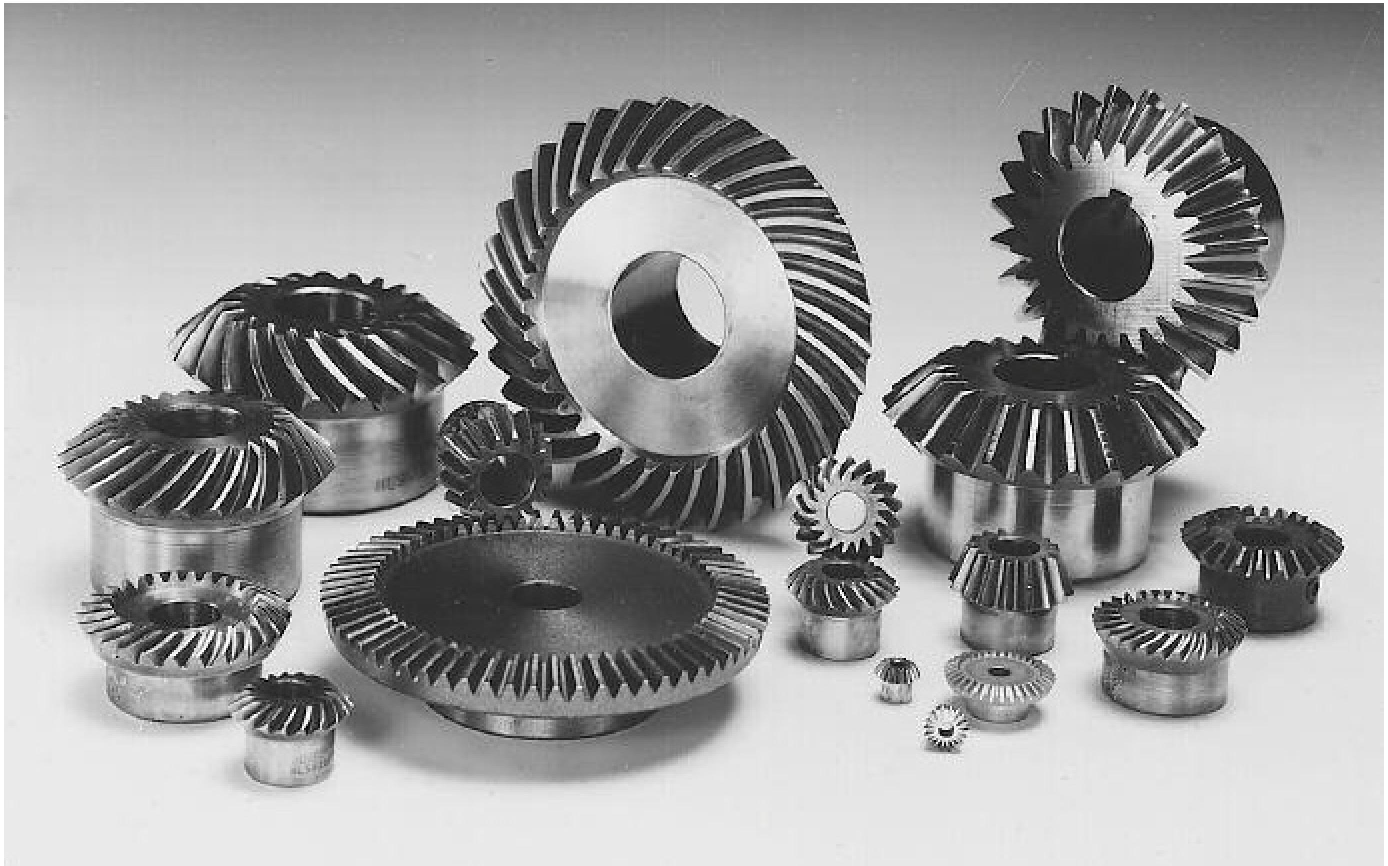
Bevel Gear



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Types of Bevel gears (1)

เฟืองดอกจอก (Bevel Gears) ใช้ส่งกำลังสำหรับเพลาที่วางทำมุมกัน (ส่วนมากเป็น 90°)

Straight bevel gear

- ราคาถูก
- ใช้กรณีที่ Noise ไม่เป็นปัจจัยที่สำคัญ
- ใช้ถึง Pitch line velocity ประมาณ 5 m/s

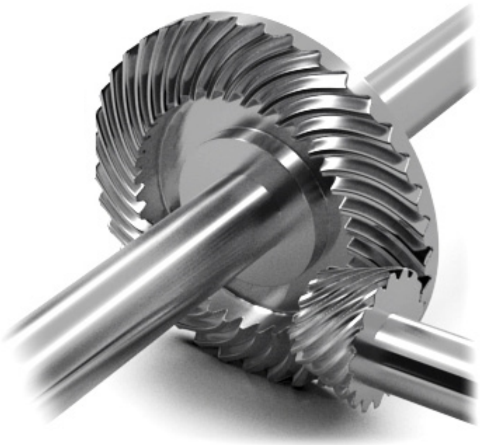
Spiral bevel gear

- ใช้เมื่อ Noise เป็นปัจจัยสำคัญ
- สามารถใช้ที่ความเร็วสูงกว่า Straight bevel gear

Types of Bevel gears (2)

Zerol bevel gear

- มีฟันโค้ง แต่มุมเอียง 0 องศา
- ใช้ทดแทน Straight bevel gear ได้ และทำงานได้เรียบกว่า

