Basic Concepts of Vibration

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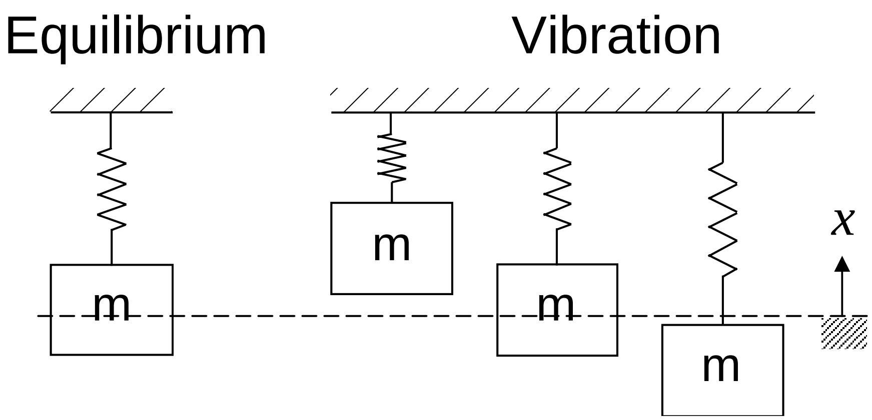
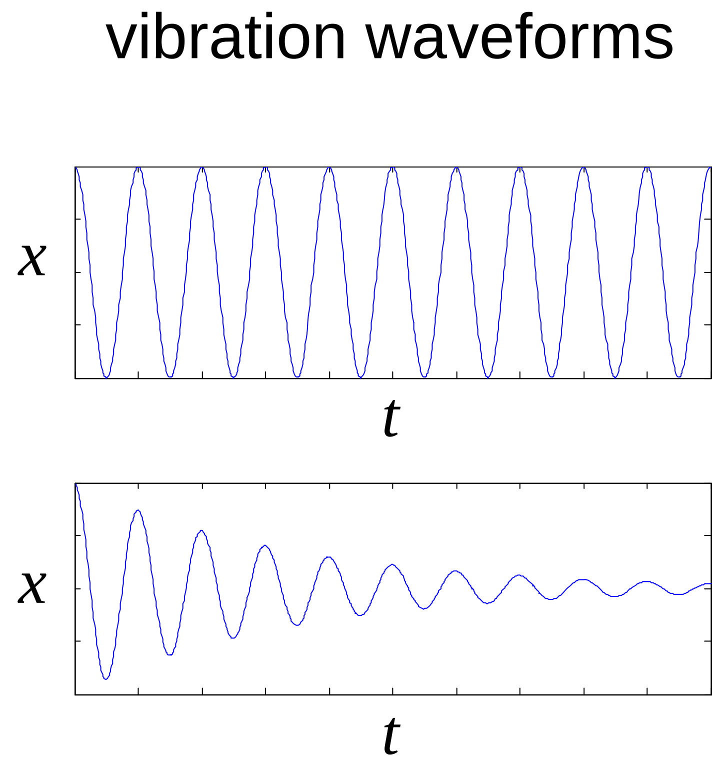
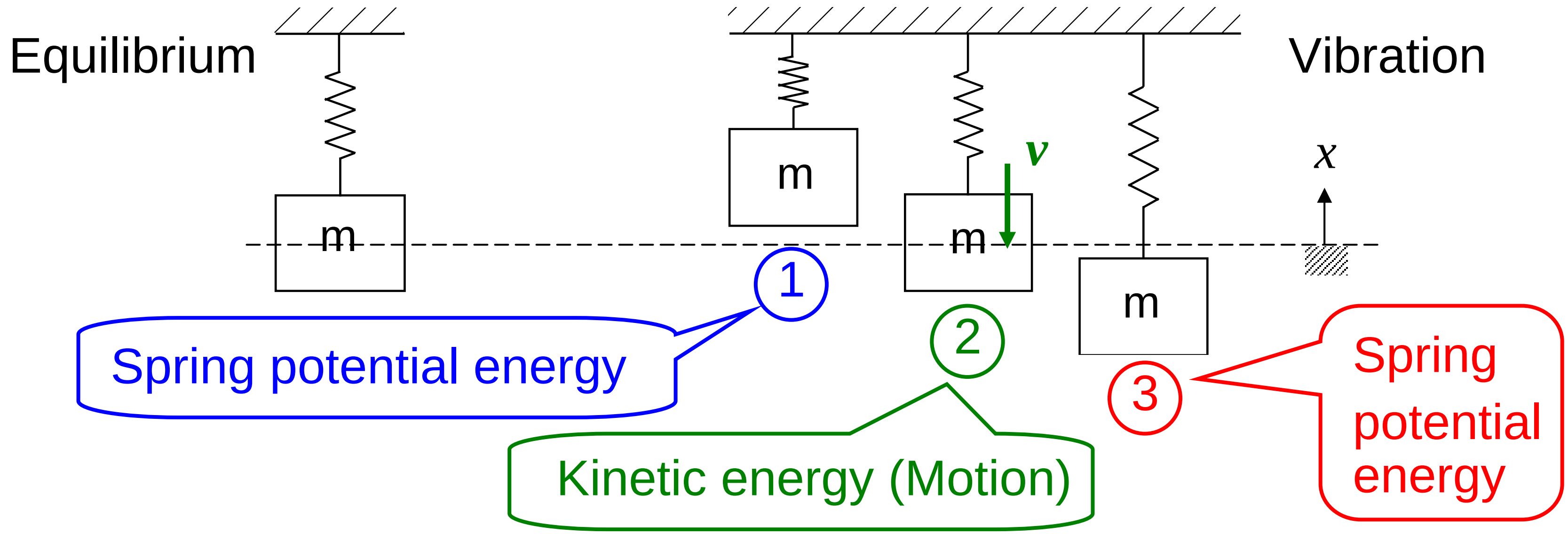
What is Vibration?

