ $\Sigma F_x = 0$

2. Concurrent at a point



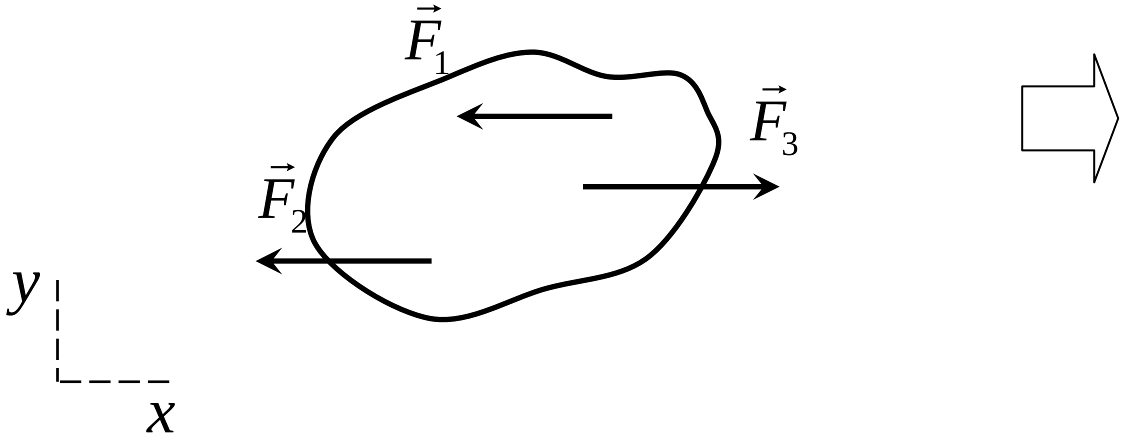
$\Rightarrow$

$\Sigma F_x = 0$

$\Sigma F_y = 0$

Categories of equilibrium(2)

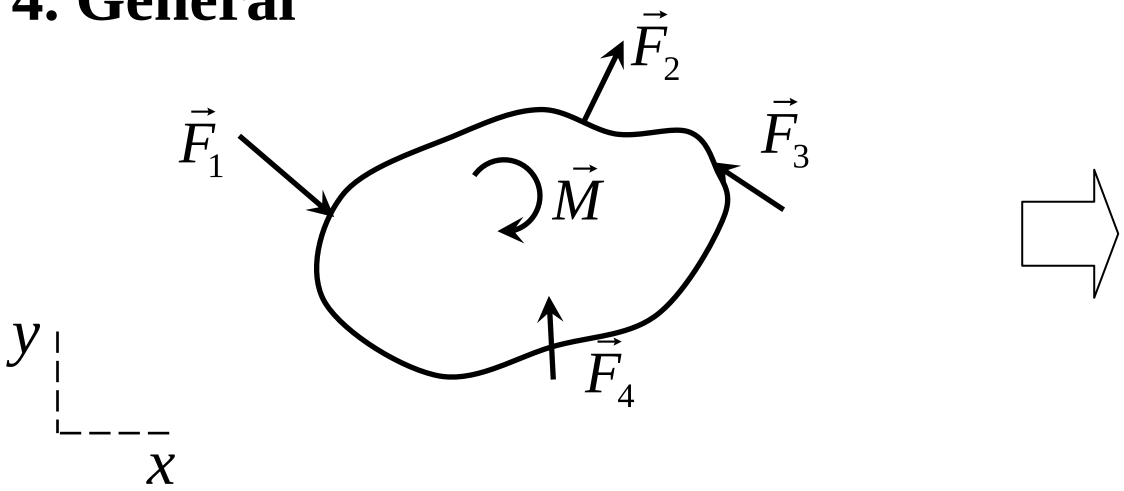
3. Parallel



$$\Sigma F_x = 0$$

$$\Sigma M_z = 0$$

4. General



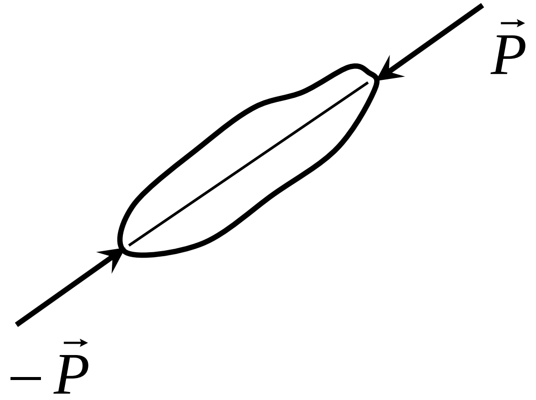
$$\Sigma F_x = 0$$

$$\Sigma F_y = 0$$

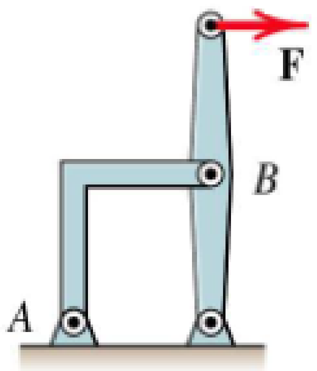
$$\Sigma M_z = 0$$

Two- and three-force members(1)