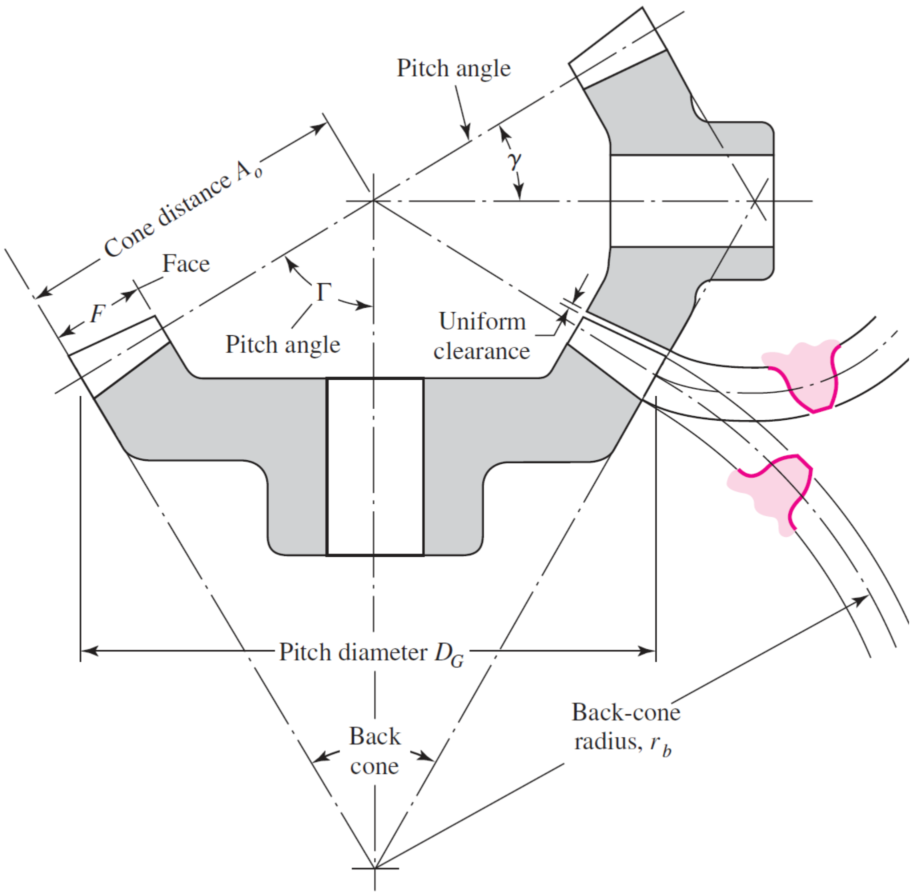


Hypoid gear

- คล้าย Spiral bevel gear แต่เพลาวางเยื้องและไม่ตัดกัน
- ใช้ในชุดเฟือง Differential ในรถยนต์

เฟืองดอกจอก (Bevel Gears) ใช้ส่งกำลังสำหรับเพลาที่วางทำมุมกัน 90° และมีอัตราทด 1:1 (มุมโคน 45° มีชื่อเรียกเฉพาะว่า **Miter gear (Mitre gear)**

Types of Bevel gears



- **พิตซ์วัดที่ปลายด้านใหญ่ของเฟือง**
- Circular pitch, Pitch diameter คิดเหมือนเฟืองตรง
- Standard straight bevel gear ใช้ 20° pressure angle
- Pitch angle หาจาก

$$\tan \gamma = \frac{N_P}{N_G}$$

$$\tan \Gamma = \frac{N_G}{N_P}$$

Tooth Proportions

Tooth Proportions for 20° Straight Bevel-Gear Teeth

Item	Formula
Working depth	$h_k = 2.0/P$
Clearance	$c = (0.188/P) + 0.002$ in
Addendum of gear	$a_G = \dfrac{0.54}{P} + \dfrac{0.460}{P(m_{90})^2}$
Gear ratio	$m_G = N_G/N_P$
Equivalent 90° ratio	$m_{90} = m_G$ when $\Gamma = 90^\circ$
	$m_{90} = \sqrt{m_G \dfrac{\cos\gamma}{\cos\Gamma}}$ when $\Gamma \neq 90^\circ$
Face width	$F = 0.3A_0$ or $F = \dfrac{10}{P}$, whichever is smaller
Minimum number of teeth	Pinion: 16, 15, 14, 13 / Gear: 16, 17, 20, 30

Minimum number of teeth:

Pinion	16	15	14	13
Gear	16	17	20	30

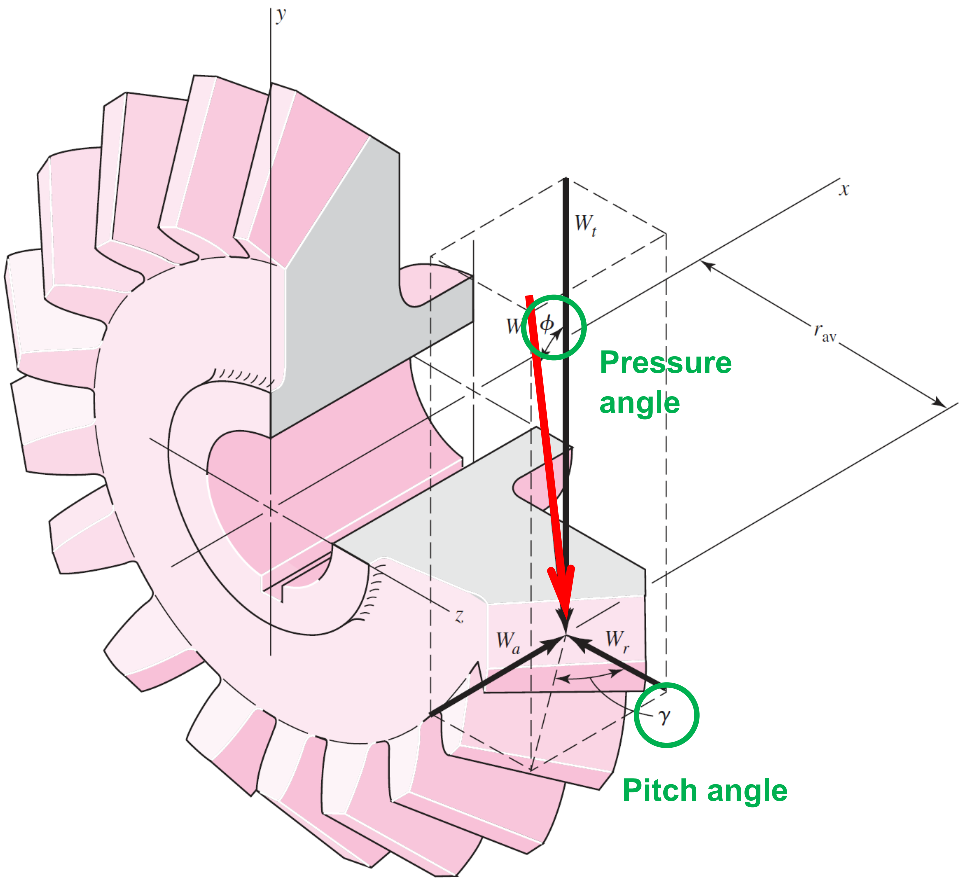
P = diametral pitch, teeth per inch

→ ปลายด้านใหญ่ของ Bevel gear

→ เลือกที่คำนวณได้น้อยกว่า

หน่วยเป็นนิ้ว

Gear force analysis



สมมุติให้แรงกระทำที่ตรงกึ่งกลางหน้าฟัน และอยู่บนผิวพิตซ์

แรงส่งกำลัง $W_t = T/r_{av}$

$W_t = W\cos\phi$

แรงแนวรัศมี $W_r = W\sin\phi\cos\gamma$

แรงแนวแกน $W_a = W\sin\phi\sin\gamma$

แรงเหล่านี้ใช้คำนวณภาระที่กระทำกับเพลาต่อไป

AGMA Stress Equation (bending)

