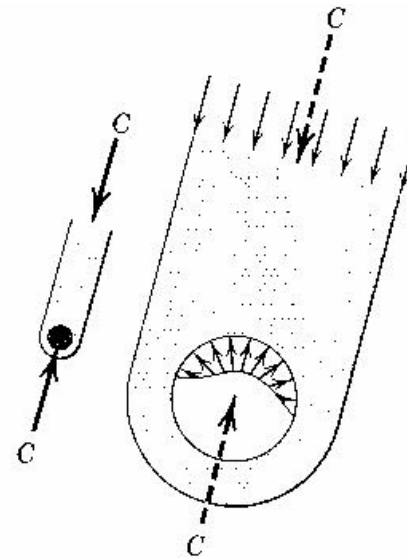
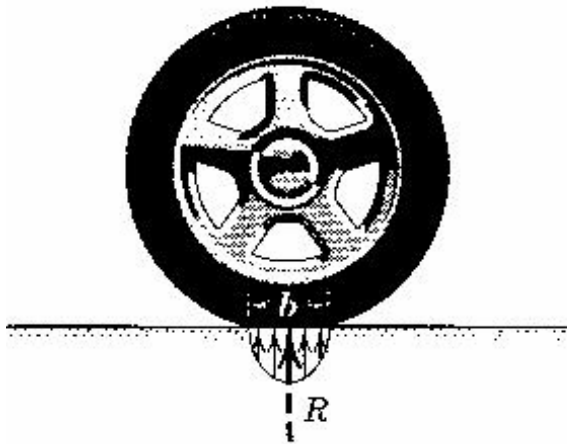


Ch. 5 Distributed forces

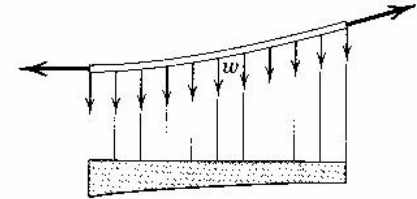
- Earlier in this class, forces are considered “**concentrated**” (each force is assumed to act at a single point).
- In reality, forces are actually distributed over a finite areas.



Categories of problems

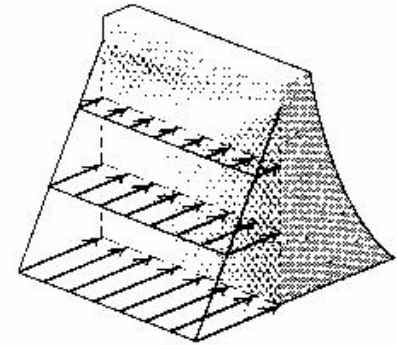
Line distribution: suspended cable

- a force is distributed along a line
- Intensity w : force per unit length, N/m



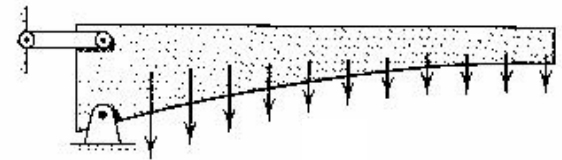
Area distribution: pressure of water

- Intensity: force per unit area (pressure);
 N/m^2 , Pa



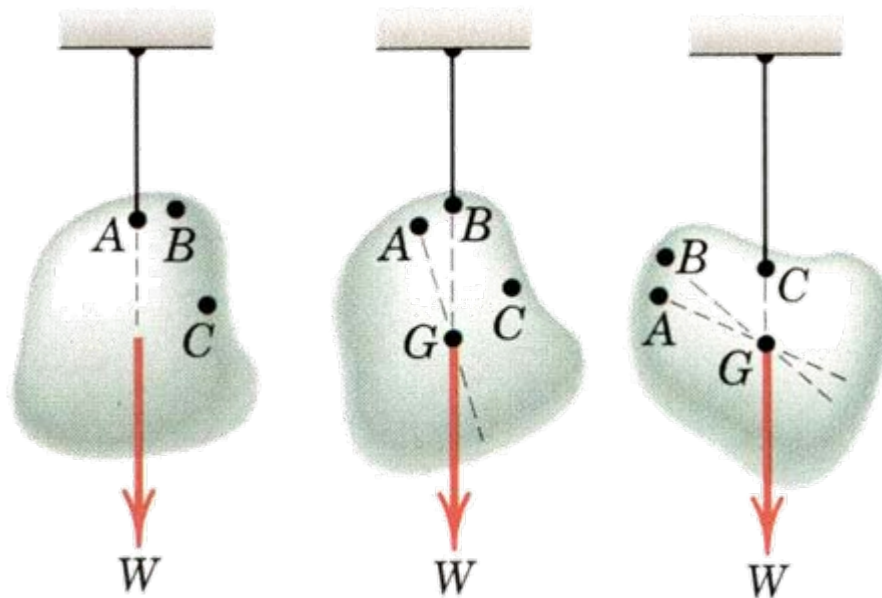
Volume distribution: body force

- Intensity: force per unit volume (specific weight = $\rho \cdot g$), N/m^3

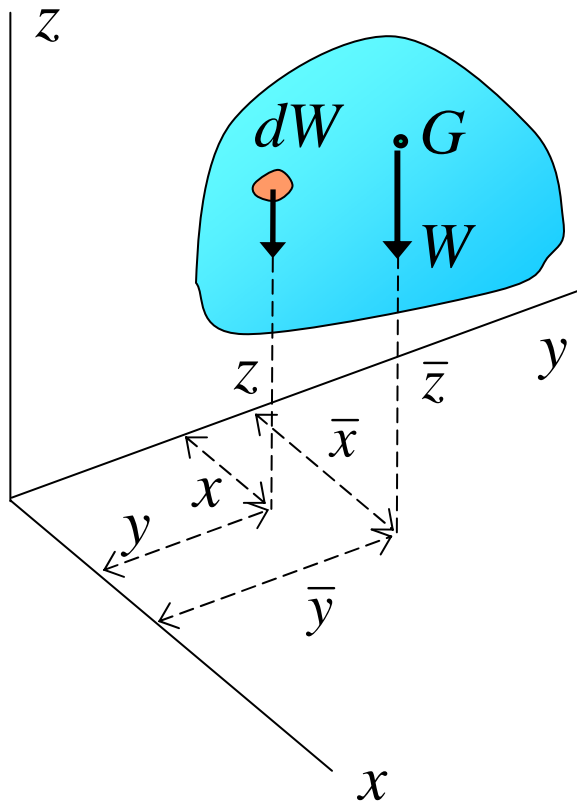


5/2 Center of mass

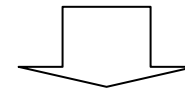
- Center of mass (gravity): mass or weight of a whole body is considered to be concentrated at this point
- For each body, there is a unique center of gravity, G .
- Center of gravity can be found as shown in the figure.



Determining the center of gravity (1)



Weight of a whole body is considered to be concentrated at the center of gravity



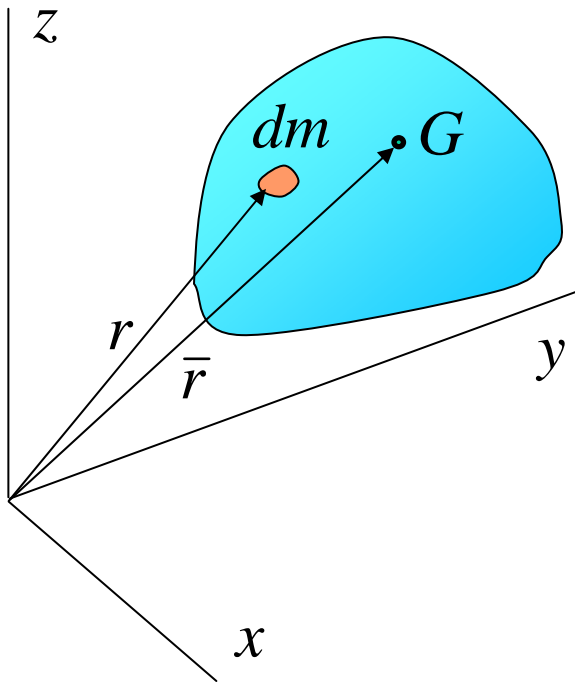
Sum of the moments of small components = Moment of the whole weight

$$\int x dW = \bar{x} \int dW = \bar{x} W$$

Center of Gravity

$$\bar{x} = \frac{\int x dW}{W}, \quad \bar{y} = \frac{\int y dW}{W}, \quad \bar{z} = \frac{\int z dW}{W}$$

