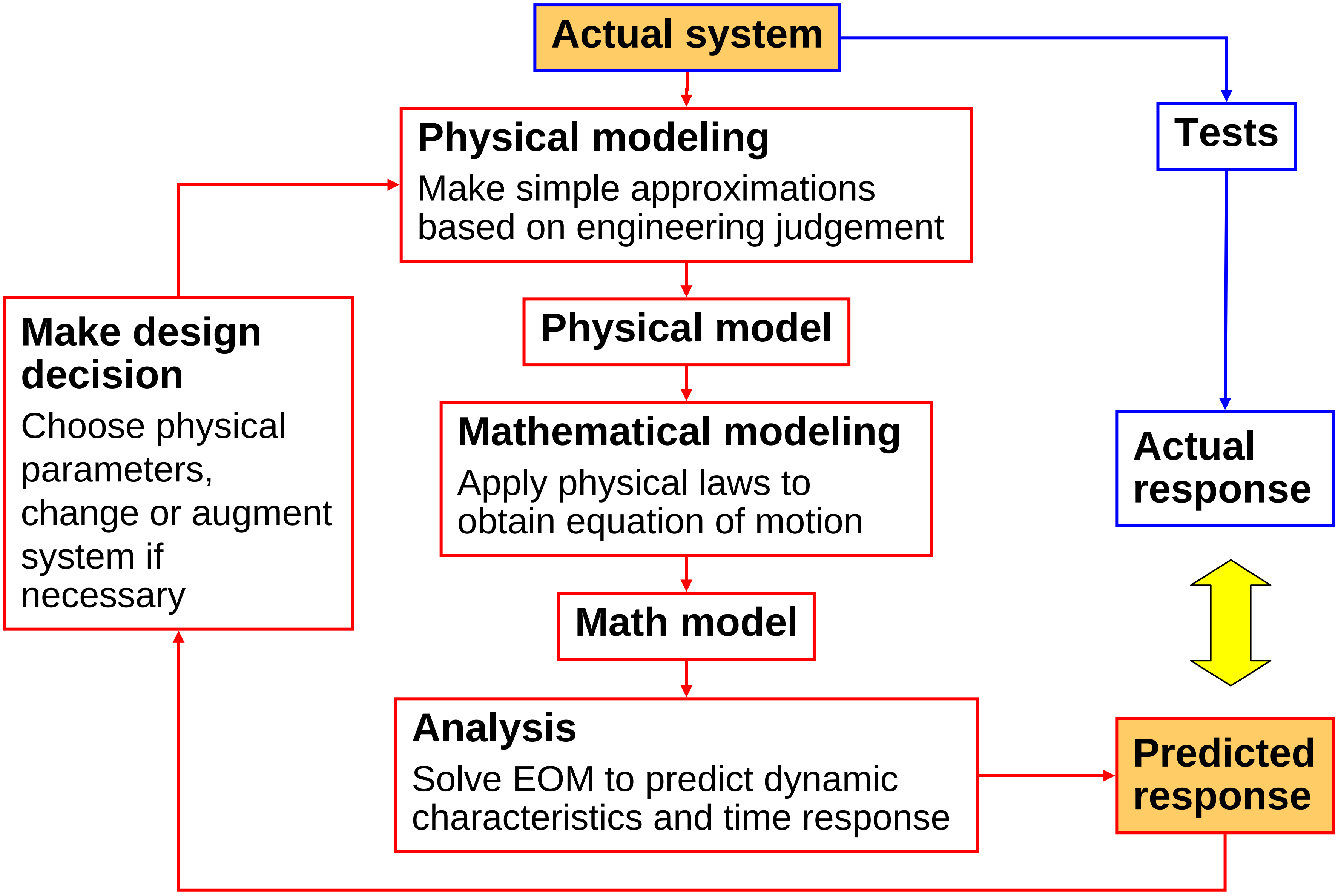
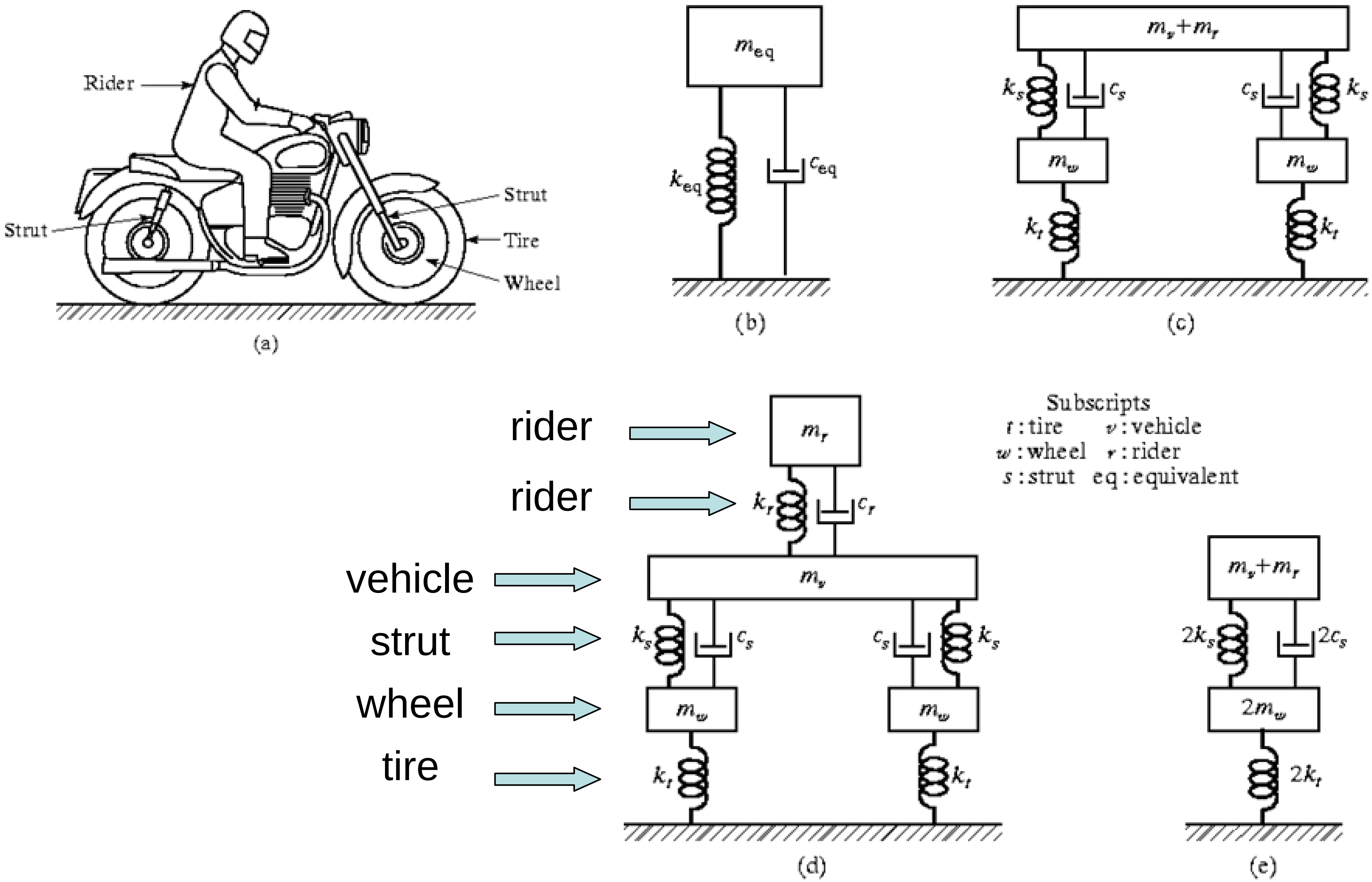
Modeling of vibration systems