American Gear Manufacturers Association (AGMA) ได้แนะนำการออกแบบเฟืองดอกจอกดังนี้

AGMA Equation (bending)

$$s_t = \frac{W_t P_d}{F Y_J} \cdot \frac{K_O K_s K_m K_v}{K_x}$$

U.S. customary units

- W_t : Tangential force
- P_d : Outer transverse diametral pitch
- F : Face width

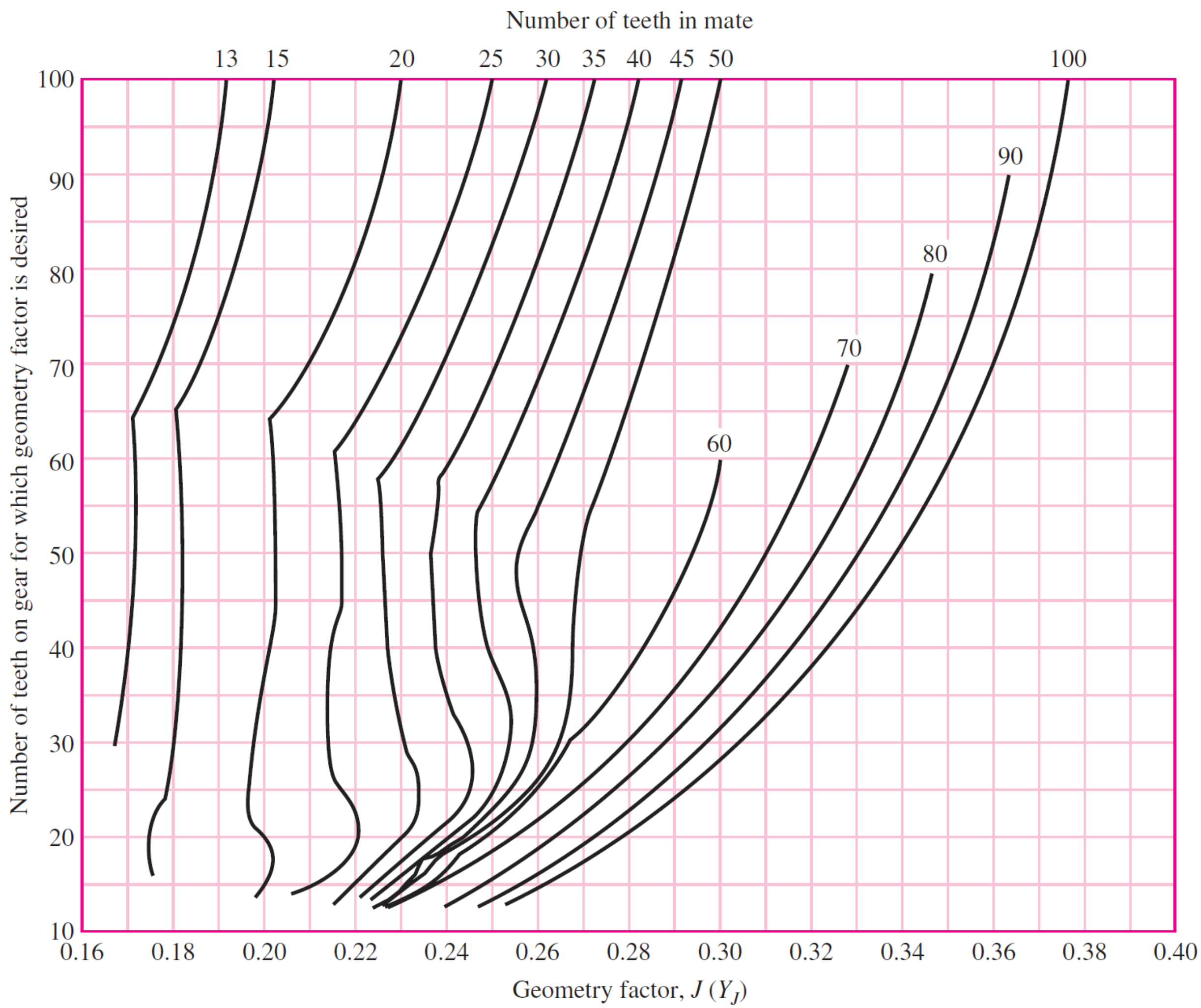
- Y_J : Geometry factor for bending strength
- K_O : Overload factor
- K_s : Size factor
- K_m : Load-distribution factor
- K_v : Dynamic factor
- K_x : Lengthwise curvature factor

P_d คิดที่ปลายด้านใหญ่ของเฟืองดอกจอก
ดังนั้น W_t จะเป็นแรงที่ทำที่ปลายด้านใหญ่เช่นกัน

$$W_t = 2T/d_p$$

Geometry factor (Y_J)

$$s_t = \frac{W_t P_d}{F Y_J} \cdot \frac{K_O K_s K_m K_v}{K_x}$$



Straight-bevel gears with a 20° normal pressure angle and 90° shaft angle

K_O and K_S

$$s_t = \frac{W_t P_d}{F Y_J} \cdot \frac{K_O K_s K_m K_v}{K_x}$$

Overload Factor K_O

Character of Prime Mover	Character of Load on Driven Machine: Uniform	Light Shock	Medium Shock	Heavy Shock
Uniform	1.00	1.25	1.50	1.75 or higher
Light shock	1.10	1.35	1.60	1.85 or higher
Medium shock	1.25	1.50	1.75	2.00 or higher
Heavy shock	1.50	1.75	2.00	2.25 or higher

Note: This table is for speed-decreasing drives. For speed-increasing drives, add $0.01(N/n)^2$ or $0.01(z_2/z_1)^2$ to the above factors.