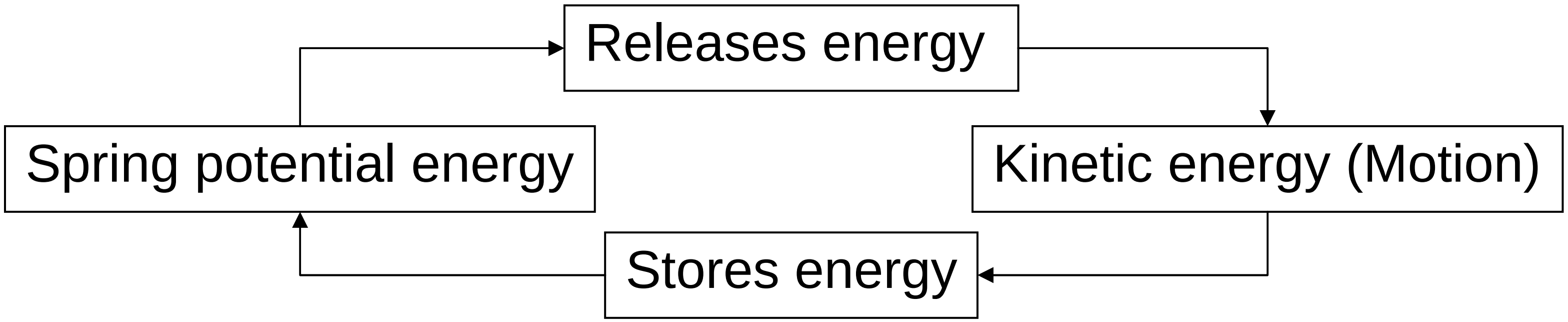
Vibration is the study of the repetitive motion of objects relative to a stationary frame of reference or nominal position.





Physical Explanation





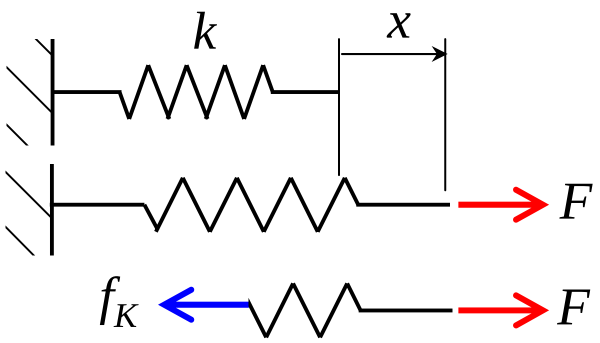
A component that stores and release potential energy is required.