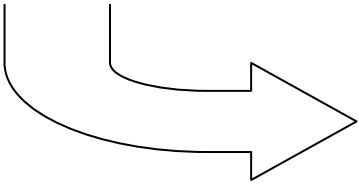
Determining the center of gravity (2)



Because $W = mg$ and $dW = gdm$,

$$\bar{x} = \frac{\int xdm}{m}, \quad \bar{y} = \frac{\int ydm}{m}, \quad \bar{z} = \frac{\int zdm}{m}$$

Combined into one Eq.


$$\vec{\bar{r}} = \frac{\int \vec{r}dm}{m}$$

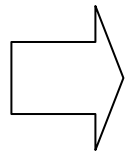
- The point $(\bar{x}, \bar{y}, \bar{z})$ is called the “*center of mass*”
- When the gravity field is uniform and parallel, the center of mass is the same as the center of gravity

5/3 Centroids

From
$$\bar{x} = \frac{\int x dW}{W}, \quad \bar{y} = \frac{\int y dW}{W}, \quad \bar{z} = \frac{\int z dW}{W}$$

if the density (ρ) of the body is uniform,

$$\bar{x} = \frac{\int x dm}{m} = \frac{\int x \rho dV}{\rho V} = \frac{\int x dV}{V}$$



$$\bar{x} = \frac{\int x dV}{V}, \quad \bar{y} = \frac{\int y dV}{V}, \quad \bar{z} = \frac{\int z dV}{V}$$

- The point $(\bar{x}, \bar{y}, \bar{z})$ is called the “*centroid*”
- Centroid of a body depends on its geometric shape only.
- When density is uniform, centroid is the same as the center of mass

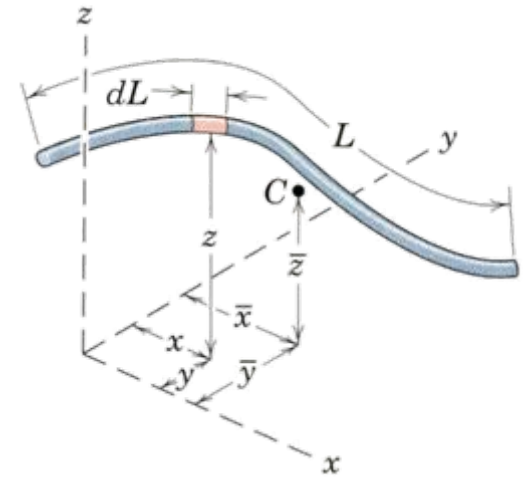
Centroids of lines and areas

From $\bar{x} = \frac{\int x dV}{V}$, $\bar{y} = \frac{\int y dV}{V}$, $\bar{z} = \frac{\int z dV}{V}$

Centroids of lines

$$dV = AdL \quad \text{and} \quad V = AL$$

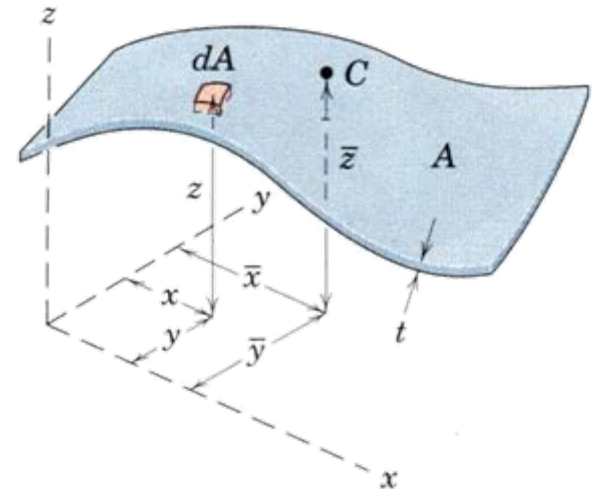
$$\bar{x} = \frac{\int x dL}{L}, \quad \bar{y} = \frac{\int y dL}{L}, \quad \bar{z} = \frac{\int z dL}{L}$$



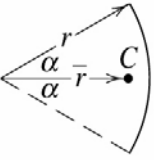
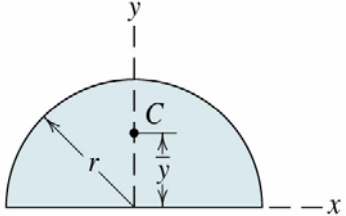
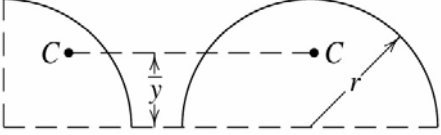
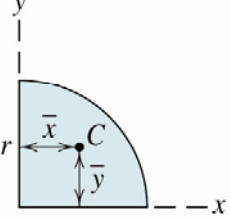
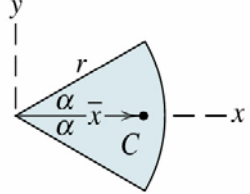
Centroids of areas

$$dV = t dA \quad \text{and} \quad V = tA$$

$$\bar{x} = \frac{\int x dA}{A}, \quad \bar{y} = \frac{\int y dA}{A}, \quad \bar{z} = \frac{\int z dA}{A}$$

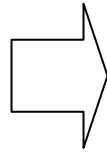


Centroid of some plane figures

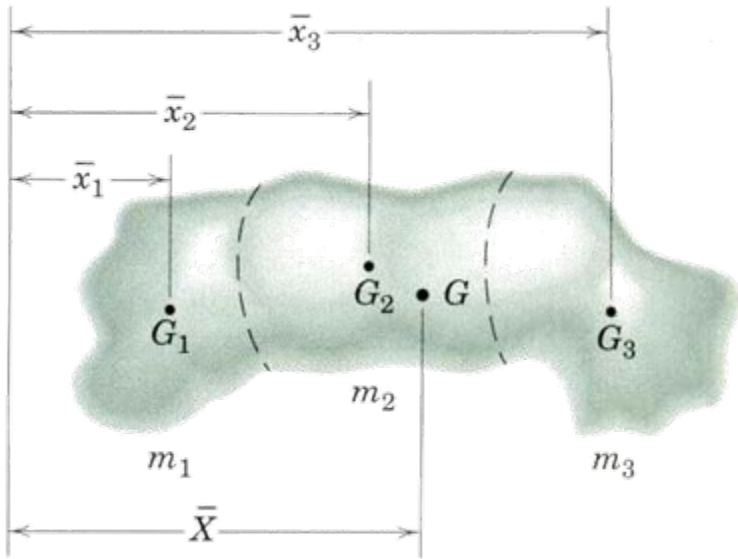
FIGURE	CENTROID		
<p data-bbox="211 454 382 486">Arc Segment</p> 	$\bar{r} = \frac{r \sin \alpha}{\alpha}$	<p data-bbox="1039 425 1210 486">Semicircular Area</p> 	$\bar{y} = \frac{4r}{3\pi}$
<p data-bbox="186 629 592 662">Quarter and Semicircular Arcs</p> 	$\bar{y} = \frac{2r}{\pi}$	<p data-bbox="1058 768 1287 829">Quarter-Circular Area</p> 	$\bar{x} = \bar{y} = \frac{4r}{3\pi}$
		<p data-bbox="1068 1072 1287 1133">Area of Circular Sector</p> 	$\bar{x} = \frac{2}{3} \frac{r \sin \alpha}{\alpha}$