Size Factor for Bending K_S

U.S. customary units

$$K_S = 0.4867 + 0.2132/P_d$$

$0.5 \leq P_d \leq 16$ teeth/in

$$K_S = 0.5$$

$P_d > 16$ teeth/in

K_m and K_x

$$s_t = \frac{W_t P_d}{F Y_J} \cdot \frac{K_O K_s K_m K_v}{K_x}$$

Load-Distribution Factor K_m

U.S. customary units

$$K_m = K_{mb} + 0.0036 F^2$$

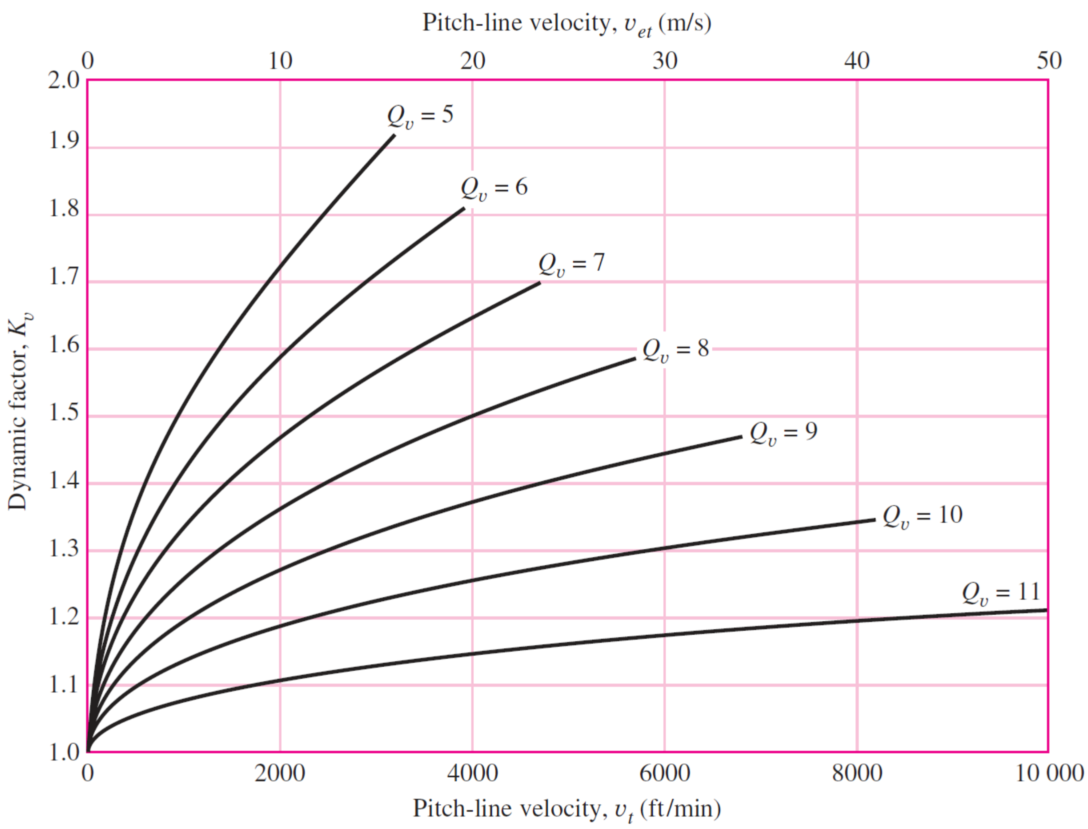
$$K_{mb} = \begin{cases} 1.00 : \text{both members straddle-mounted} \\ 1.10 : \text{one member straddle-mounted} \\ 1.25 : \text{neither member straddle-mounted} \end{cases}$$

Lengthwise Curvature Factor K_x

$K_x = 1$ for straight-bevel gears

Dynamic factor (K_v)

$$s_t = \frac{W_t P_d}{F Y_J} \cdot \frac{K_O K_s K_m K_v}{K_x}$$



Q_v = Transmission accuracy number

$$K_v = \left(\frac{A + \sqrt{v_t}}{A} \right)^B$$

$$A = 50 + 56(1 - B)$$

$$B = 0.25(12 - Q_v)^{2/3}$$

Pitch line velocity (v_t) at **outside pitch diameter** in **ft/min**

$$v_t = \pi d_P n_P / 12$$

Maximum pitch-line velocity (ft/min)

$$v_{t\,\max} = [A + (Q_v - 3)]^2$$

Selection of material (bending stress)

AGMA Equation (bending)

คำนวณจากภาระที่เฟืองต้องรับ

$$s_t = \frac{W_t P_d}{F Y_J} \cdot \frac{K_O K_s K_m K_v}{K_x}$$

$<$

Permissible Bending Stress Numbers

ขึ้นกับสมบัติวัสดุ

$$s_{wt} = s_{at} \frac{K_L}{SF \cdot K_T \cdot K_R}$$

s_{at} : Allowable bending stress

K_L : Stress cycle number for bending strength

K_T : Temperature factor

K_R : Reliability factor

SF : factor of safety (design decision)

Allowable bending stress, S_{at}

$$s_{wt} = s_{at}\frac{K_L}{SF \cdot K_T \cdot K_R}$$

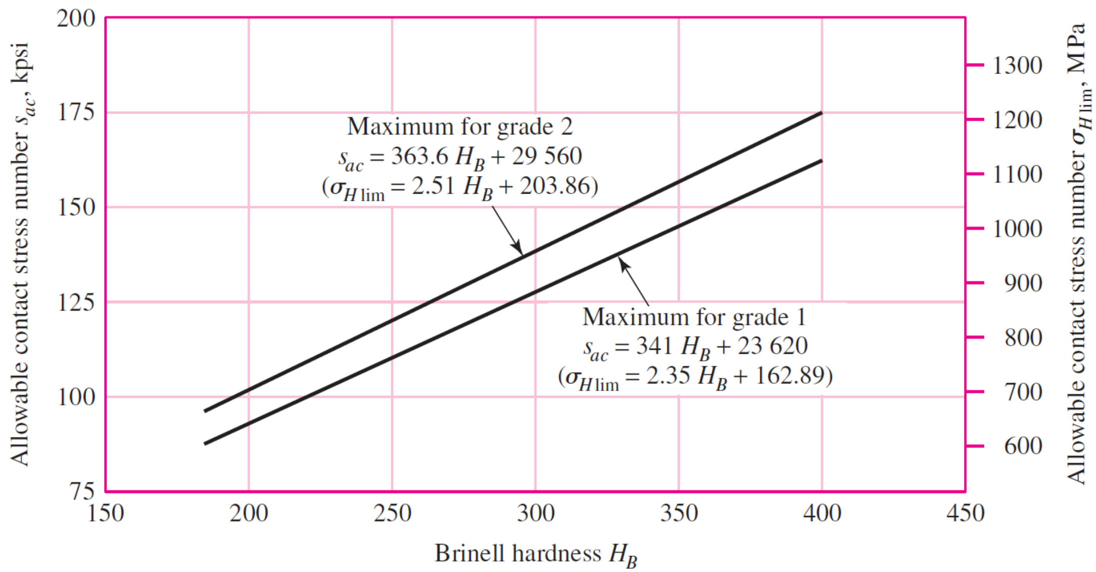
Material Designation	Heat Treatment	Minimum Surface Hardness	Bending Stress Number (Allowable), s_{at} ($\sigma_{F\,\lim}$) lbf/in² (N/mm²)		
			Grade 1*	Grade 2*	Grade 3*
Steel	Through-hardened	Fig. 15–13	Fig. 15–13	Fig. 15–13	
	Flame or induction hardened				
	Unhardened roots	50 HRC	15 000 (85)	13 500 (95)	
	Hardened roots		22 500 (154)		
	Carburized and case hardened†	2003-B97 Table 8	30 000 (205)	35 000 (240)	40 000 (275)
AISI 4140	Nitrided†,‡	84.5 HR15N		22 000 (150)	
Nitralloy 135M	Nitrided†,‡	90.0 HR15N		24 000 (165)	