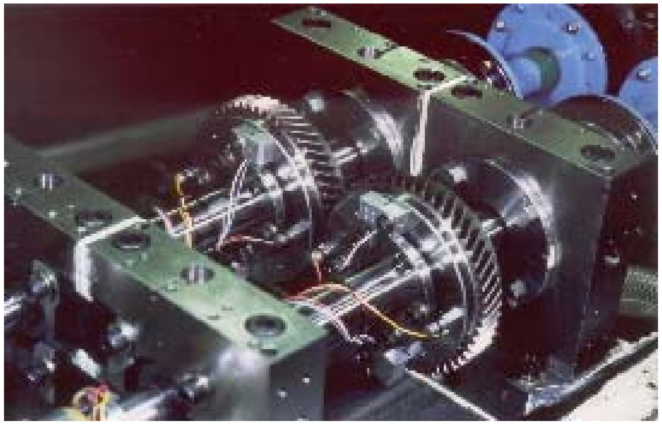


Modeling example (1)



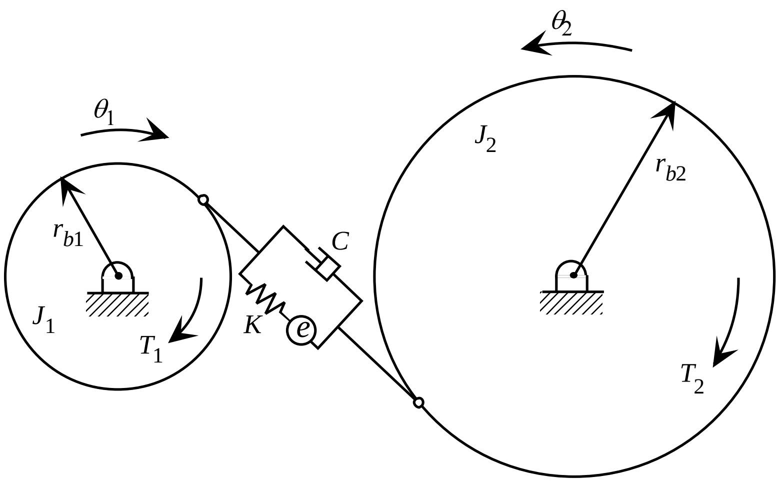
Modeling example (2)