5/4 Composite bodies

m_1, m_2, m_3, G_1, G_2 and G_3
are known



G of the whole body can also
calculated using principle of
moment



$$(m_1 + m_2 + m_3)\bar{X} = m_1\bar{x}_1 + m_2\bar{x}_2 + m_3\bar{x}_3$$

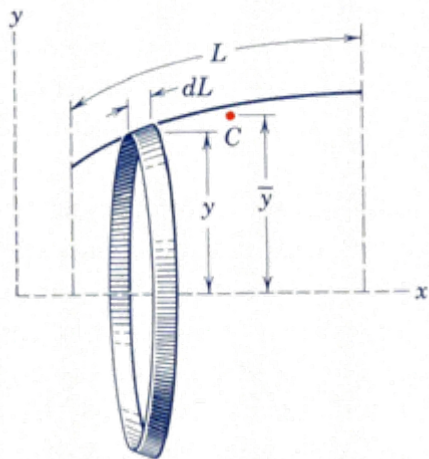
or

$$\bar{X} = \frac{\sum m\bar{x}}{\sum m}$$

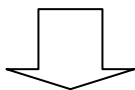
- For an irregular body, divides it into parts and approximate each part with a regular body
- Parts with negative volume/area may be used to simplify the calculation

5/5 Theorems of Pappus

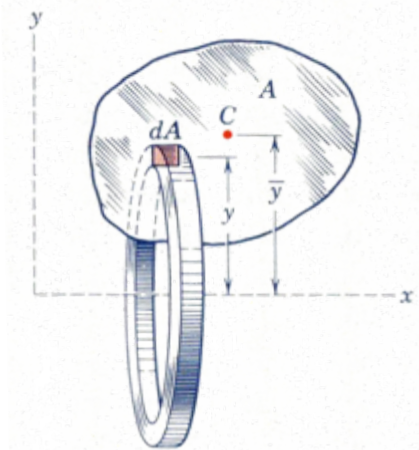
Use to find area of surface (or volume) generated by revolving a plane curve (or an area) about a non-intersecting axis in the plane of a curve (or an area)



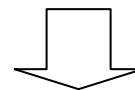
$$A = 2\pi \int y dL = 2\pi \bar{y} L$$



$$A = \theta \bar{y} L$$



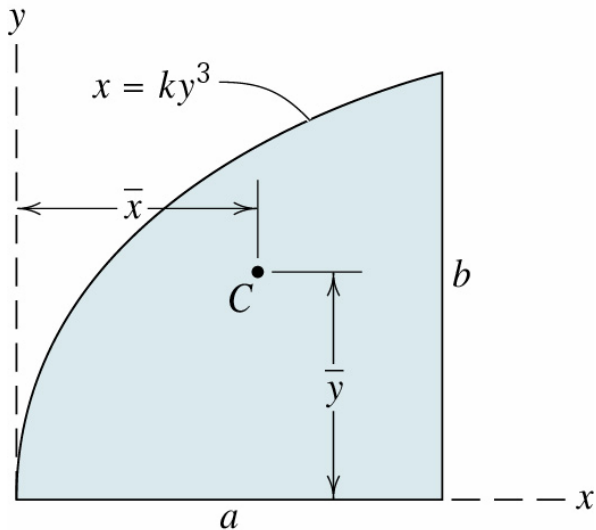
$$V = 2\pi \int y dA = 2\pi \bar{y} A$$



$$V = \theta \bar{y} A$$

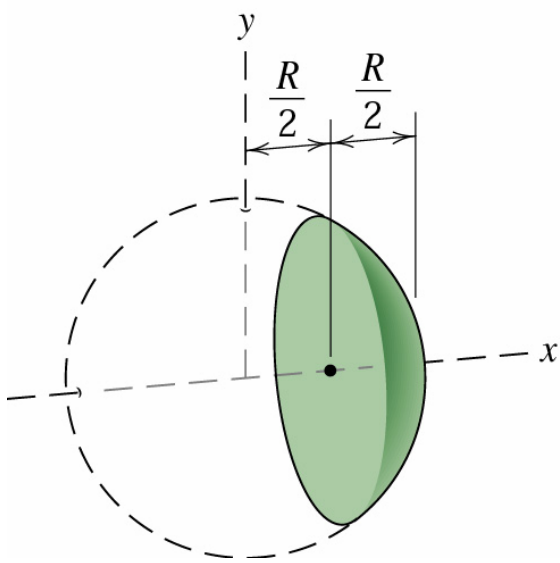
Sample 1

Locate the centroid of the area under the curve $x = ky^3$ from $x = 0$ to $x = a$.



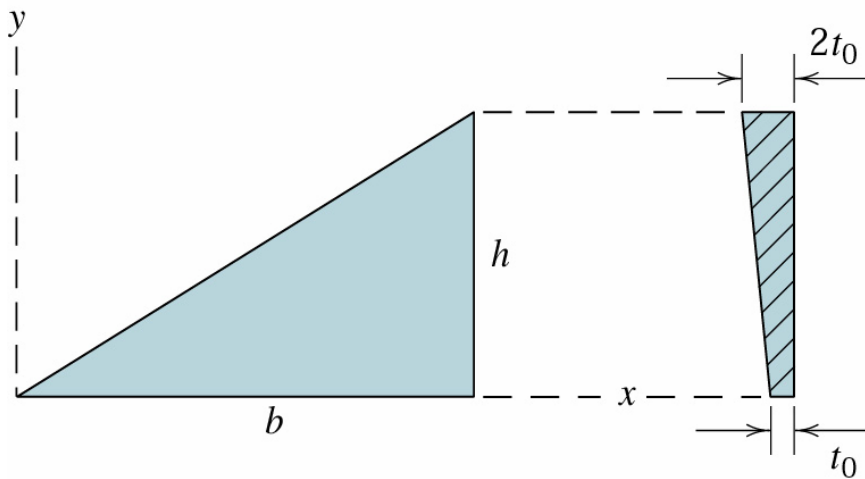
Sample 2

Determine the x-coordinate of the centroid of the solid spherical segment.



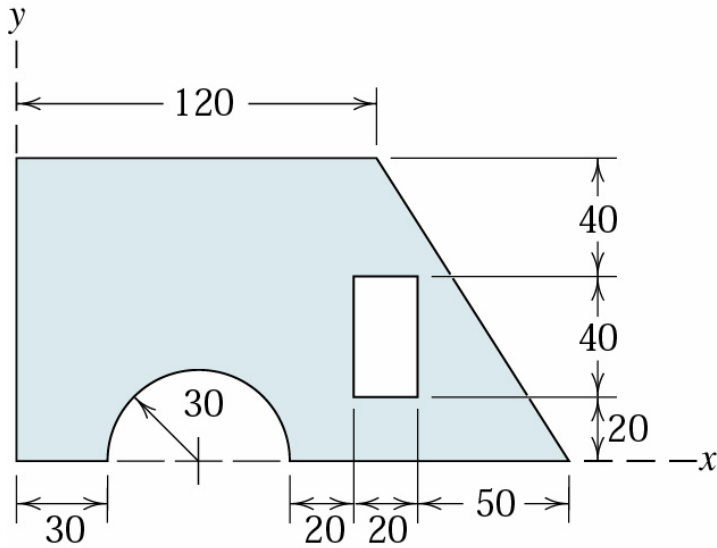
Sample 3

The thickness of the triangular plate varies linearly with y from a value t_0 along its base $y = 0$ to $2t_0$ at $y = h$. Determine the y -coordinate of the center of mass of the plate.