*See ANSI/AGMA 2003-B97, Tables 8–11, for metallurgical factors for each stress grade of steel gears.

†The allowable stress numbers indicated may be used with the case depths prescribed in 21.1, ANSI/AGMA 2003-B97.

‡The overload capacity of nitrided gears is low. Since the shape of the effective S-N curve is flat, the sensitivity to shock should be investigated before proceeding with the design.

Allowable Bending Stress Numbers for Steel Gears

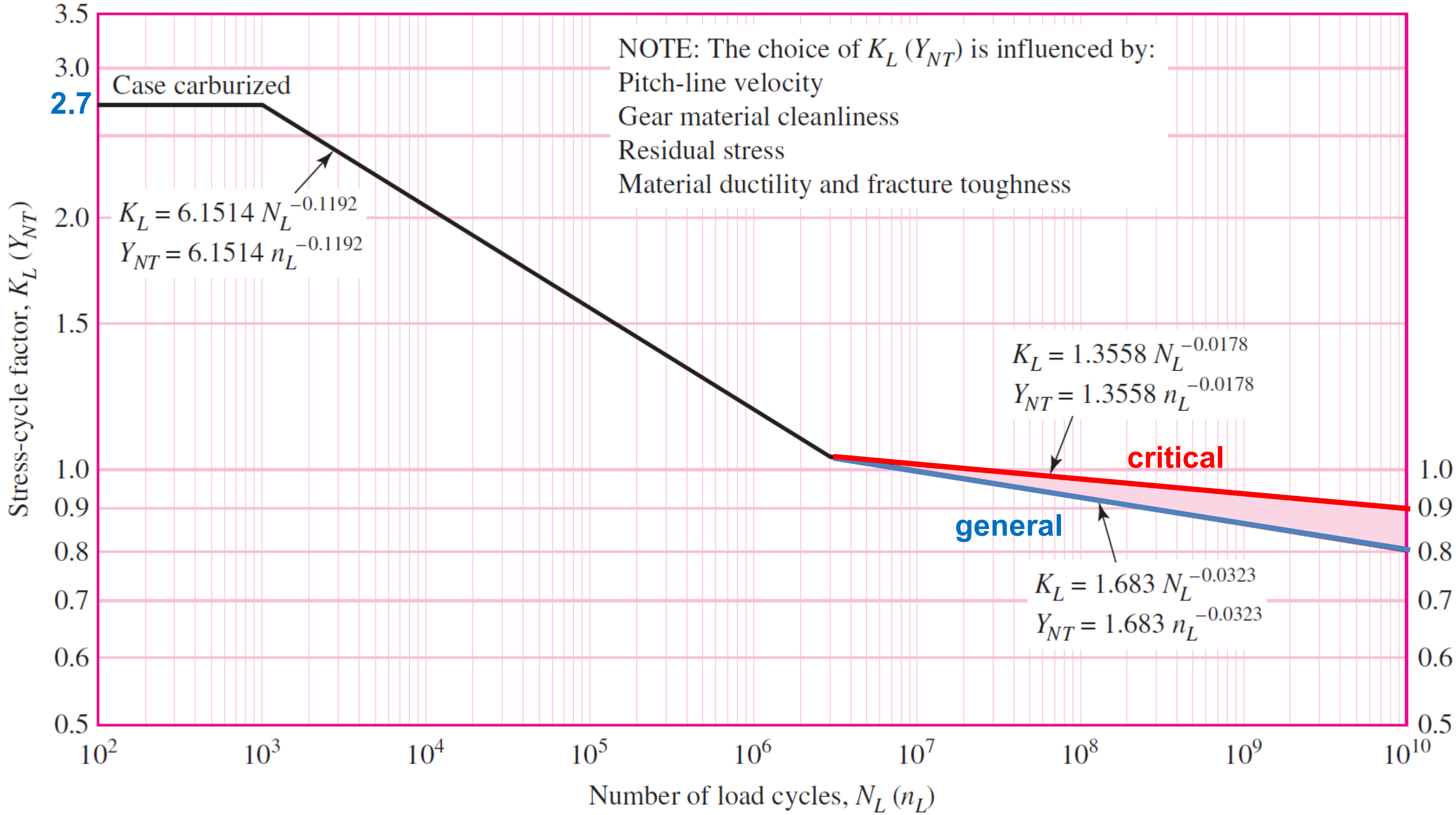


Allowable Bending Stress Numbers for through-hardened steel gears

Stress-Cycle Factor, K_L

$$s_{wt} = s_{at} \frac{K_L}{SF \cdot K_T \cdot K_R}$$

Stress-cycle factor for bending strength KL for carburized case-hardened steel bevel gears



K_T and K_R

$$s_{wt} = s_{at} \frac{K_L}{SF \cdot K_T \cdot K_R}$$

K_T : Temperature factor

$$K_T = \begin{cases} 1.00 & : 32^{\circ}F \le t \le 250\ ^{\circ}F \\ (460+t)/710 & : t > 250\ ^{\circ}F \end{cases}$$

K_R : Reliability factor

Requirements of Application	Reliability Factors for Steel* C_R (Z_Z)	K_R (Y_Z)†
Fewer than one failure in 10 000	1.22	1.50
Fewer than one failure in 1000	1.12	1.25
Fewer than one failure in 100	1.00	1.00
Fewer than one failure in 10	0.92	0.85‡
Fewer than one failure in 2	0.84	0.70§

*At the present time there are insufficient data concerning the reliability of bevel gears made from other materials.