Helical gear pair

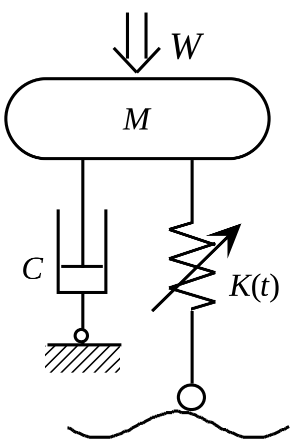


Equation of motion

$$M\ddot{x} + C\dot{x} + K(t)(x - e(t)) = W$$



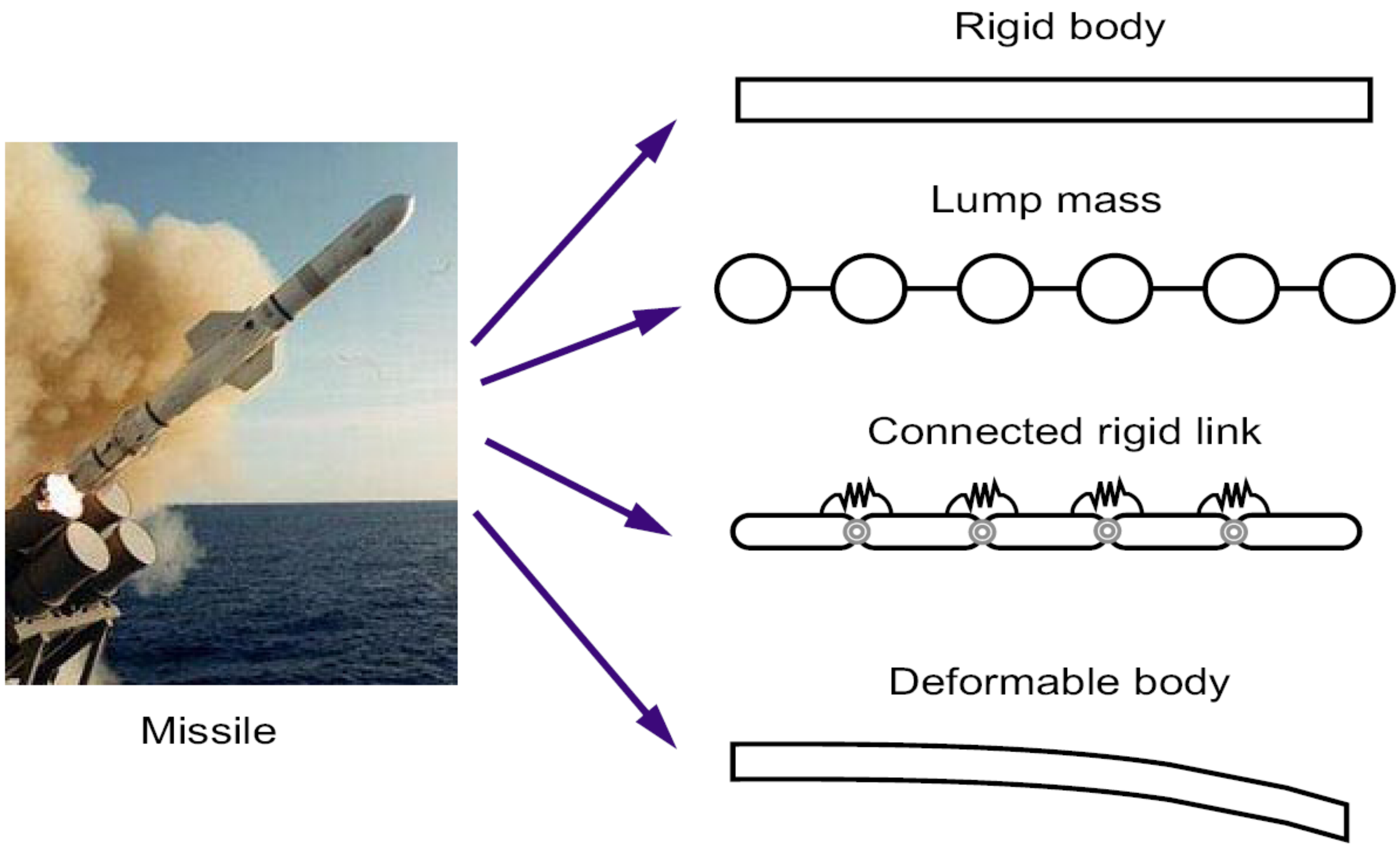
Physical model



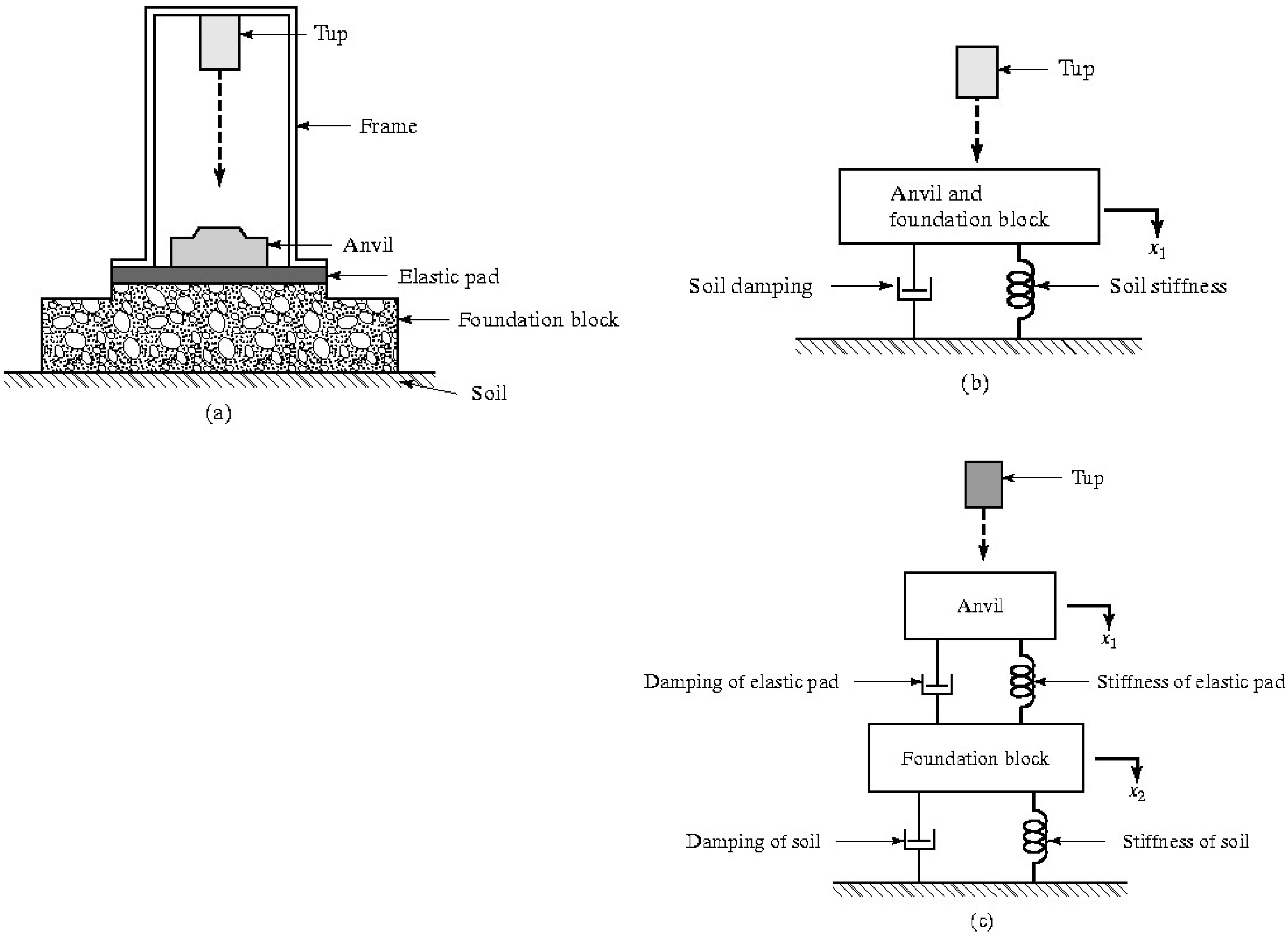
Physical model

Modeling example (3)