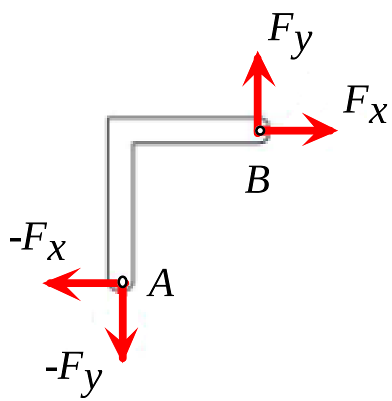
Two-force members



If there are only 2 forces acting on a body, and the body is in equilibrium, <u>two forces must be equal, opposite, and collinear.</u>

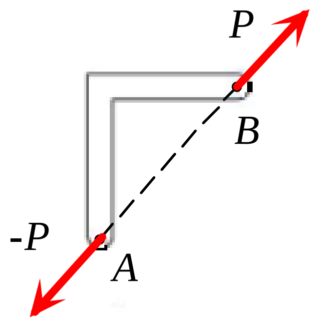


Negligible weight



2 unknowns

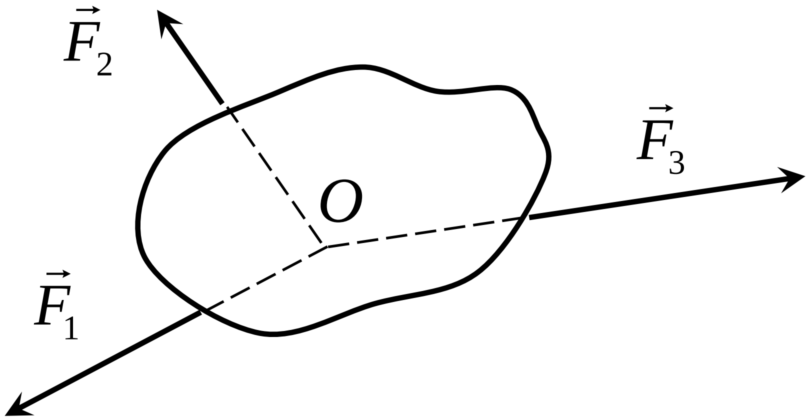
<u>Two-force members</u>



Only 1 unknown

Two- and three-force members(2)

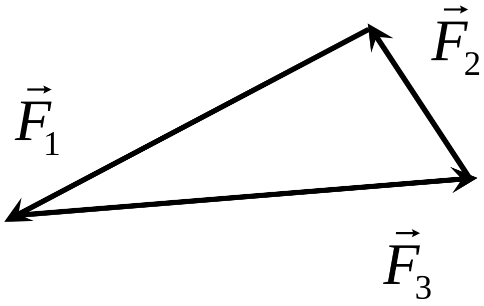
Three-force members



Three force must be concurrent.

$$\sum M_O = 0$$

Exception: three forces are parallel



Closed polygon satisfies

$\sum \vec{F} = 0$ can be drawn

Sine and cosine law

Alternative equilibrium equations

In general, these three equilibrium equations are used

$$\Sigma F_x = 0 \qquad \Sigma F_y = 0 \qquad \Sigma M_o = 0$$

Alternatively, the following sets of equilibrium equations can also be used

$$\Sigma M_A = 0 \qquad \Sigma F_x = 0 \qquad \Sigma M_B = 0$$

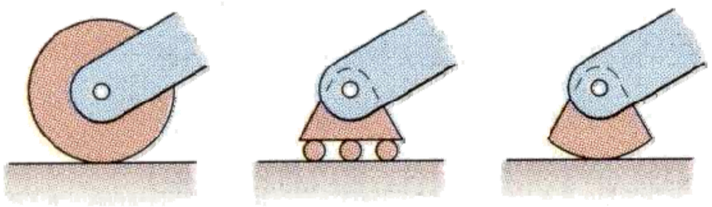
⇦ 2-moments, 1-force

$$\Sigma M_A = 0 \qquad \Sigma M_B = 0 \qquad \Sigma M_c = 0$$

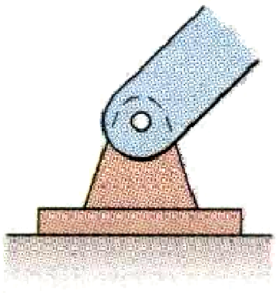
⇦ 3-moments

Constraints

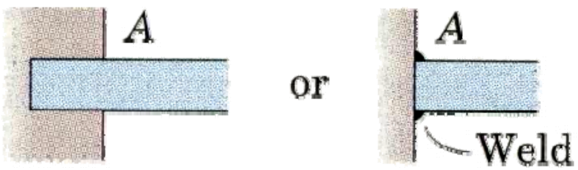
Constraint: the restriction of movement



- Constraint normal to the surface of contact
- Tangential force cannot be supported