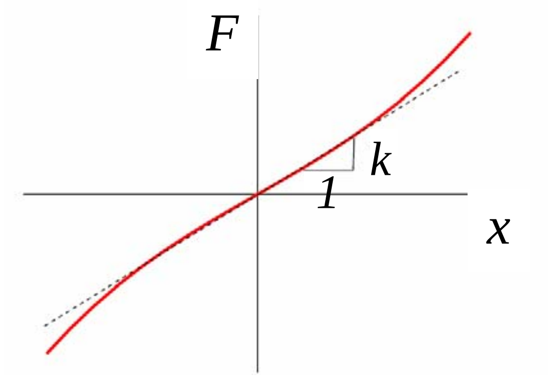
Elementary parts of vibrating systems (1)

1. Elastic components

Elastic components store or release potential (strain) energy as displacements increase or decrease.

e.g., helical spring, elastic bar & beam.





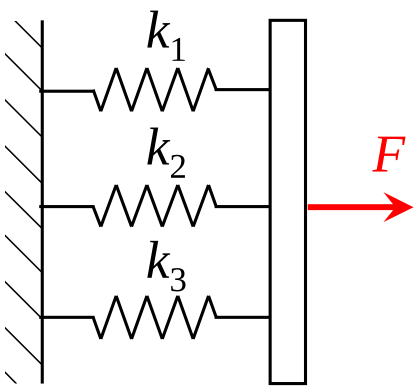
Restoring force $$f_k = -F = -kx$$

Potential energy $$V = \int_0^x kx\,dx = \frac{1}{2}kx^2$$

Elementary parts of vibrating systems (2)

Combination of springs

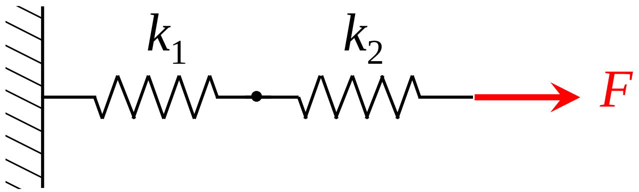
Parallel



$$k_{eq} = k_1 + k_2 + k_3$$

$$k_{eq} = \sum_{i=1}^{n} k_i$$ n springs

Series



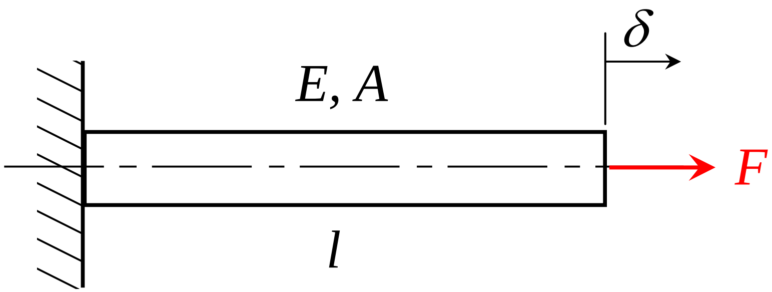
$$k_{eq} = \left(\frac{1}{k_1} + \frac{1}{k_2} \right)^{-1}$$

$$k_{eq} = \left(\sum_{i=1}^{n} \frac{1}{k_i} \right)^{-1}$$ n springs

Elementary parts of vibrating systems (3)

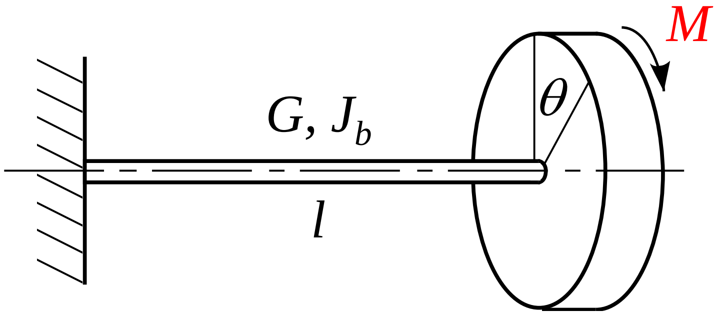
Elastic elements as springs

1. Thin rod



$$k_{eq} = \frac{F}{\delta} = \frac{EA}{l}$$

2. Torsional bar

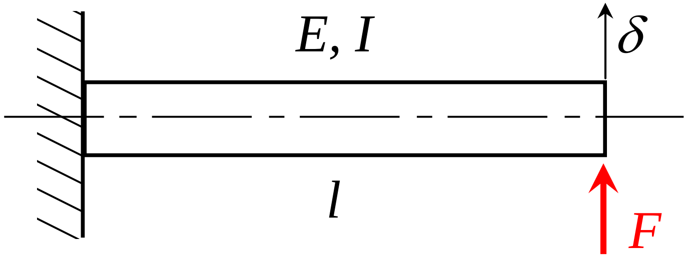


$$k_{eq} = \frac{M}{\theta} = \frac{GJ_b}{l}$$

Elementary parts of vibrating systems (4)