Sample 4

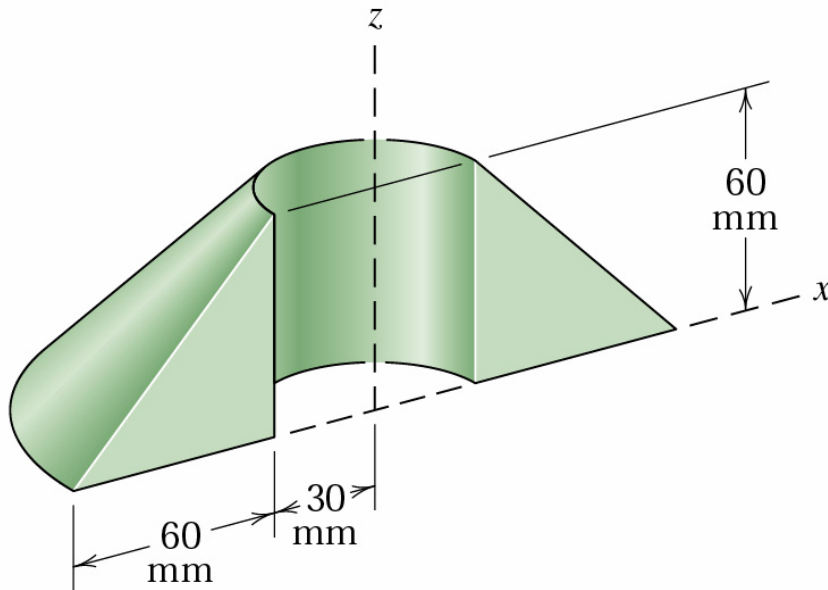
Locate the centroid of the shaded area.



Dimensions in millimeters

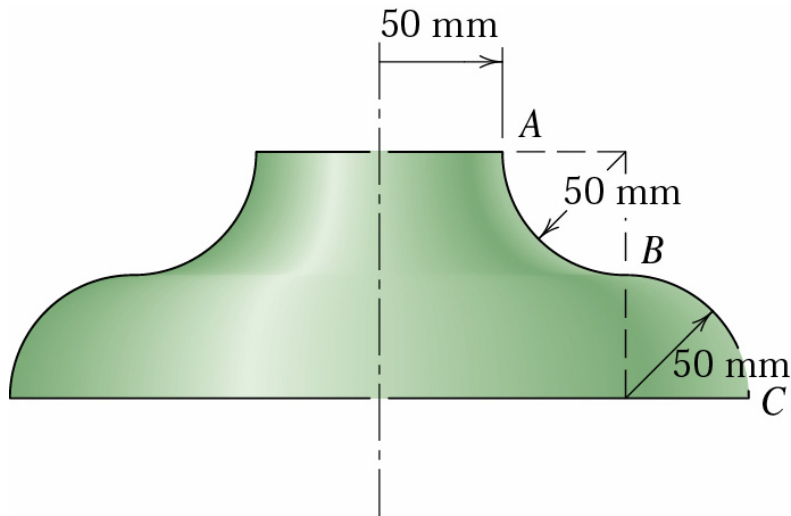
Sample 5

Calculate the volume V of the solid generated by revolving the 60-mm right triangular area through 180° about the z -axis. If this body were constructed of steel, what would be its mass m ? ($\rho_{\text{steel}} = 7830 \text{ kg/m}^3$)



Sample 6

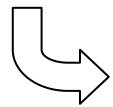
The two circular arcs AB and BC are revolved about the vertical axis to obtain the surface of revolution shown. Compute the area A of the outside of this surface.



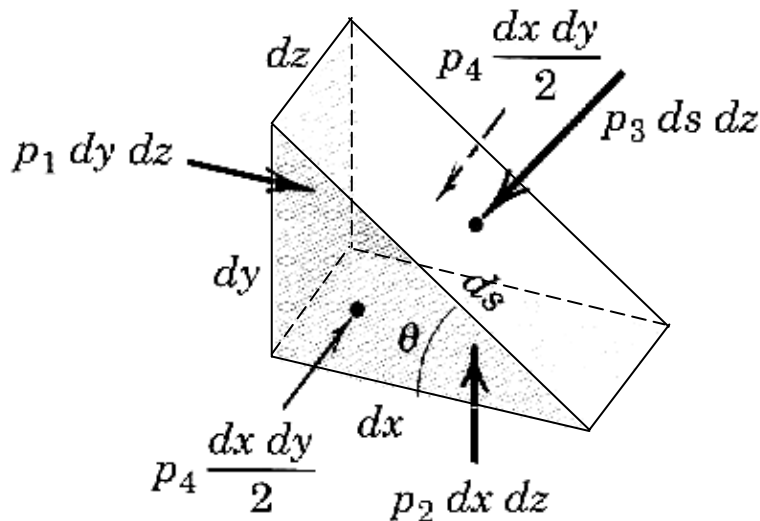
5/9 Fluid statics

Fluid pressure

- Fluids at rest cannot support shear forces



Pressure force is always perpendicular to the surface.



$$p_1 dy dz = p_3 ds dz \sin \theta$$

$$p_2 dx dz = p_3 ds dz \cos \theta$$

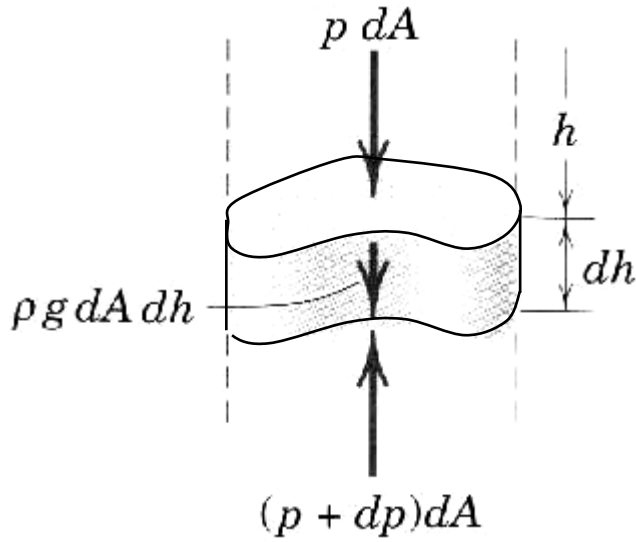
Since $ds = dy/\sin \theta = dz/\cos \theta$,

$$p_1 = p_2 = p_3$$

Pressure at any given point in a fluid is the same in all directions

Fluid pressure

In all fluids at rest the pressure is a function of the vertical dimension



$$p dA + \rho g dA dh - (p + dp) dA = 0$$

$$dp = \rho g dh$$

When ρ is constant,

$$p = p_o + \rho gh$$

p_o = pressure on the surface of the fluid ($h = 0$)

When p_o = atmospheric pressure (101.3 kPa),

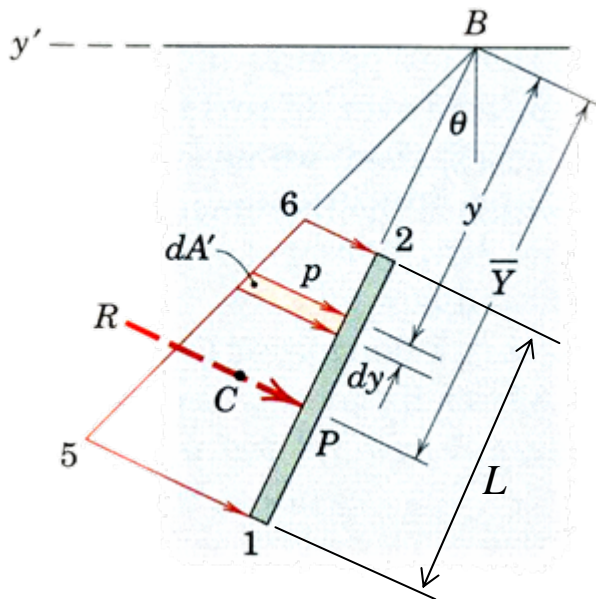
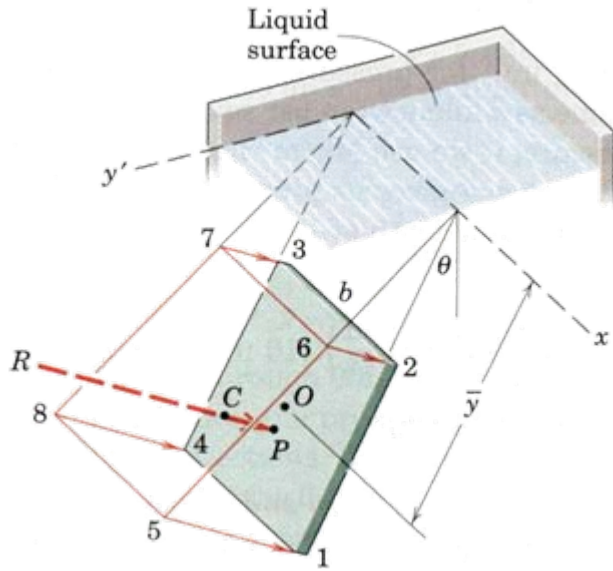
ρgh = increment above atmospheric pressure (**gage pressure**)