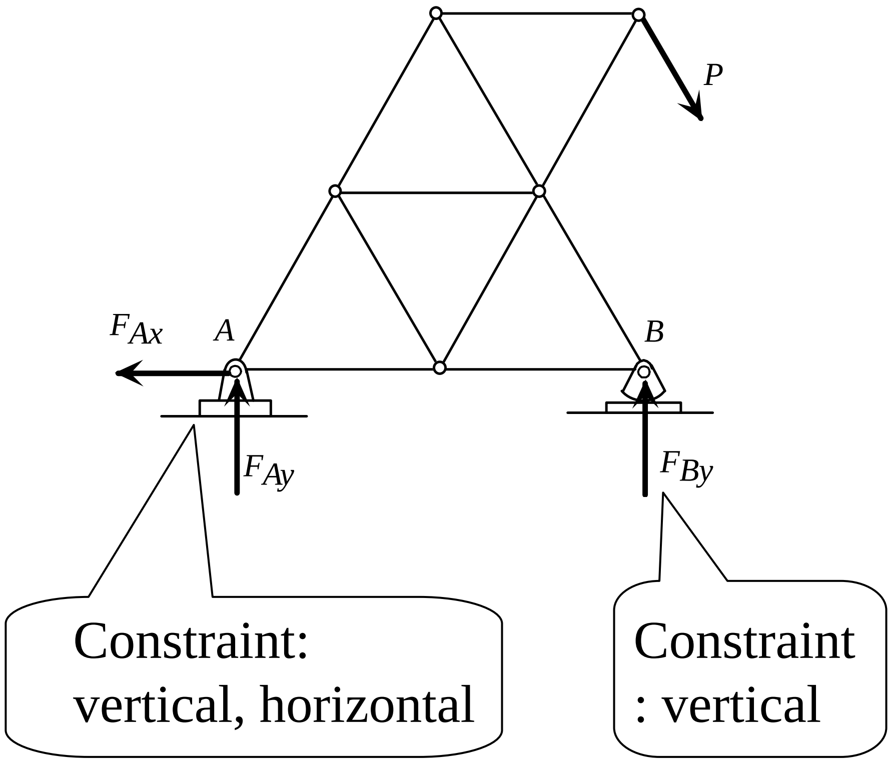


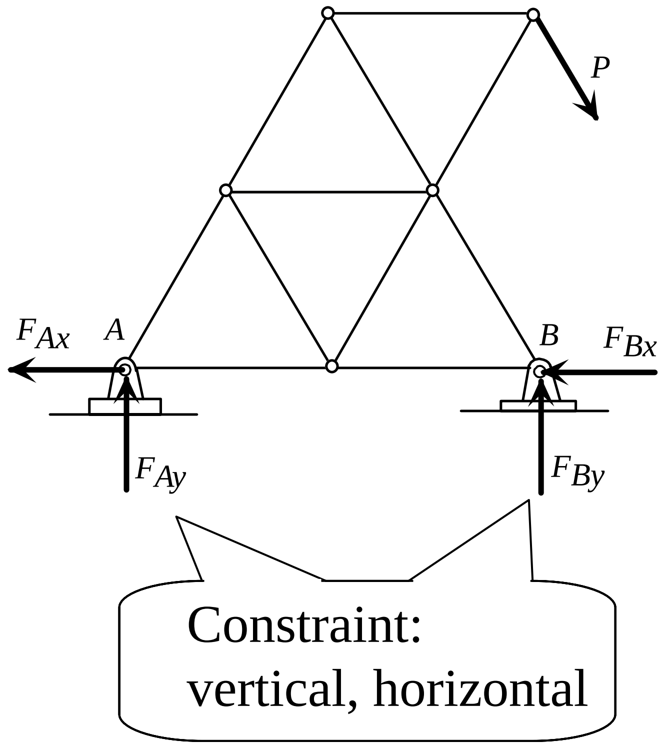
- Constraint in both vertical and horizontal direction
- Moment cannot be supported



- Constraint in vertical and horizontal direction and moment

Statical determinacy





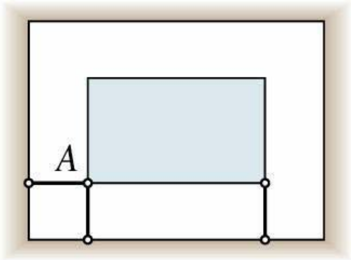
- 3 Unknowns
- 3 Equations ($\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma M_o = 0$)