There is “*no unique*” physical models for one particular hardware

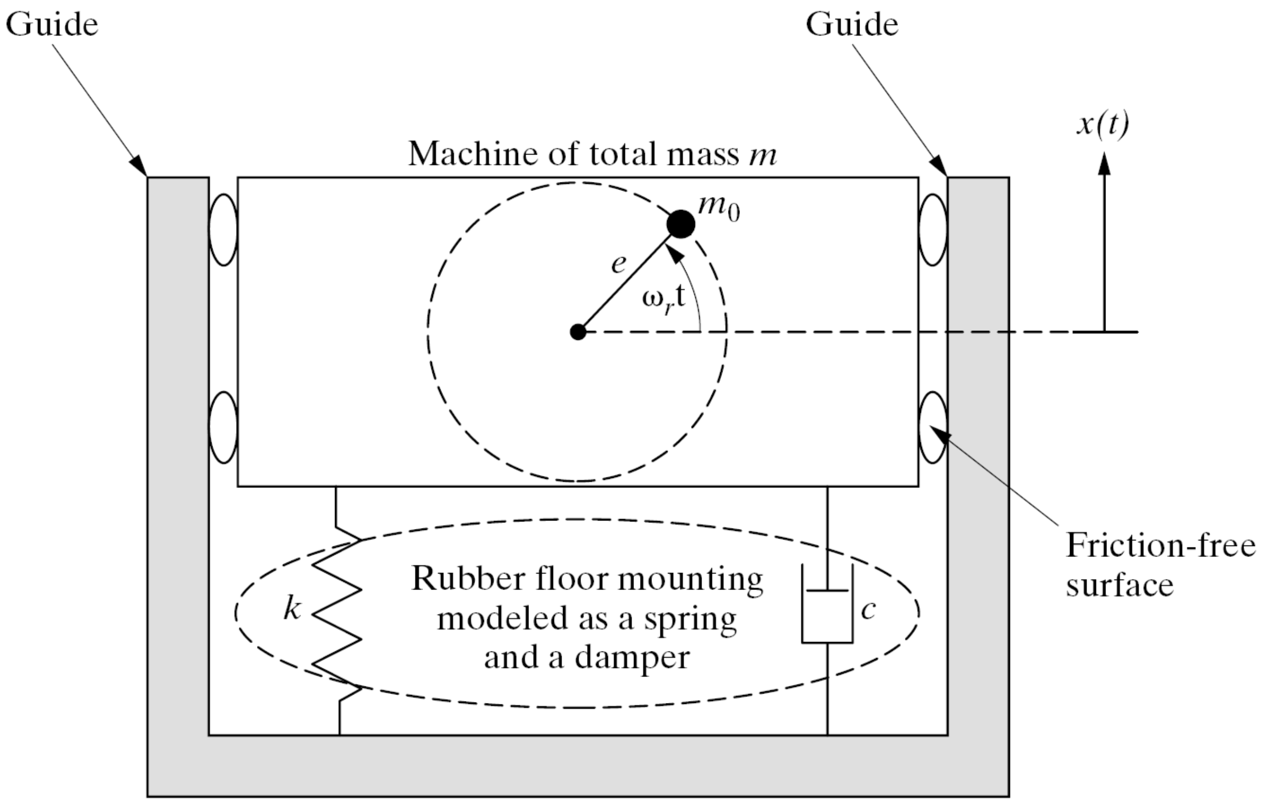


Example: A Forging hammer

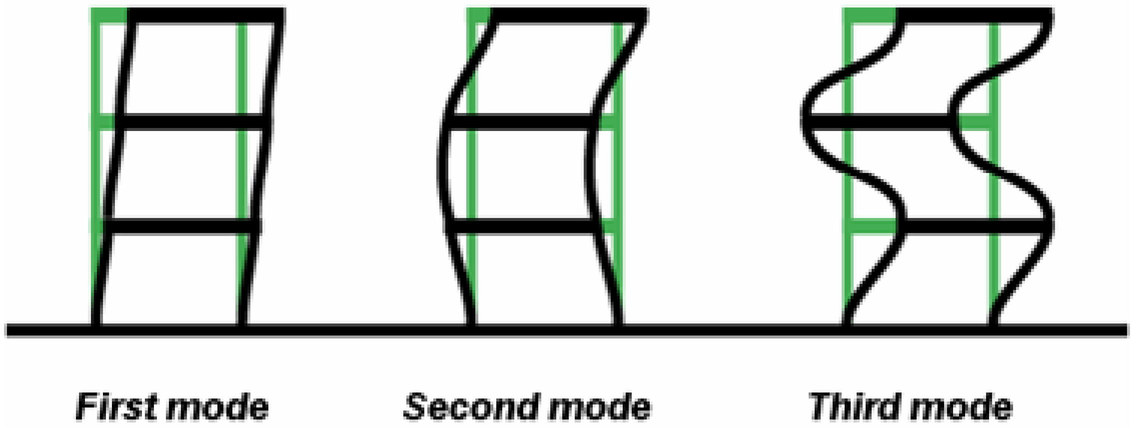


Example: Rotating unbalance

Cause: Small irregularities in the distribution of the mass in the rotating component

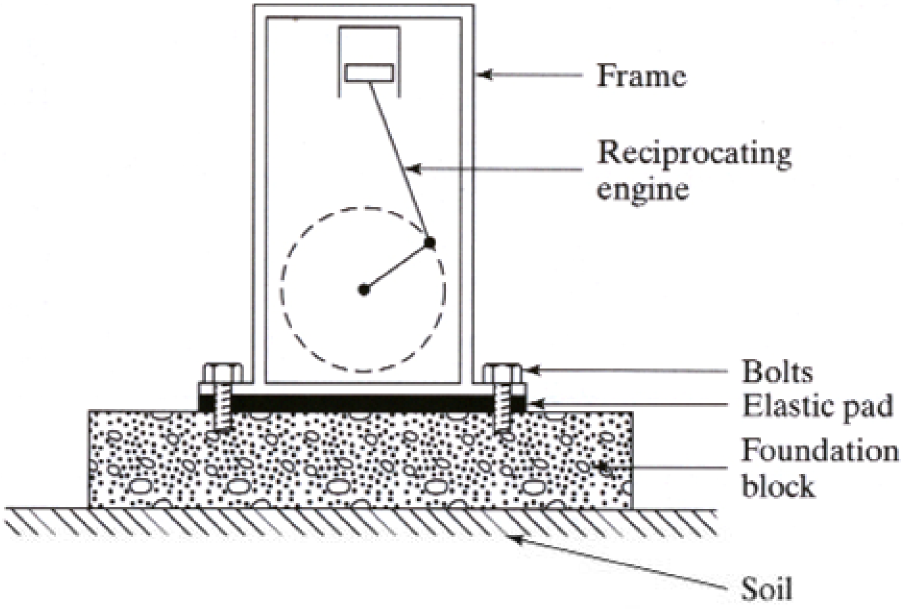


Example: Building



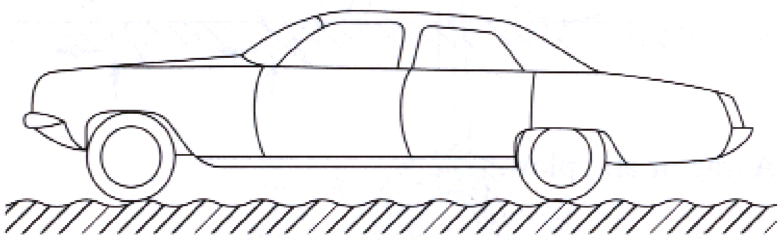
Example: Reciprocating engine

A reciprocating engine is mounted on a foundation as shown. The unbalanced forces and moments developed in the engine are transmitted to the frame and the foundation. An elastic pad is placed between the engine and the foundation block to reduce the transmission of vibration. Develop the vibration model.



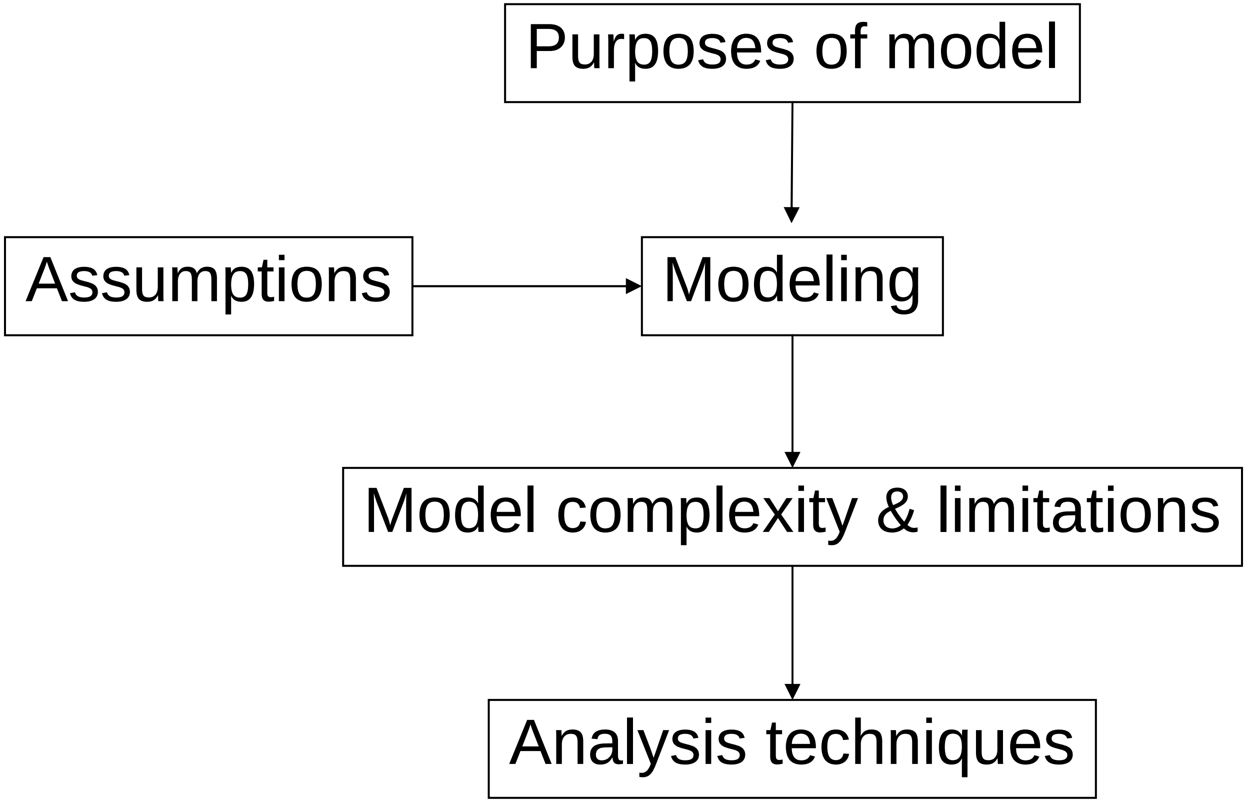
Example: An automobile

An automobile moving over a rough road can be modeled considering (a) weight of the car body, passengers, seats, front wheels, and rear wheels; (b) elasticity of tires, suspension; (c) damping of seats, front and rare suspensions.