Fluid pressure on submerged surfaces

1. Rectangular surfaces

For systems open to the atmosphere,

- p_o acts over all surface, zero resultant
- consider only gage pressure



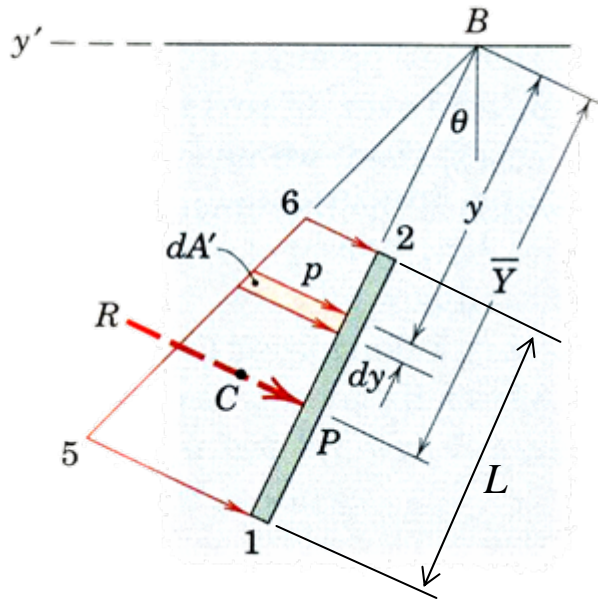
$$\text{Resultant} \quad R = \int dR = \int p dA$$

$$R = \int \rho g h b dy = \int_{y_1}^{y_2} (\rho g b y \cos \theta) dy$$

$$R = \rho g \left(\frac{h_1 + h_2}{2} \right) bL = \rho g \bar{h} A$$

$$R = \left(\frac{p_1 + p_2}{2} \right) bL = p_{av} A$$

Rectangular surfaces (2)



Alternative method to find resultant R

Resultant $R = \int dR = \int p dA$

$$R = \int p b dy = b \int p dy$$

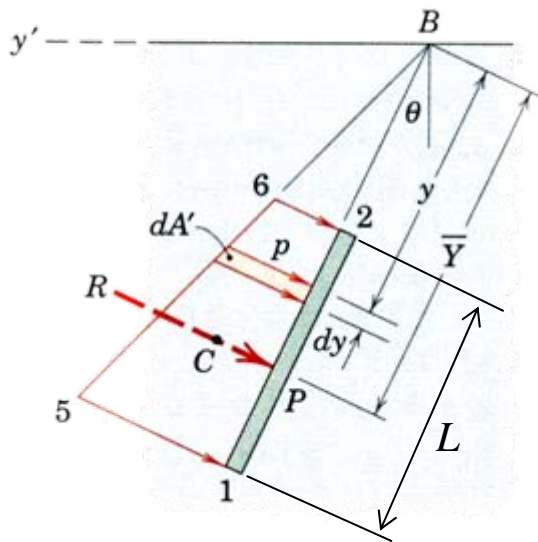
Area of trapezoid 1265 (A')

$$R = \left(\frac{p_1 + p_2}{2} \right) bL = p_{av} A$$

$$R = \rho g \left(\frac{h_1 + h_2}{2} \right) bL = \rho g \bar{h} A$$

Volume of prism
(trapezoid 1265,
depth b)

Location of the resultant



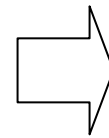
Apply principle of moment

$$\bar{Y}R = \int y p dA = b \int y p dy = b \int y dA'$$

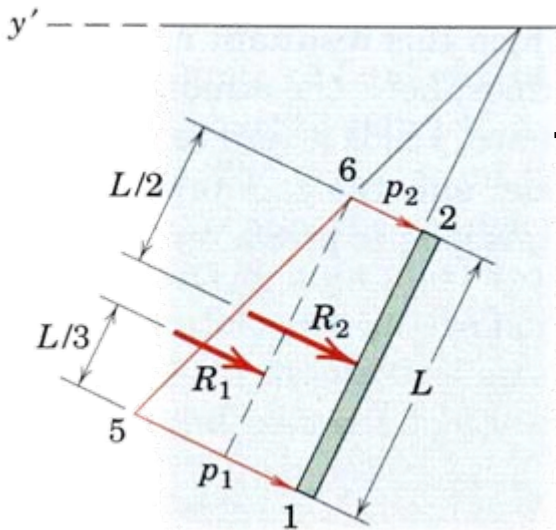
A' = Area of trapezoid 1265

$$\bar{Y} \left(b \int dA' \right) = b \int y dA'$$

$$\bar{Y} = \frac{\int y dA'}{\int dA'}$$



Centroid of trapezoid 1265

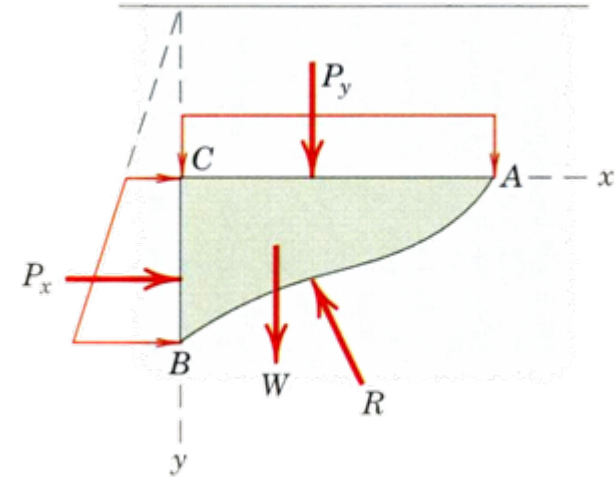
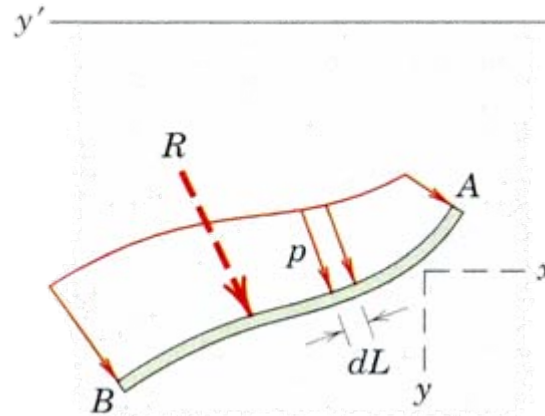
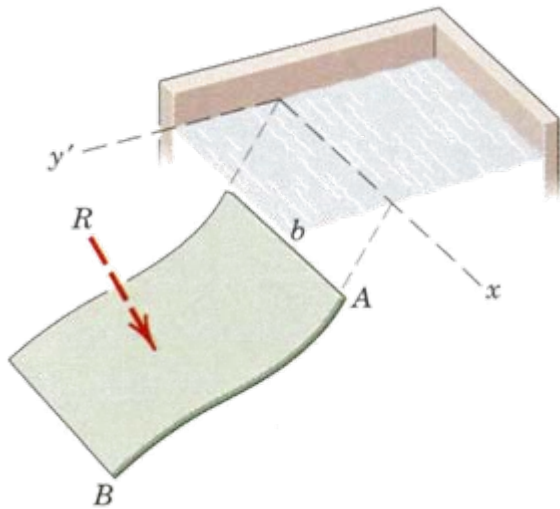