†Tooth breakage is sometimes considered a greater hazard than pitting. In such cases a greater value of K_R (Y_Z) is selected for bending.

‡At this value plastic flow might occur rather than pitting.

§From test data extrapolation.

AGMA Stress Equation (contact)

AGMA Equation (Contact)

$$s_c = C_p\left[\frac{W_t}{Fd_P I}K_O K_v K_m C_s C_{xc}\right]^{1/2}$$

U.S. customary units

W_t : Tangential force
d_P : Pitch diameter (pinion)
F : Face width
I : Geometry factor for pitting resistance

C_s : Size factor for pitting resistance
C_{xc} : Crown factor for pitting resistance

K_O : Overload factor
K_m : Load-distribution factor
K_v : Dynamic factor

C_p : Elastic coefficient for pitting resistance

หาได้เช่นเดียวกับกรณี Spur/ helical gear

หาได้เช่นเดียวกับกรณี Bending stress

P_d คิดที่ปลายด้านใหญ่ของเฟืองดอกจอก
ดังนั้น W_t จะเป็นแรงที่ทำที่ปลายด้านใหญ่เช่นกัน

$$W_t = 2T/d_p$$

C_p, C_s and C_{xc}

$$s_c = C_p \left[\frac{W_t}{F d_P I} K_O K_v K_m C_s C_{xc} \right]^{1/2}$$

C_p : Elastic coefficient for pitting resistance

U.S. customary units

$$C_p = \left[\frac{1}{\pi[(1-\nu_P^2)/E_P + (1-\nu_G^2)/E_G]} \right]^{1/2}$$

E_P, E_G = Young's moduli for pinion and gear, psi

= Young's moduli for pinion and gear, N/mm^2

For steel

C_p = 2290 $(psi)^{1/2}$

C_p = 190 $(N/mm^2)^{1/2}$

C_s : Size factor for pitting resistance

U.S. customary units

$$C_s = \begin{cases} 0.5 & : F < 0.5 \text{ in} \\ 0.125F + 0.4375 & : 0.5 \leq F \leq 4.5 \text{ in} \\ 1 & : F > 4.5 \text{ in} \end{cases}$$

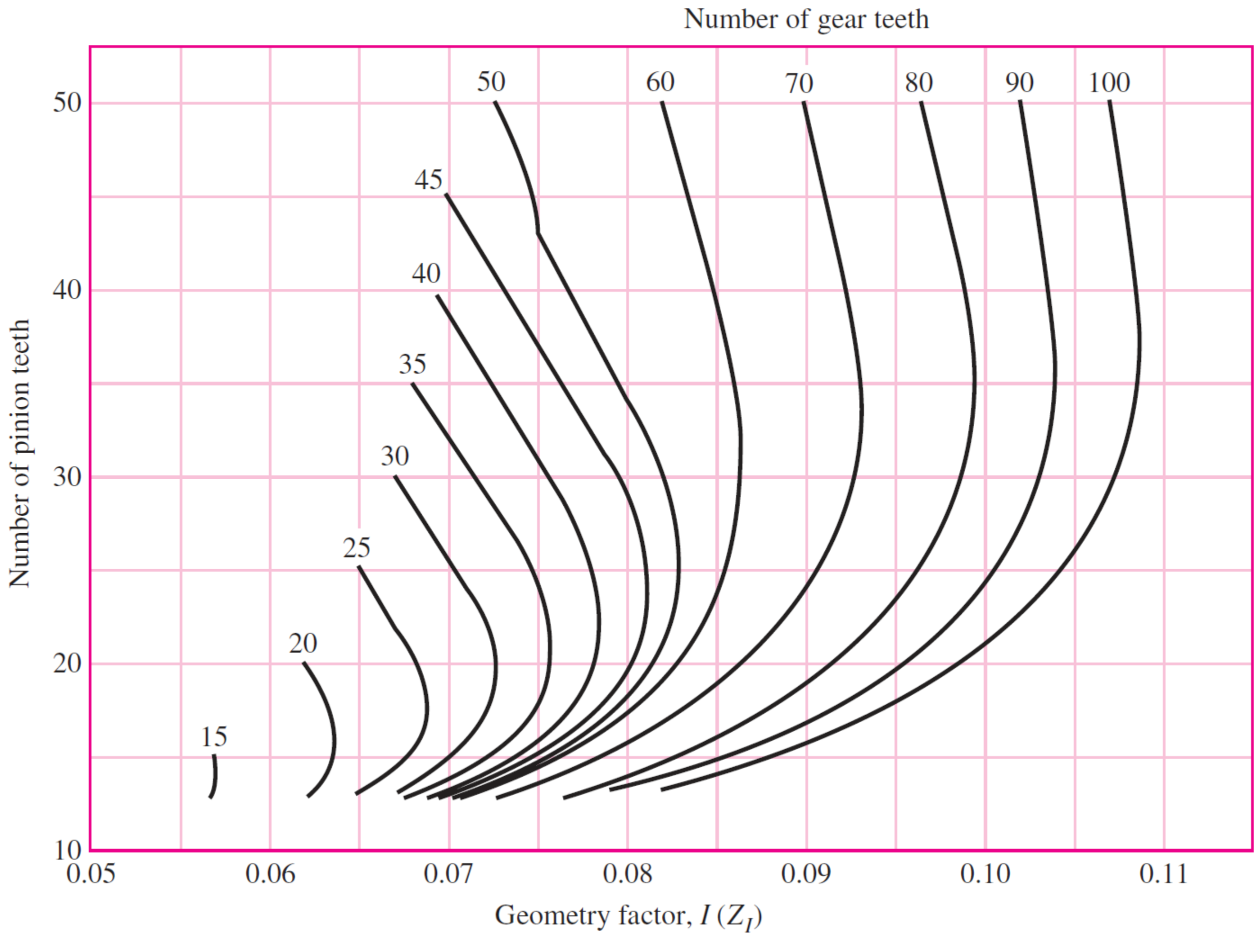
C_{xc} : Crowning factor for pitting

$$C_{xc} = \begin{cases} 1.5 & : \text{properly crowned teeth} \\ 2.0 & : \text{or larger uncrowned teeth} \end{cases}$$

Geometry factor, I

$$s_c = C_p \left[\frac{W_t}{F d_P I} K_O K_v K_m C_s C_{xc} \right]^{1/2}$$

Contact geometry factors, I, for coniflex straight-bevel gears with 20° normal pressure angle and a 90° shaft angle