Elastic elements as springs

3. Cantilever beam



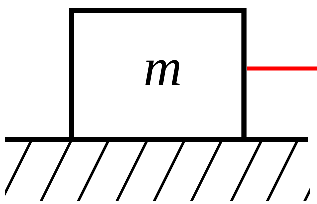
$$k_{eq} = \frac{F}{\delta} = \frac{3EI}{l^3}$$

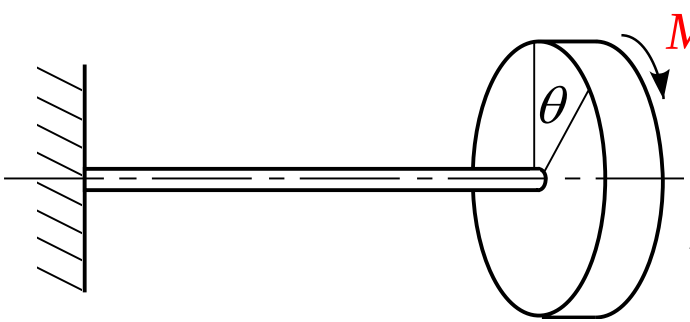
Elementary parts of vibrating systems (5)

2. Inertia (mass) components

Inertia components store or release kinetic energy as velocities increase or decrease.

e.g., mass (translation), mass moment of inertia (rotation)





Kinetic energy (translation)

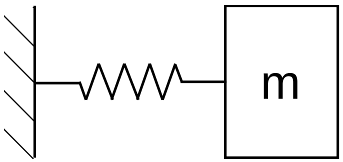
$$T = \frac{1}{2} m \dot{x}^2$$

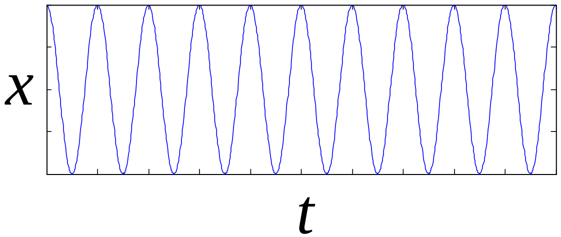
Kinetic energy (rotation)

$$T = \frac{1}{2} J \dot{\theta}^2$$

Elementary parts of vibrating systems (6)

Vibration of the spring-mass system



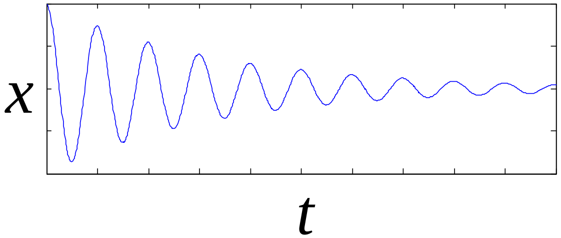


Ideal system

There is no energy loss during vibration.

The system will oscillate indefinitely.

Real system



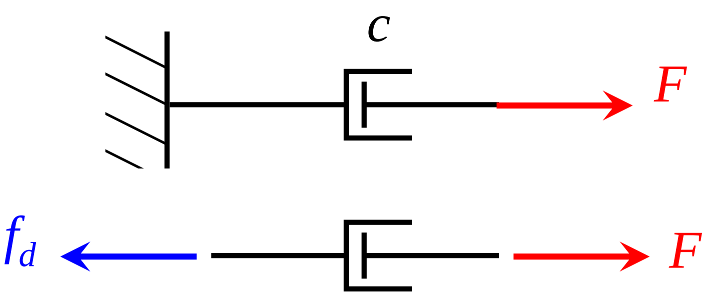
Oscillating systems eventually die out and reduce to zero motion.