Engineering judgement

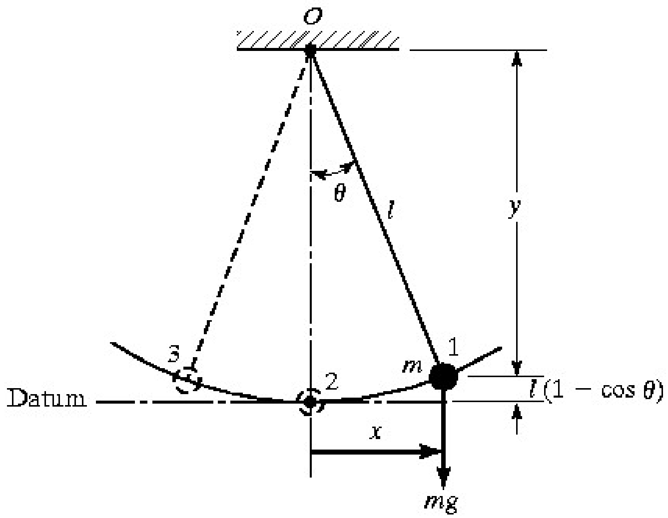
Modeling requires good “engineering judgement” and “experiences” with hardware.

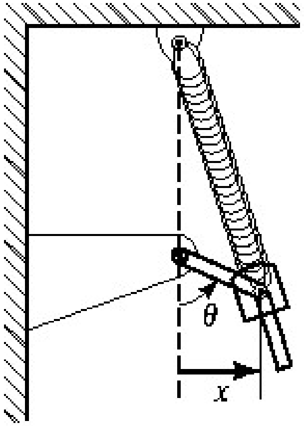


Degree of freedom (1)

Degree of freedom (DOF): The minimum number of independent coordinates required to determine completely the positions of all parts of a system at any instant of time.

Single degree of freedom systems

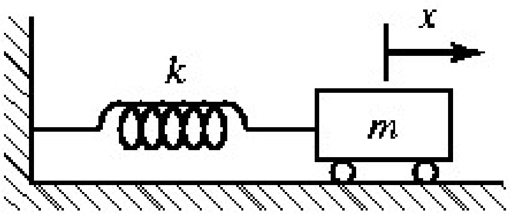




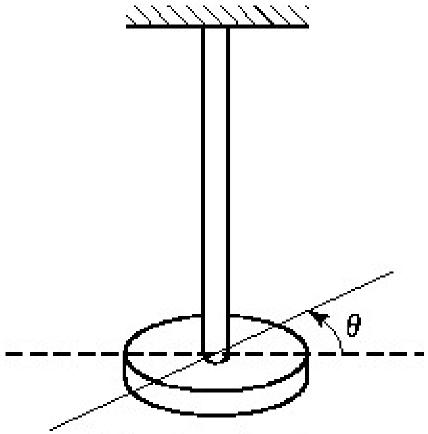
(a) Slider-crank-spring mechanism

Degree of freedom (2)

Single degree of freedom systems

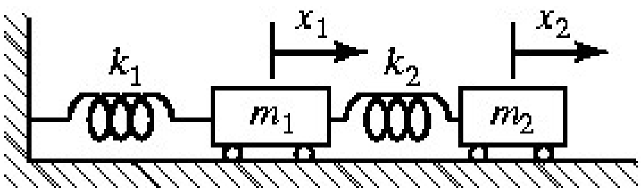


(b) Spring-mass system

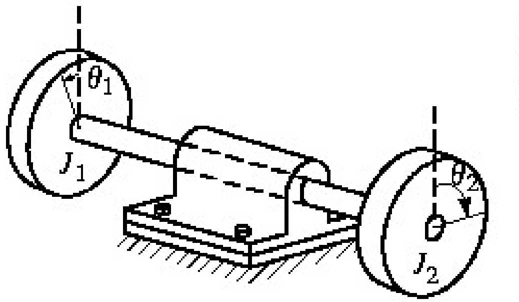


(c) Torsional system

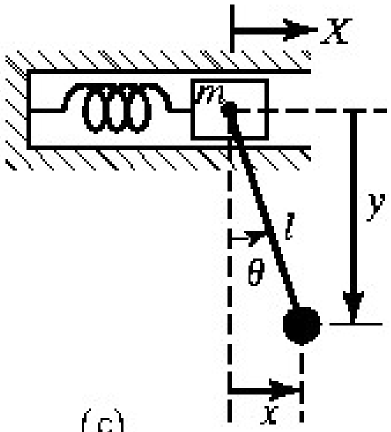
Two degree of freedom systems



(a)



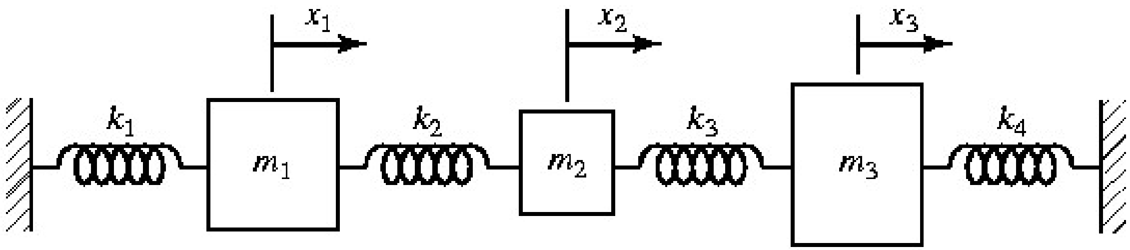
(b)



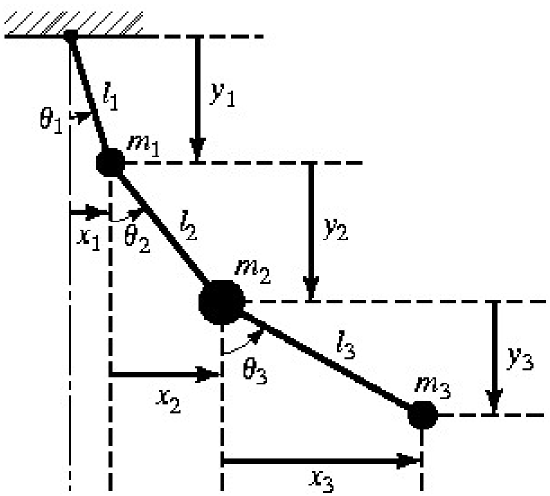
(c)