Statically determinate

- 4 Unknowns
- 3 Equations

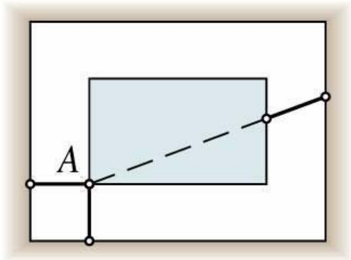
Statically indeterminate

Adequacy of constraints (1)



(a) Complete fixity
Adequate constraints

<u>Constraint</u>:
F_x, F_y, Moment ⇒ Adequate constraints

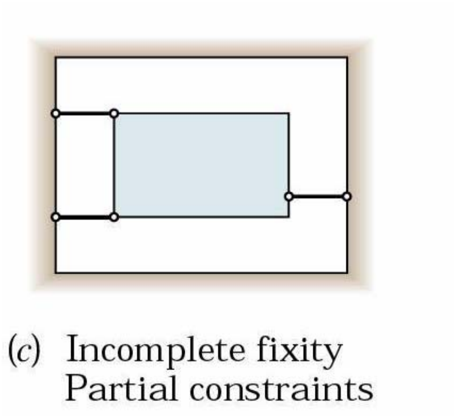


(b) Incomplete fixity
Partial constraints

<u>Constraint</u>: F_x, F_y
Moment about A cannot be supported ⇒ Partial constraints

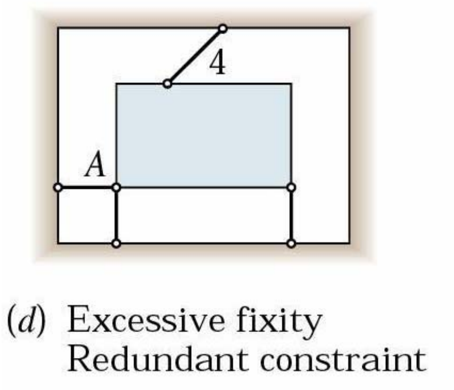
Three constraints does not always guarantee a stable equilibrium configuration

Adequacy of constraints (2)



Constraint:
F_x, Moment
F_y cannot be supported

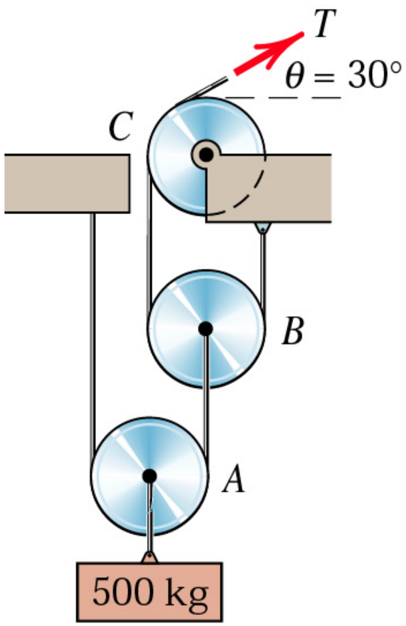
⇨ Partial constraints



Link 4 is unnecessary to maintain a fixed position

⇨ Redundant constraints

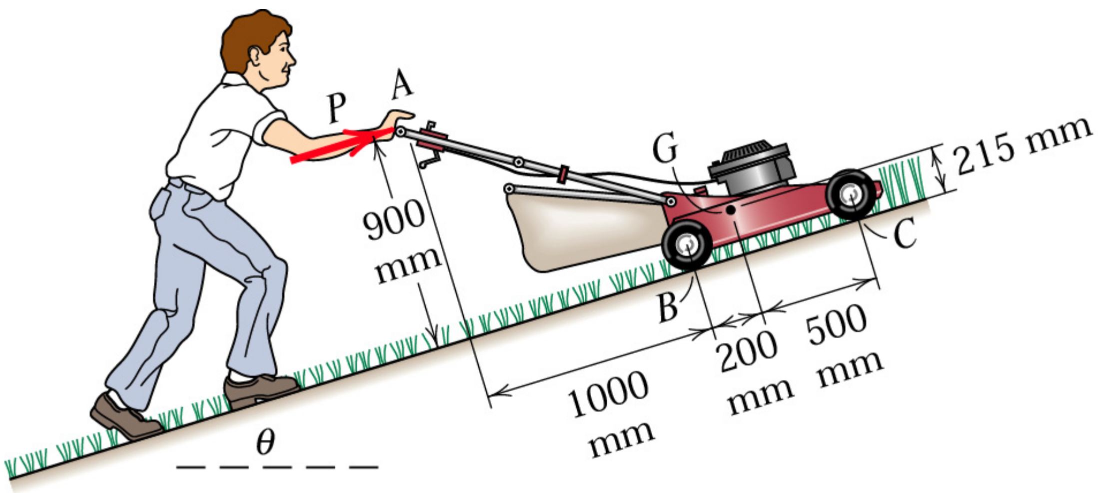
Sample 1



Calculate the tension *T* in the cable which supports the 500-kg mass with the pulley arrangement shown. Each pulley is free to rotate about it bearing, and the weights of all parts are small compared with the load. Find the magnitude of the total force on the bearing of pulley *C*.