Alternative Trapezoid = Triangle + Rectangle

Principle of moment

$$\bar{Y} (R_1 + R_2) = \bar{y}_1 R_1 + \bar{y}_2 R_2$$

Fluid pressure on cylindrical surfaces

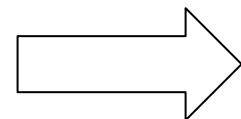


1. Integration method (R is vector, and can not be integrated directly)

$$R_x = b \int (pdL)_x = b \int p dy \quad R_y = b \int (pdL)_y = b \int p dx$$

2. Use equilibrium of the fluid

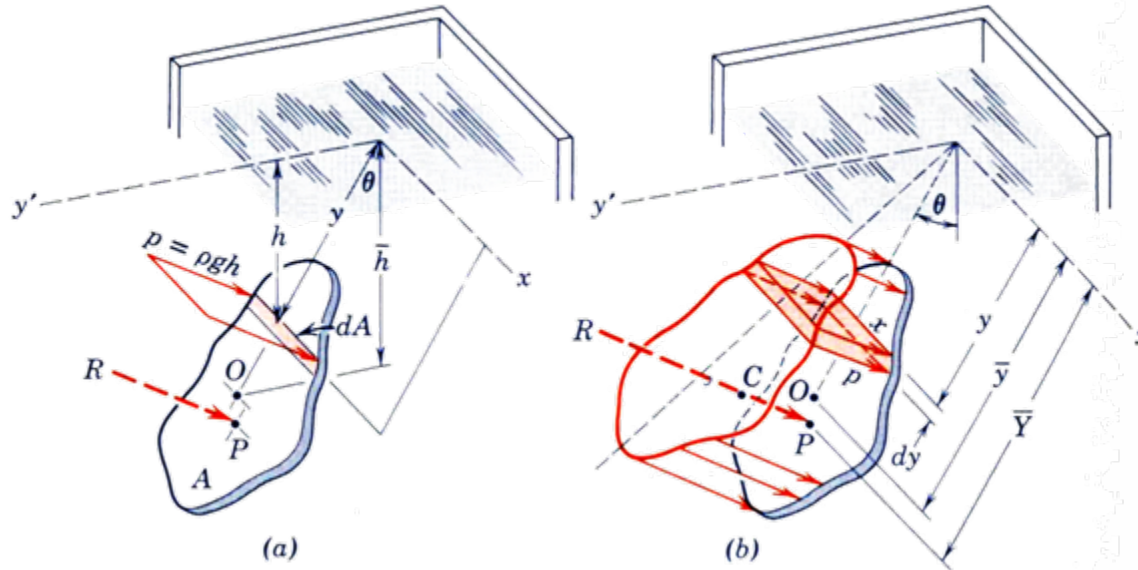
- Calculate P_x and P_y
- Calculate the weight W (area ABC)



Equi. eq.

Resultant R

Fluid pressure on flat surfaces



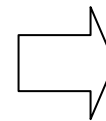
Resultant $R = \int p dA = \rho g \int h dA = \rho g \bar{h} A$

or $R = \int p dA = \int (\rho g h x) dy$

Principle of moment
 $\int h dA = \bar{h} A$

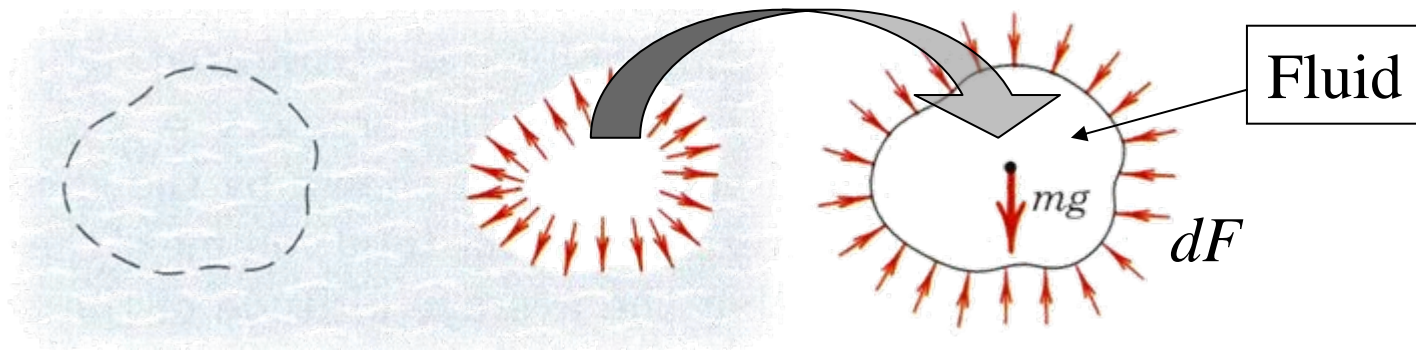
Center of pressure

$R \bar{Y} = \int y dR$



$\bar{Y} = \frac{\int (y p x) dy}{\int (p x) dy}$

Buoyancy (1)



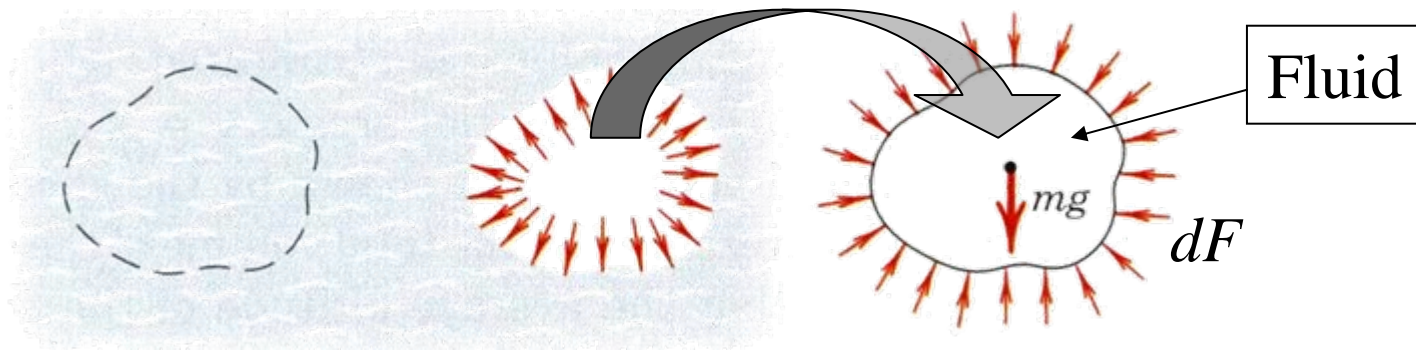
Since this fluid portion is in **equilibrium**,

$$F = \int dF = (mg)_{fluid} \quad \Rightarrow \quad F = \rho_{fluid} gV$$

The force of buoyancy is equal to the weight of fluid displaced

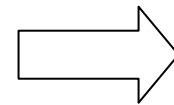
Similarly, for an object immersed in fluid

Buoyancy (2)



Fluid portion is in equilibrium with 2 forces that are

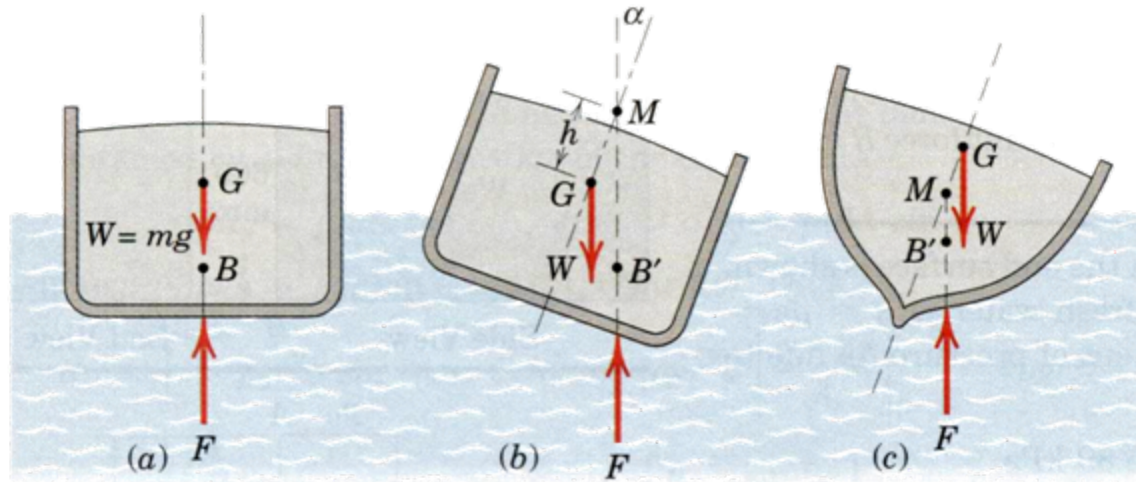
1. Resultant F of distributed force
2. Mass of fluid portion