Degree of freedom (3)

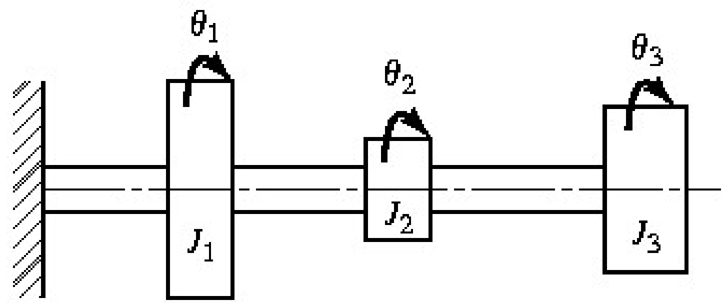
Three-degree of freedom systems



(a)



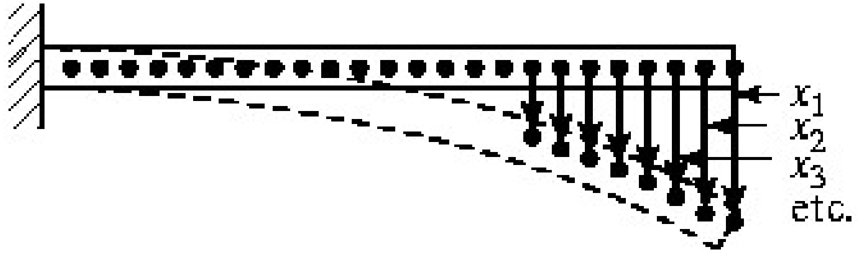
(b)



(c)

Degree of freedom (4)

Infinite-number-of-degrees-of-freedom systems (continuous or distributed systems)



Increasing number of degrees of freedom

- More accurate result
- More complexity

Equations of motion

Procedures

(1) Geometry

Define coordinates and their positive directions

Note degrees of freedom (DOF)

Write geometric constraints and compatibility

(2) Kinematics

Write necessary kinematic relations

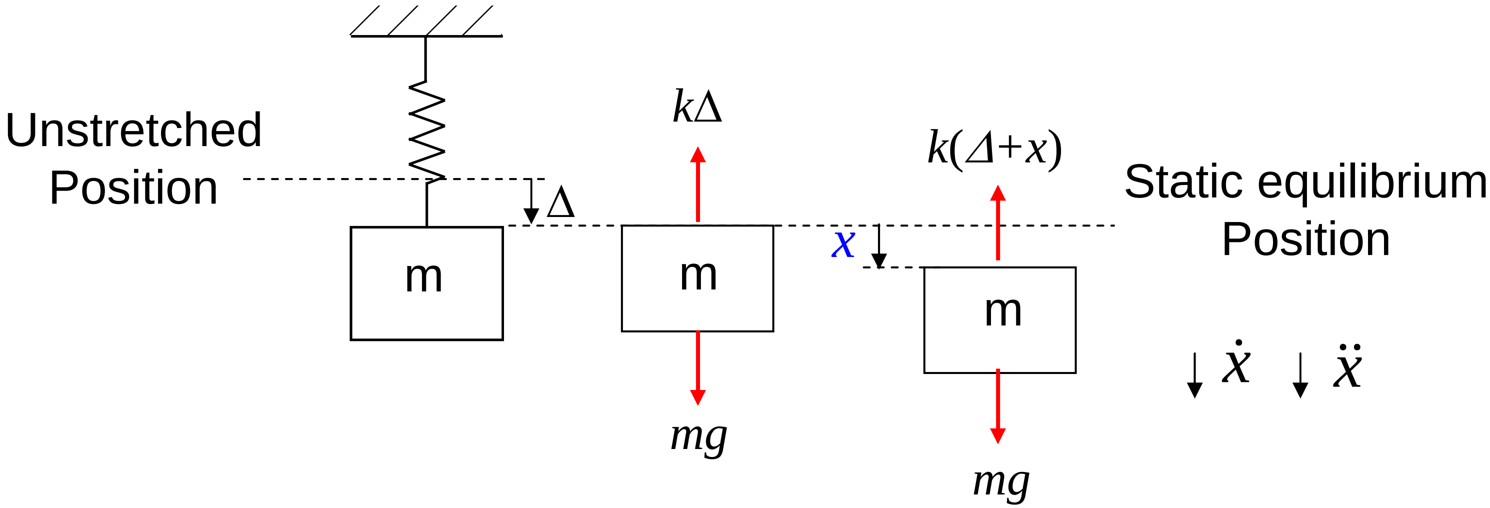
(3) Force equations

Draw free-body diagram

Apply Newton's 2^{nd} law on the free body

(4) Combine all relations

Example 1: A spring-mass system (1)



(1) Geometry

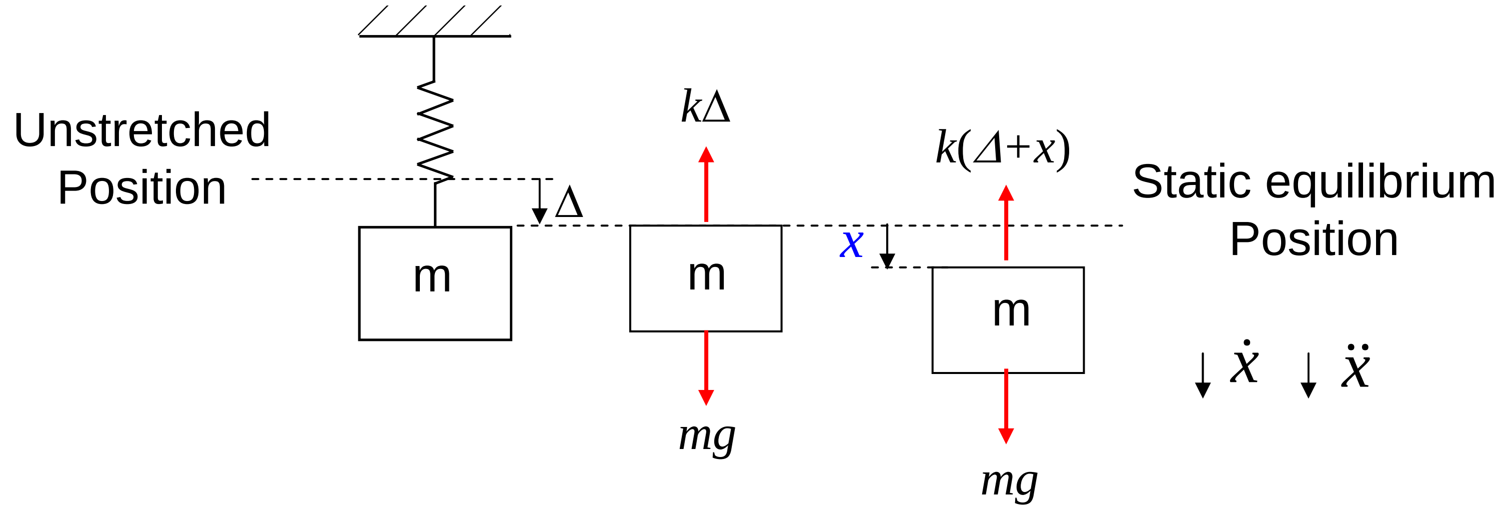
x = mass position measured from equilibrium position