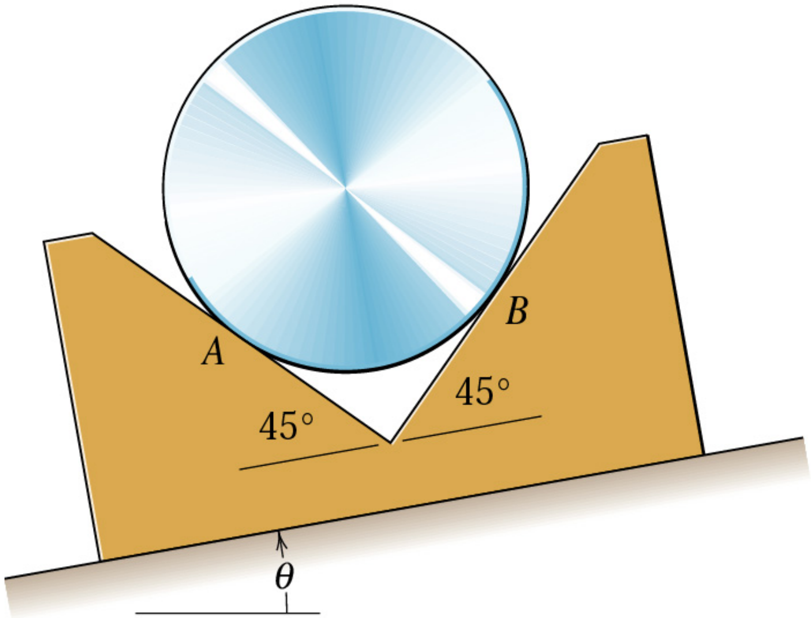
Sample 2

The man pushes the lawn mower at a steady speed with a force P that is parallel to the incline. The mass of the mower with attached grass bag is 50 kg with mass center at G. If $\theta = 15°$, determine the normal forces N_B and N_C under each pair of wheels B and C. Neglect friction. Compare with the normal forces for the conditions of $\theta = 0$ and $P = 0$



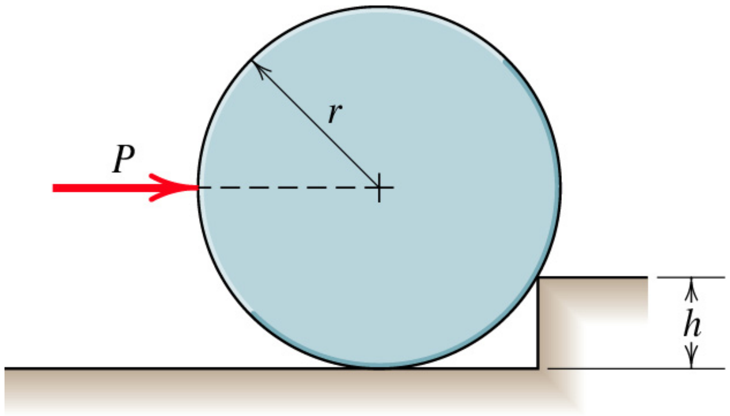
Sample 3

Find the angle of tilt θ with the horizontal so that the contact force at B will be one-half that at A for the smooth cylinder.



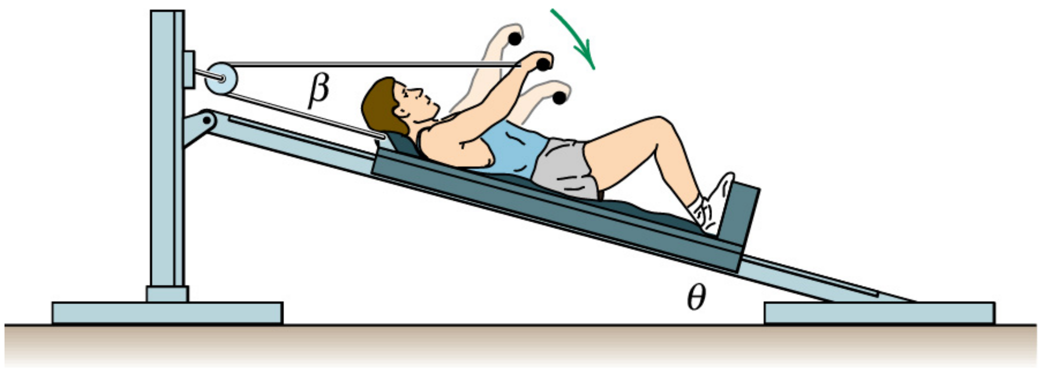
Sample 4

Determine the force P required to begin rolling the uniform cylinder of mass m over the obstruction of height h.