Two force member

The force of buoyancy (F) must pass through the center of mass of the fluid portion

Buoyancy (3)



If $B =$ Centroid of displaced volume causing the buoyancy force F

$G =$ Center of gravity of the ship with weight W

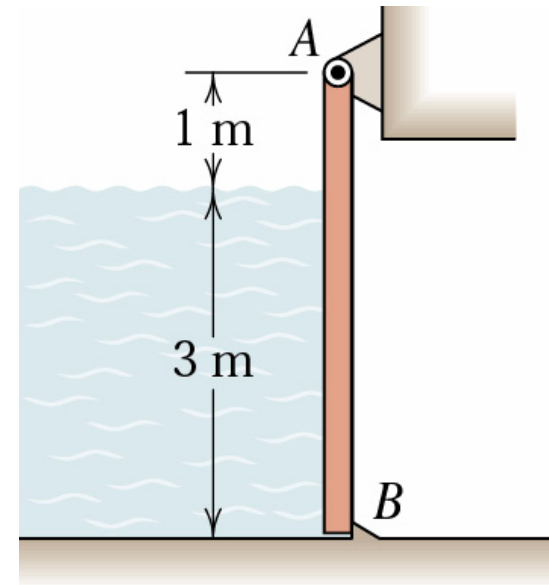
Two Possibilities

(b) \Rightarrow Moment will cause the ship to move back to the original position.

(c) \Rightarrow Moment will cause the ship to turn over.

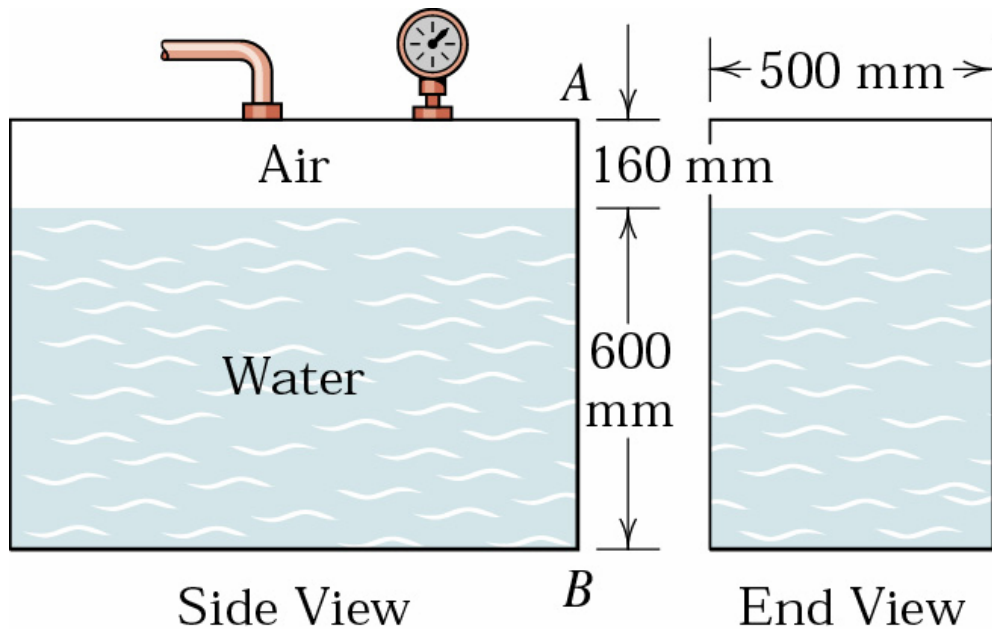
Sample 7

A rectangular plate, shown in vertical section AB , is 4 m high and 6 m wide (normal to the plane of the paper) and blocks the end of a fresh-water channel 3 m deep. The plate is hinged about a horizontal axis along its upper edge through A and is restrained from opening by the fixed ridge B which bears horizontally against the lower edge of the plate. Find the force B exerted on the plate by the ridge.



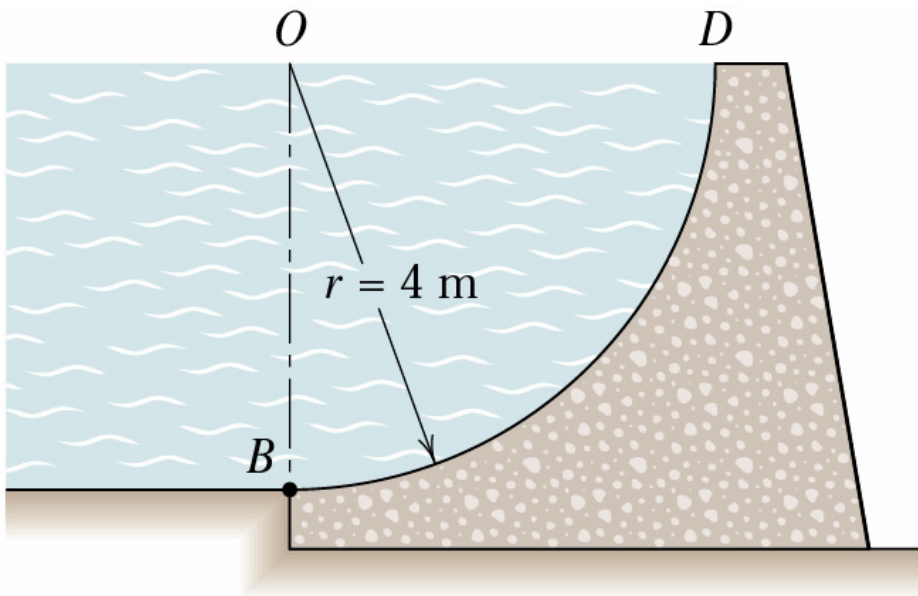
Sample 8

The air space in the closed fresh-water tank is maintained at a pressure of 5.5 kPa (above atmospheric). Determine the resultant force R exerted by the air and water on the end of the tank.



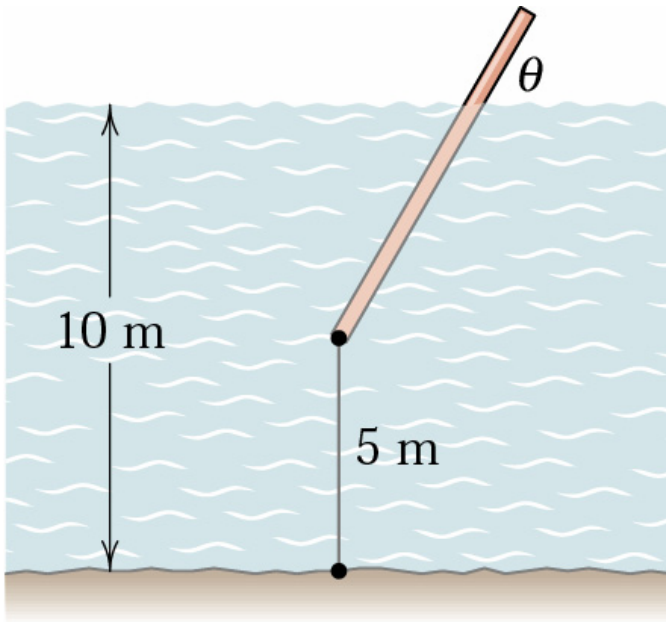
Sample 9

Determine completely the resultant force R exerted on the cylindrical dam surface by the water. The density of fresh water is 1.000 Mg/m^3 , and the dam has a length b , normal to the paper, of 30 m .