1 DOF, only 1 EOM required

(2) Kinematics

position, velocity, and acceleration are x, $\dot{x}$, $\ddot{x}$

Example 1: A spring-mass system (2)



(3) Force equations

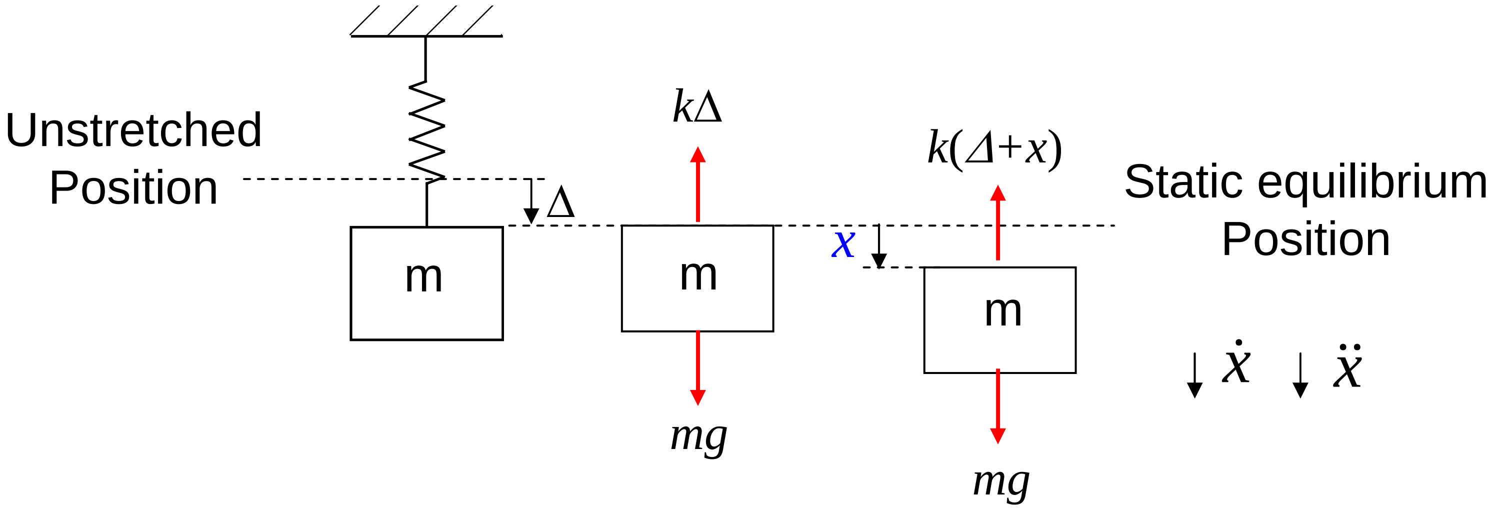
At equilibrium $\left[\sum F = 0\right]$ $\quad mg - k\Delta = 0\ ; \quad mg = k\Delta$

During vibration $\left[\sum F = ma\right]$ $\quad mg - k(\Delta + x) = m\ddot{x}$

(3) Combine all relations

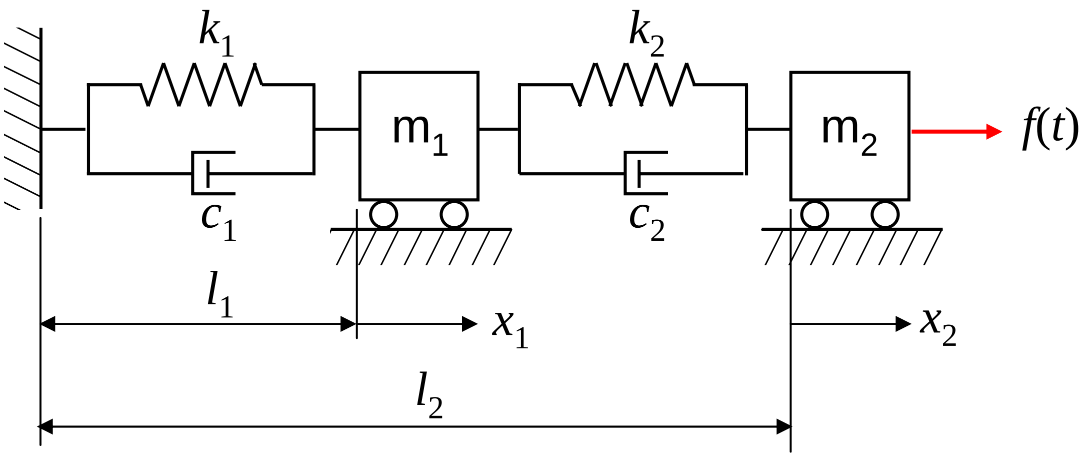
EOM: $m\ddot{x} + kx = 0$

Example 1: A spring-mass system (3)



- What if x is measured from the other positions?
- What if there are the other forces applied to the system?
- What if a damper is added to the system?

Example 2: m-c-k systems (2DOF) (1)



(1) Geometry

l_1, l_2 = positions of m_1 and m_2 measured when both springs are unstretched

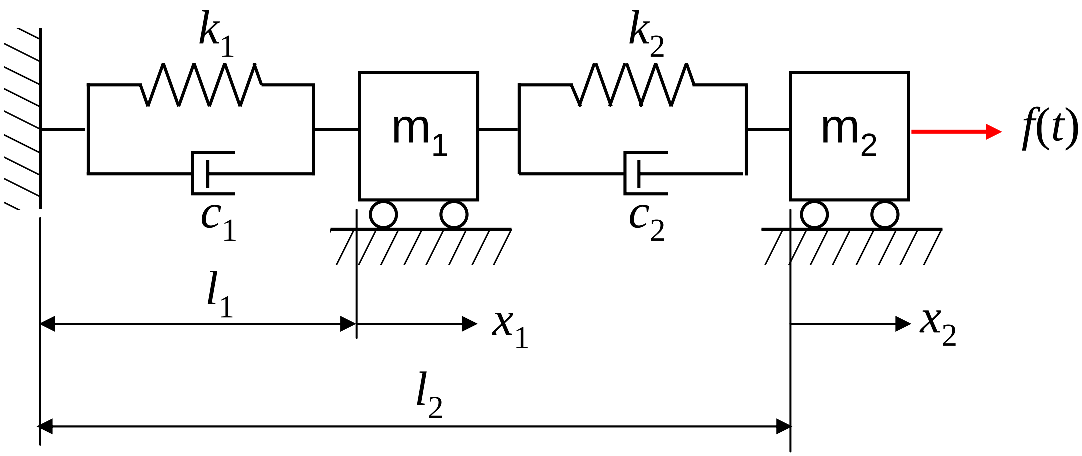
x_1, x_2 = positions of m_1 and m_2 measured from their unstretched positions