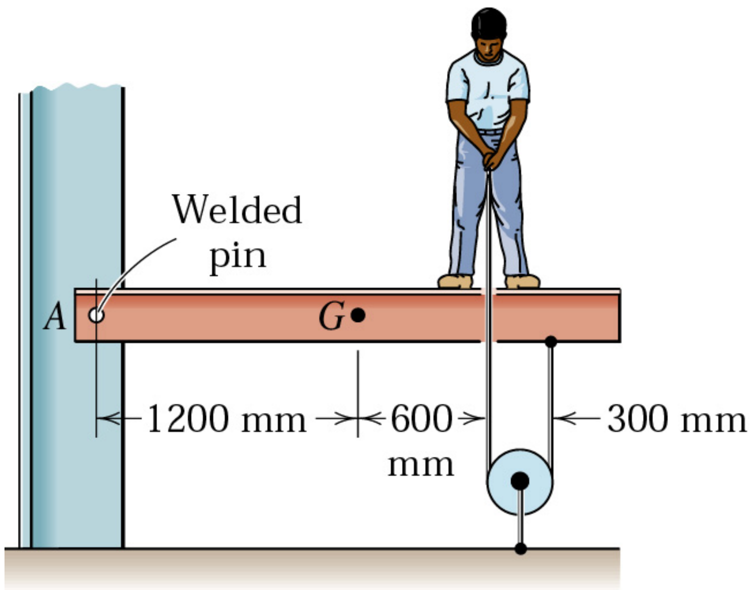
Sample 5

The exercise machine is designed with a lightweight cart which is mounted on small rollers so that it is free to move along the inclined ramp. Two cables are attached to the cart-one for each hand. If the hands are together so that the cables are parallel and if each cable lies essentially in a vertical plane, determine the force P which each hand must exert on its cable in order to maintain an equilibrium position. The mass of the person is 70 kg, the ramp angle θ is 15°, and the angle β is 18°. In addition, calculate the force R which the ramp exerts on the cart.



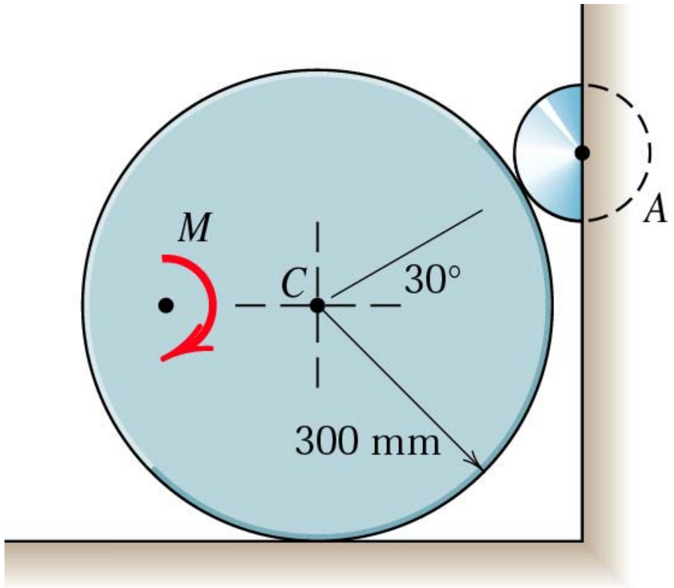
Sample 6

The pin *A*, which connects the 200-kg steel beam with center of gravity at *G* to the vertical column, is welded both to the beam and to the column. To test the weld, the 80-kg man loads the beam by exerting a 300-N force on the rope which passes through a hole in the beam as shown. Calculate the torque (couple) *M* supported by the pin.



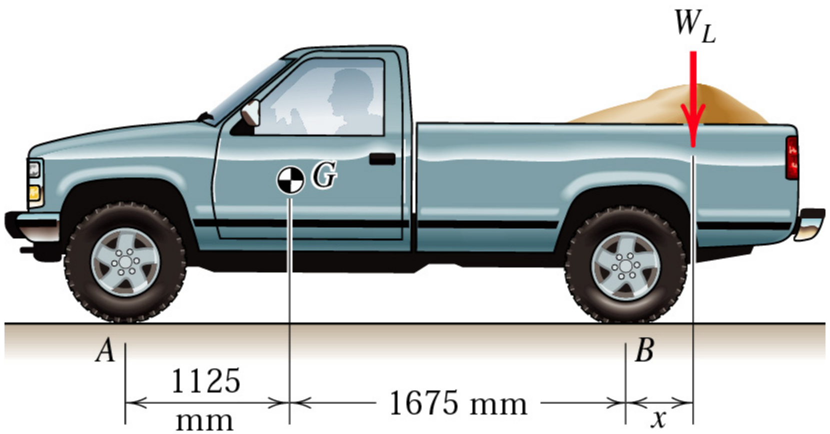
Sample 7

The 100-kg wheel rests on a rough surface and bears against the roller *A* when the couple *M* is applied. If *M* = 60 Nm and the wheel does not slip, compute the reaction on the roller *A*.