Selection of material (contact stress)

AGMA Equation (contact)
คำนวณจากภาระที่เฟืองต้องรับ

$$s_c = C_p \left[\frac{W_t}{F d_P I} K_O K_v K_m C_s C_{xc} \right]^{1/2}$$

$<$

Permissible Contact Stress Numbers
ขึ้นกับสมบัติวัสดุ

$$s_{wc} = s_{ac} \frac{C_L \cdot C_H}{S_H \cdot K_T \cdot C_R}$$

s_{ac} : Allowable contact stress
C_L : Stress cycle factor for pitting resistance
C_H : Hardness ratio factor for pitting resistance
K_T : Temperature factor
C_R : Reliability factor for pitting
S_H : Contact factor of safety (design decision)

หาได้เช่นเดียวกับกรณี **Bending stress**

Allowable contact stress, S_{ac}

$$s_{wc} = s_{ac} \frac{C_L \cdot C_H}{S_H \cdot K_T \cdot C_R}$$

Allowable Contact Stress Numbers for Steel Gears

Material Designation	Heat Treatment	Minimum Surface* Hardness	Allowable Contact Stress Number, s_{ac} ($\sigma_{H\,\lim}$) lbf/in² (N/mm²) Grade 1†	Grade 2†	Grade 3†
Steel	Through-hardened‡	Fig. 15–12	Fig. 15–12	Fig. 15–12	
	Flame or induction hardened§	50 HRC	175 000 (1210)	190 000 (1310)	
	Carburized and case hardened§	2003-B97 Table 8	200 000 (1380)	225 000 (1550)	250 000 (1720)
AISI 4140	Nitrided§	84.5 HR15N		145 000 (1000)	
Nitralloy 135M	Nitrided§	90.0 HR15N		160 000 (1100)	

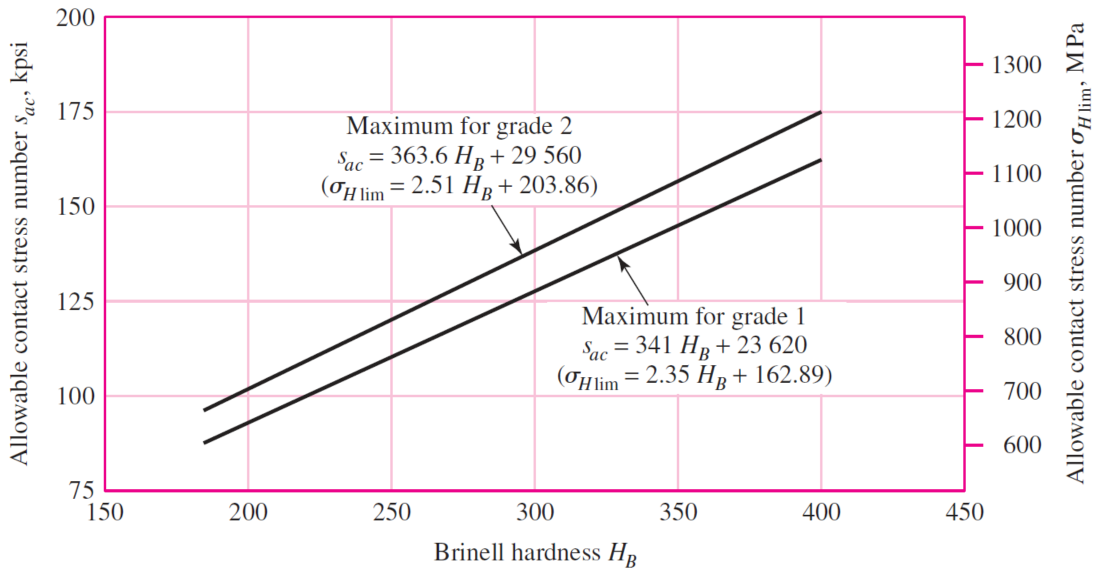
*Hardness to be equivalent to that at the tooth middepth in the center of the face width.

†See ANSI/AGMA 2003-B97, Tables 8 through 11, for metallurgical factors for each stress grade of steel gears.

‡These materials must be annealed or normalized as a minumum.

§The allowable stress numbers indicated may be used with the case depths prescribed in 21.1, ANSI/AGMA 2003-B97.

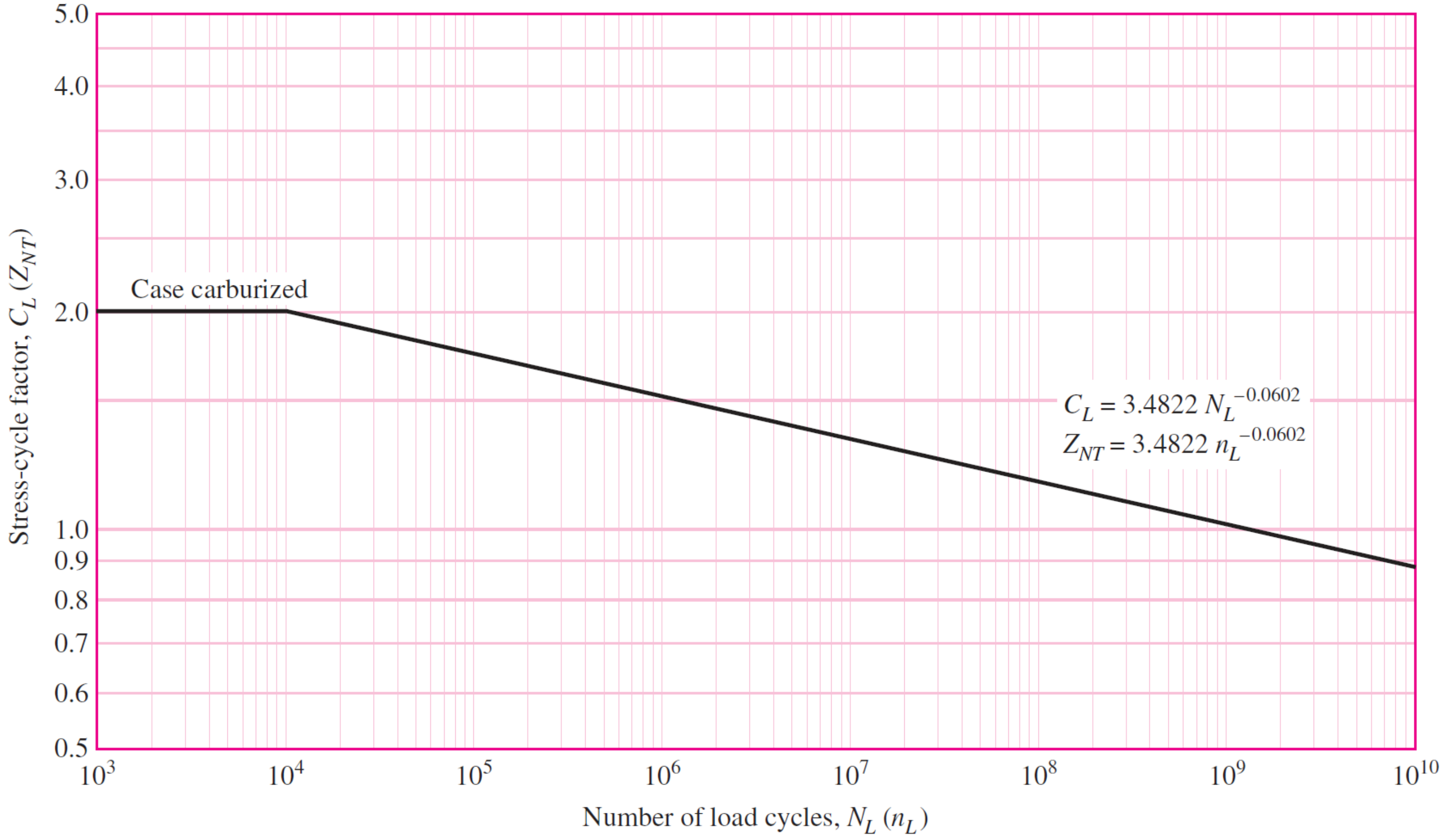
Allowable Contact Stress Numbers for through-hardened steel gears