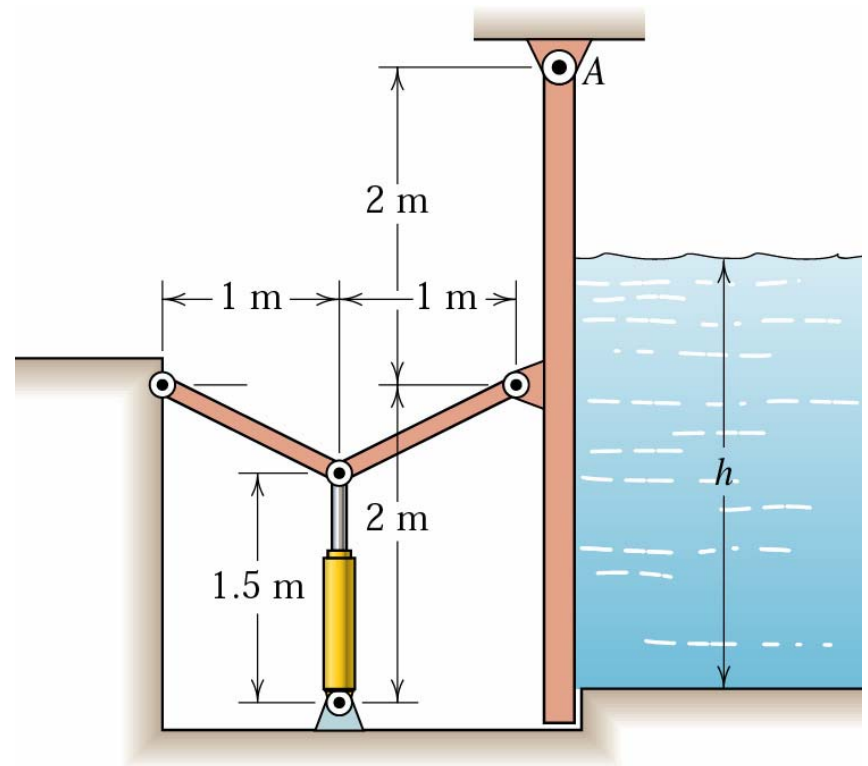
Sample 10

A buoy in the form of a uniform 8-m pole 0.2 m in diameter has a mass of 200 kg and is secured at its lower end to the bottom of a fresh-water lake with 5 m of cable. If the depth of the water is 10 m, calculate the angle θ made by the pole with the horizontal.



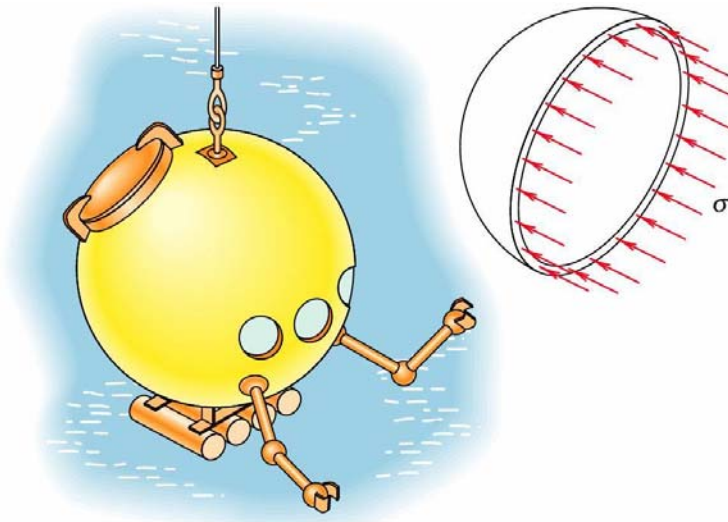
Sample 11

The hydraulic cylinder operates the toggle which closes the vertical gate against the pressure of fresh water on the opposite side. The gate is rectangular with a horizontal width of 2 m perpendicular to the paper. For a depth $h = 3$ m of water, calculate the required oil pressure p which acts on the 150-mm-diameter piston of the hydraulic cylinder.



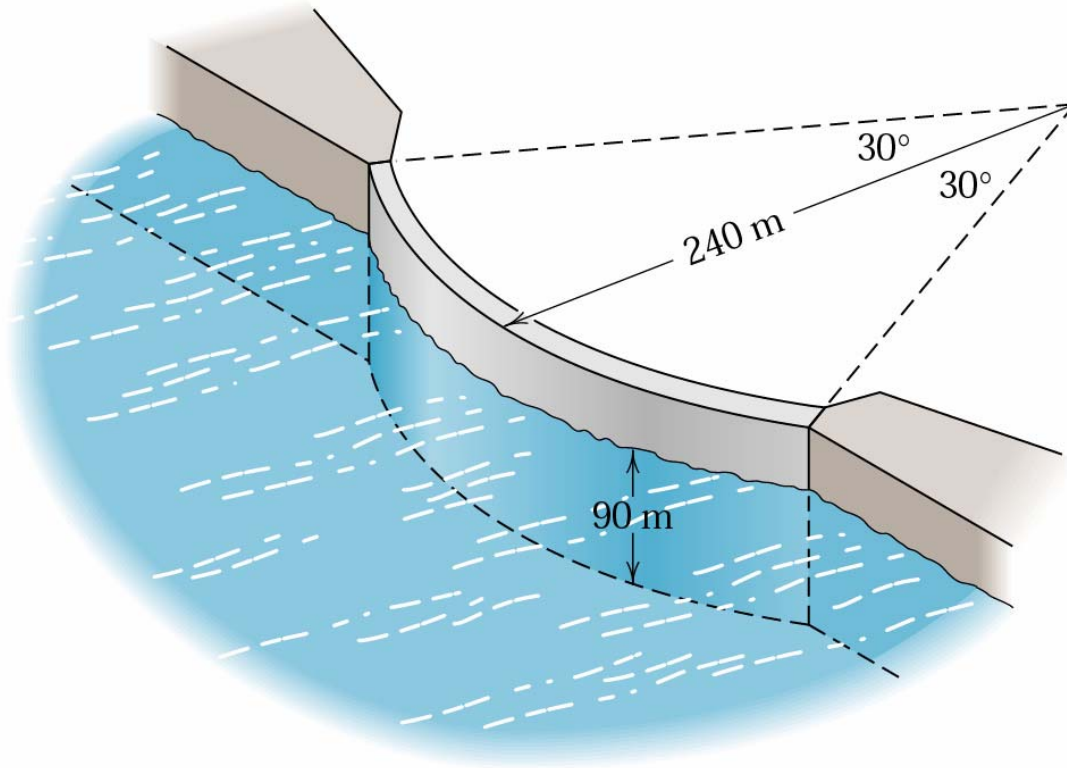
Sample 12

A deep-submersible diving chamber designed in the form of a spherical shell 1500 mm in diameter is ballasted with lead so that its weight slightly exceeds its buoyancy. Atmospheric pressure is maintained within the sphere during an ocean dive to a depth of 3 km. The thickness of the shell is 25 mm. For this depth calculate the compressive stress σ which acts on a diametral section of the shell, as indicated in the right-hand view.



Sample 13

The upstream side of an arched dam has the form of a vertical cylindrical surface of 240-m radius and subtends an angle of 60° . If the fresh water is 90 m deep, determine the total force R exerted by the water on the dam face.



Sample 14

A block of wood in the form of a waterproofed 400-mm cube is floating in a tank of salt water with a 150-mm layer of oil floating on the water. Assume that the cube floats in the attitude shown, and calculate the height h of the block above the surface of the oil. The densities of oil, salt water, and wood are 900, 1030, and 800 kg/m³, respectively.