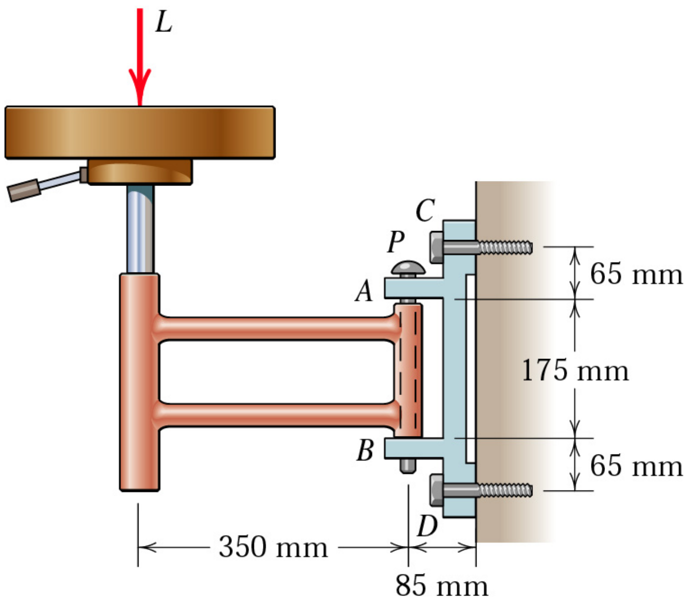
Sample 8

The indicated location of the center of mass of the 1600-kg pickup truck is for the unladen condition. If a load whose center of mass is x = 400 mm behind the rear axle is added to the truck, determine the mass m_L of the load for which the normal forces under the front and rear wheels are equal.



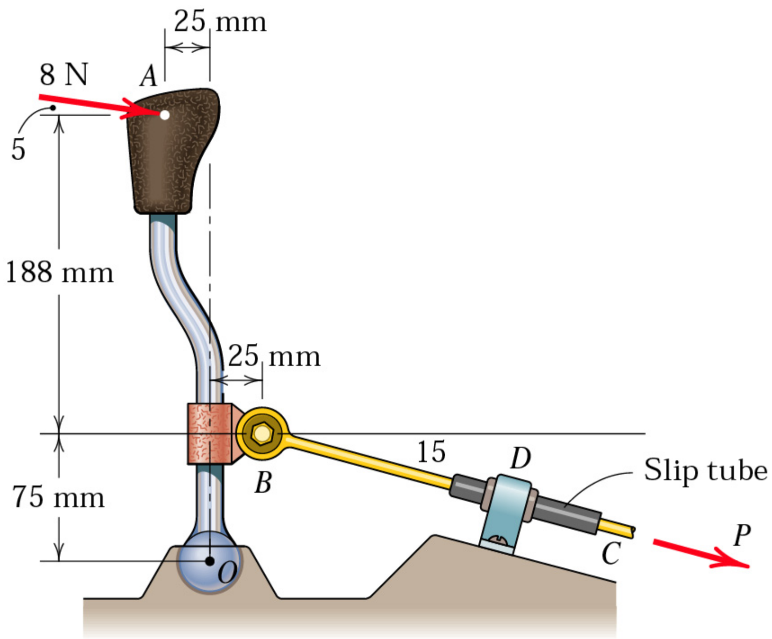
Sample 9

The elements of a wall-mounted swing-away stool are shown in the figure. The hinge pin *P* fits loosely through the frame tube, and the frame tube has a slight clearance between the supports *A* and *B*. Determine the reactions on the frame tube at *A* and *B* associated with the weight *L* of an 80-kg person. Also, calculate the changes in the horizontal reactions at *C* and *D* due to the same load *L*. State any assumptions.