Stress-Cycle Factor, C_L

$$s_{wc} = s_{ac}\frac{C_L \cdot C_H}{S_H \cdot K_T \cdot C_R}$$

Contact Stress-cycle factor for pitting resistance C_L for carburized case-hardened steel bevel gears

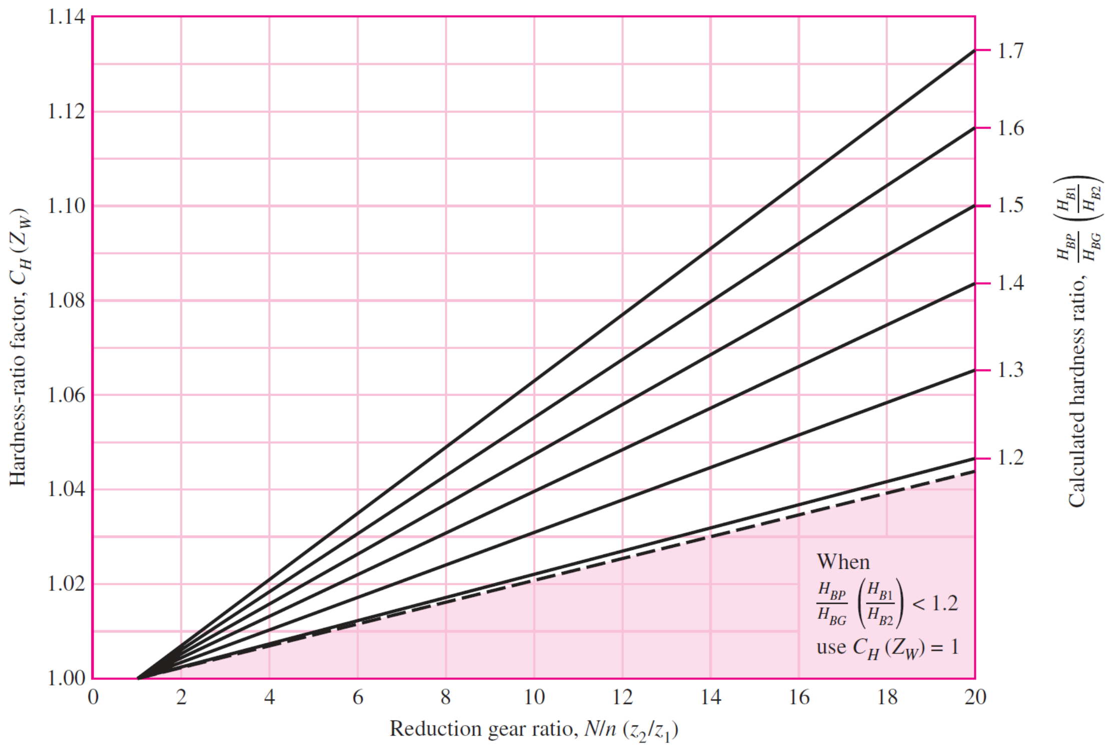


Hardness-Ratio Factor, C_H

$$s_{wc} = s_{ac} \frac{C_L \cdot C_H}{S_H \cdot K_T \cdot C_R}$$

- โดยปกติ Pinion มีขนาดเล็ก จึงมีรอบการหมุนขบมากกว่า Gear
- ถ้าให้ Pinion มีผิวแข็งกว่า Gear จะทำเพิ่ม capacity ของ pitting resistance
- H_{BP} : Brinell hardness of pinion
- H_{BG} : Brinell hardness of gear
- ถ้าใช้ความแข็ง Pinion - Gear เท่ากัน C_H = 1

Hardness-ratio factor CH for through-hardened pinion and gear



K_T and K_R

$$s_{wc} = s_{ac}\frac{C_L \cdot C_H}{S_H \cdot K_T \cdot C_R}$$

K_T : Temperature factor

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K_R : Reliability factor

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*At the present time there are insufficient data concerning the reliability of bevel gears made from other materials.

†Tooth breakage is sometimes considered a greater hazard than pitting. In such cases a greater value of K_R (Y_Z) is selected for bending.

‡At this value plastic flow might occur rather than pitting.

§From test data extrapolation.

Example

Design a straight-bevel gear mesh for shaft centerlines that intersect perpendicularly, to deliver 6.85 hp at 900 rev/min with a gear ratio of 3:1, temperature of 300°F, normal pressure angle of 20°, using a design factor of 2. The load is uniform-uniform. Although the minimum number of teeth on the pinion is 13, which is mesh with 31 or more teeth without interference, use a pinion of 20 teeth. The material is to be AGMA grade 1 and the teeth are to be crowned. The reliability goal is 0.995 with a pinion life of 10^9 revolutions [Ex.15-2 Shigley's Mechanical Engineering Design 9th, Richard G. Budynas, J. Keith Nisbett]