There is a component that dissipates energy.

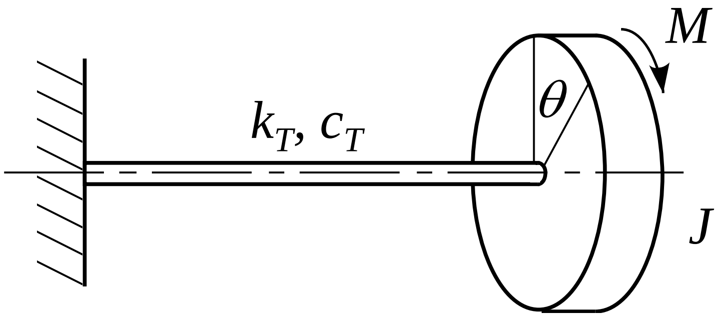
Elementary parts of vibrating systems (7)

3. Viscous damper

Viscous damper or dashpot dissipates energy. Energy is converted to heat or sound.



$$f_d = -F = -c\dot{x}$$



k, m, c for rotational motion

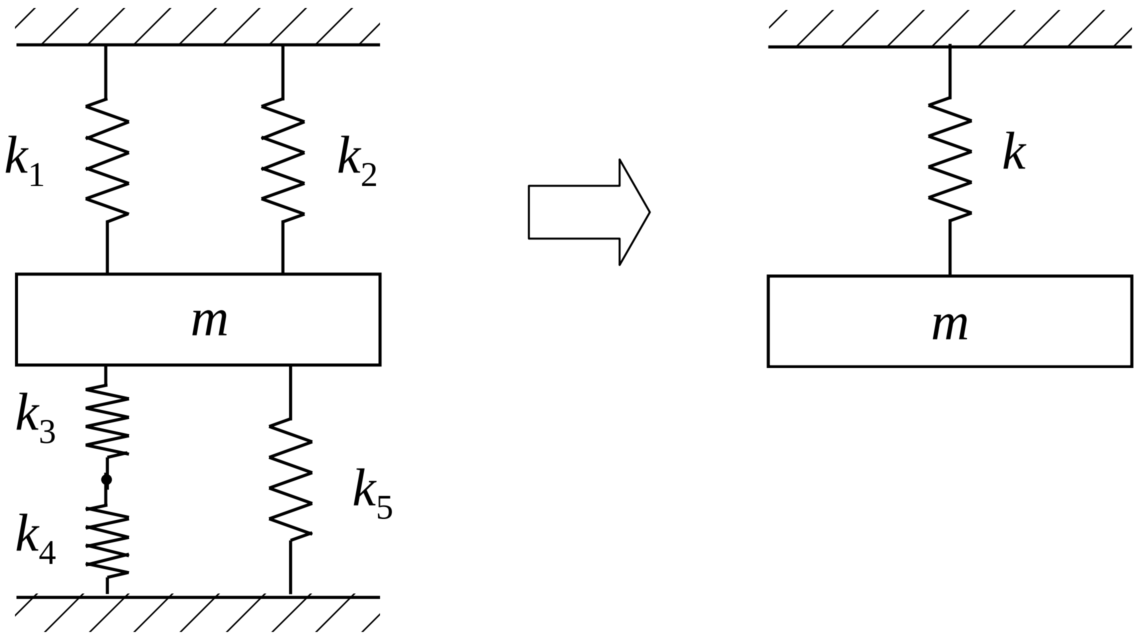
$$F_k = kx \quad \Rightarrow \quad M_k = k_T\theta$$