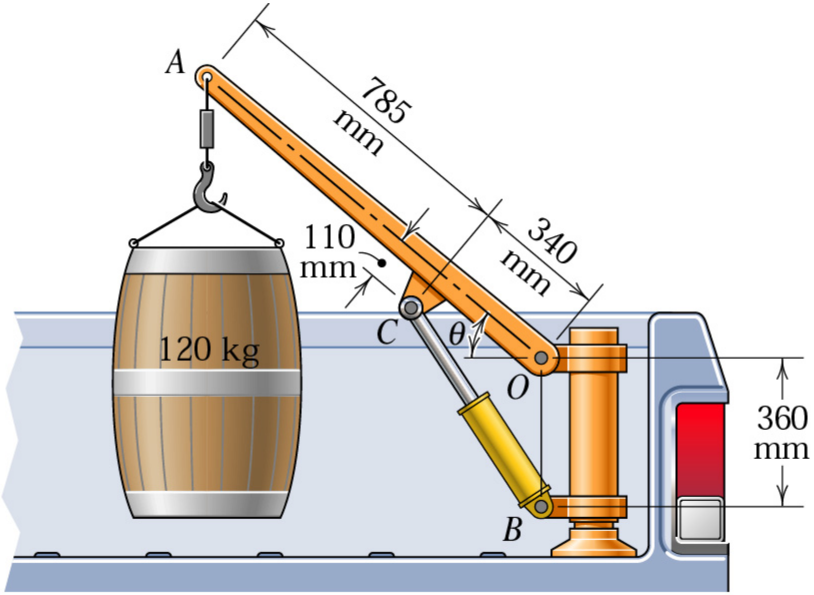
Sample 10

A portion of the shifter mechanism for a manual car transmission is shown in the figure. For the 8-N force exerted on the shift knob, determine the corresponding force *P* exerted by the shift link *BC* on the transmission (not shown). Neglect friction in the ball-and – socket joint at *O*, in the joint *B*, and in the slip tube near support *D*. Note that a soft rubber bushing at *D* allows the slip tube to self-align with link *BC*.



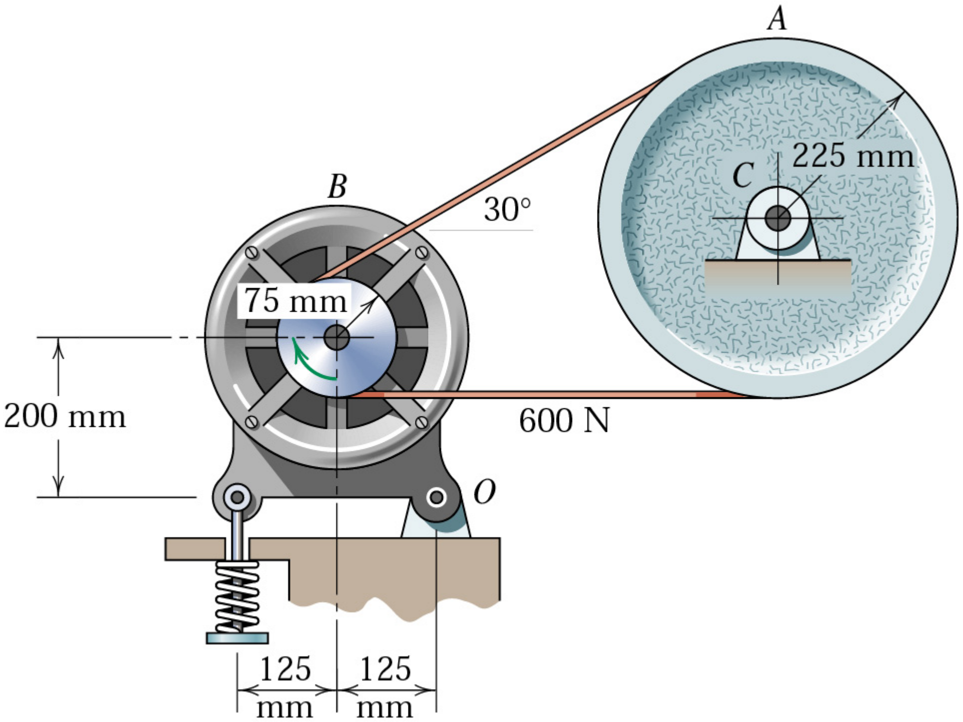
Sample 11

The small crane is mounted on one side of the bed of a pickup truck. For the position $\theta = 40^{\circ}$, determine the magnitude of the force supported by the pin at *O* and the oil pressure *p* against the 50-mm-diameter piston of the hydraulic cylinder *BC*.



Sample 12

Pulley *A* delivers a steady torque (moment) of 100 Nm to a pump through its shaft at *C*. The tension in the lower side of the belt is 600 N. The driving motor *B* has a mass of 100 kg and rotates clockwise. As a design consideration, determine the magnitude *R* of the force on the supporting pin at *O*..



Sample 13

The uniform 400-kg drum is mounted on a line of rollers at *A* and a line of rollers at *B*. An 80-kg man moves slowly a distance of 700 mm from the vertical centerline before the drum begins to rotate. All rollers are perfectly free to rotate, except one of them at *B* which must overcome appreciable friction in its bearing. Calculate the friction force *F* exerted by that one roller tangent to the drum and find the magnitude *R* of the force exerted by all rollers at *A* on the drum for this condition.