$$F_d = c\dot{x} \quad \Rightarrow \quad M_d = c_T\dot{\theta}$$

$$F = m\ddot{x} \quad \Rightarrow \quad M = J\ddot{\theta}$$

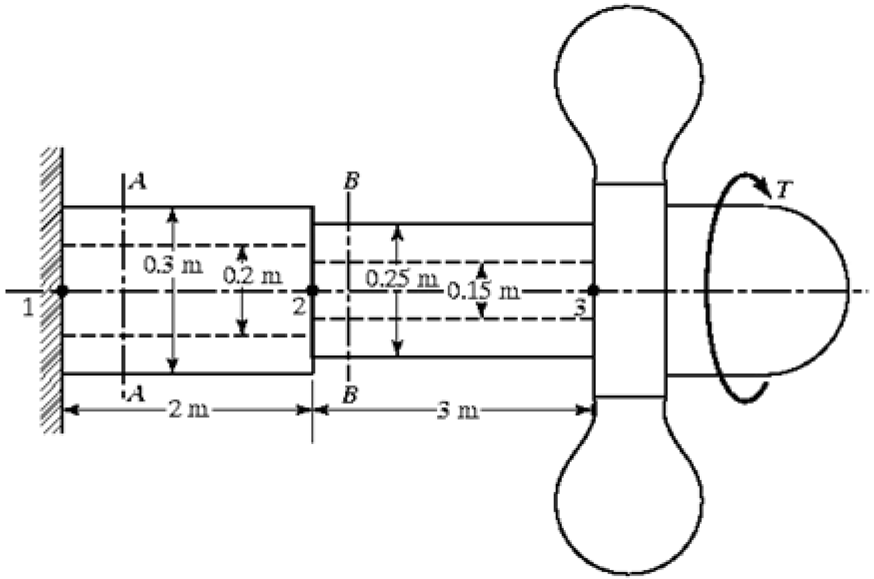
Combination of springs (Example)

Find the equivalent single stiffness representation of the five-spring system shown in the figure.



Springs (Example)

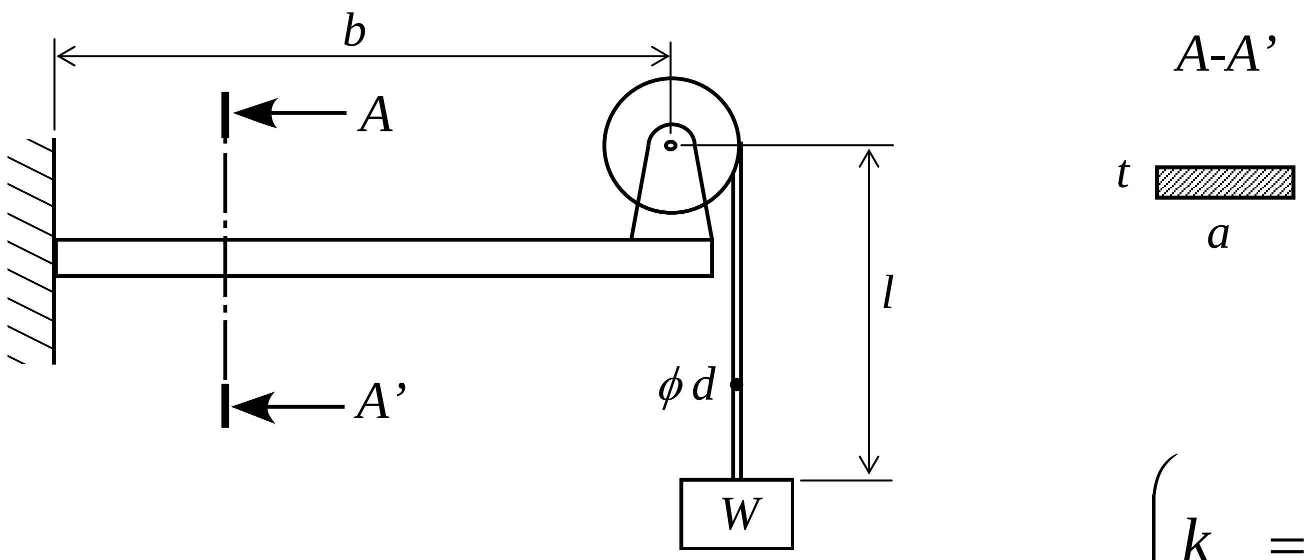
Determine the torsional spring constant of the steel propeller shaft shown in the figure. (Shear modulus $G = 80$ GPa)



($k_{eq} = 6.6\times10^6$ N-m/rad)

Springs (Example)

A hoisting drum, carrying a steel wire rope, is mounted at the end of a cantilever beam as shown in the figure. Determine the equivalent spring constant of the system when the suspended length of the wire rope is l. Assume that the net cross-sectional diameter of the wire rope is d and the Young's modulus of the beam and the wire rope is E.



$$\left(k_{eq} = \frac{E}{4}\left(\frac{\pi a t^3 d^2}{\pi d^2 b^3 + l a t^3} \right) \right)$$