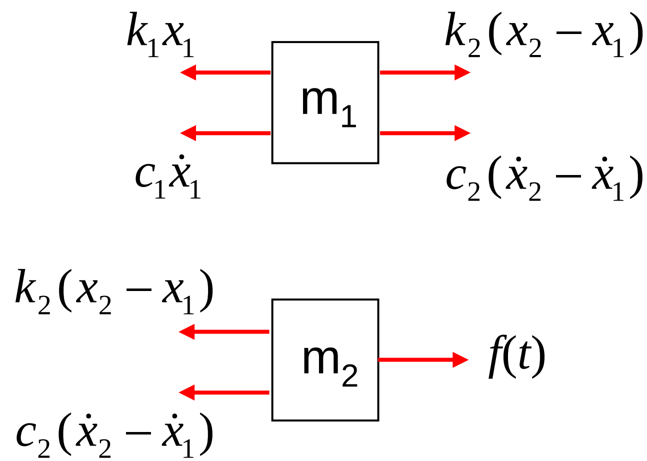
2 DOFs, 2 EOMs required

(2) Kinematics

$x_1, \dot{x}_1, \ddot{x}_1$ and $x_2, \dot{x}_2, \ddot{x}_2$ for mass m_1 and m_2

Example 2: m-c-k systems (2DOF) (2)





(3) Force equations

$$\left[\sum F_x = ma_x\right] \qquad k_2(x_2 - x_1) + \dot{c}_2(\dot{x}_2 - \dot{x}_1) - k_1 x_1 - c_1 \dot{x}_1 = m_1 \ddot{x}_1$$

$$f(t) - k_2(x_2 - x_1) - \dot{c}_2(\dot{x}_2 - \dot{x}_1) = m_2 \ddot{x}_2$$

In matrix form, EOM is

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} c_1 + c_2 & -c_2 \\ -c_2 & c_2 \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} + \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ f(t) \end{bmatrix}$$

Example 2: m-c-k systems (2DOF) (3)

In matrix form, EOM is

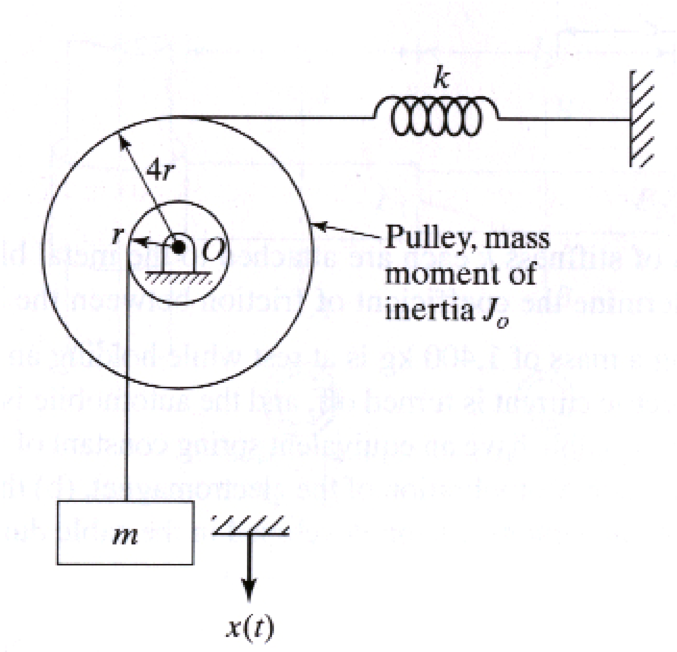
$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} c_1 + c_2 & -c_2 \\ -c_2 & c_2 \end{bmatrix}\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} + \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ f(t) \end{bmatrix}$$

or simply

$$\mathrm{M}\ddot{\mathrm{x}} + \mathrm{C}\dot{\mathrm{x}} + \mathrm{K}\mathrm{x} = \mathrm{F}(t)$$

where **M** is “mass or inertia matrix”
C is “damping matrix”
K is “stiffness matrix”
x is position vector
F is input vector

Example 3



Draw the free-body diagram and derive the EOM using Newton's second law of motion

Example (4)

Derive the EOM of an airplane's steering-gear mechanism for the nose wheel of its landing gear. The mechanism is modeled as the single-degree-of-freedom system illustrated in the figure. [Inman/1.49]

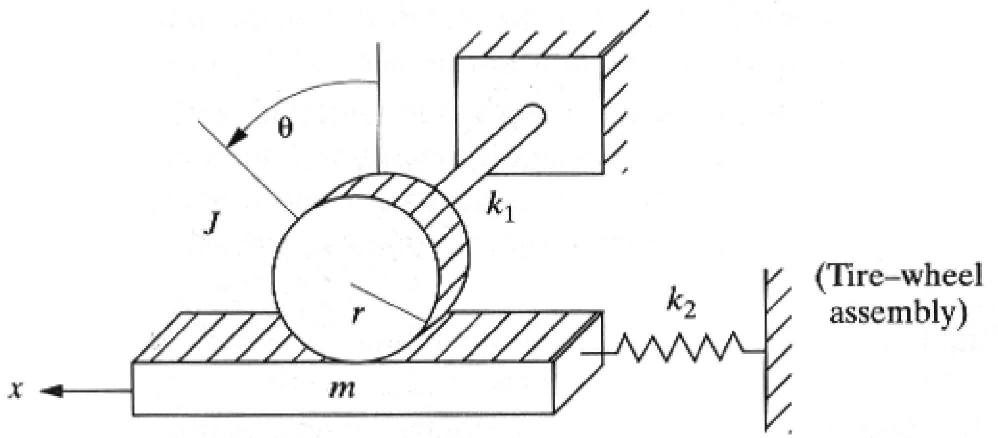
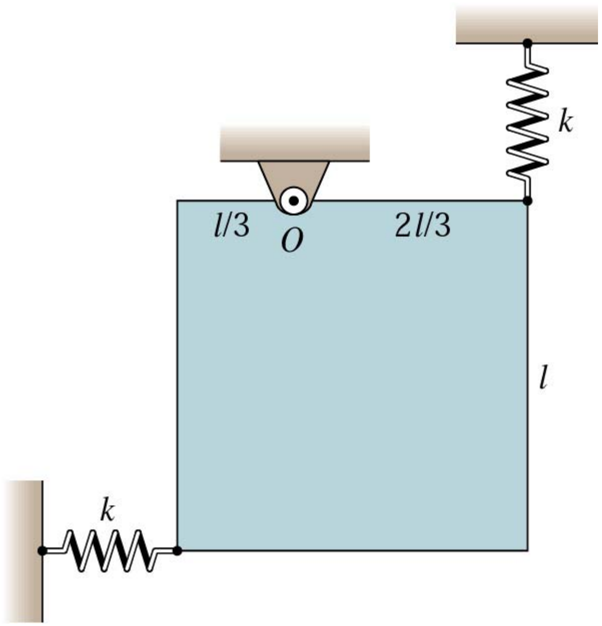


Figure P1.49 Single-degree-of-freedom model of a steering mechanism.

Example (5)

A Spring-loaded homogeneous plate of mass m pivots freely about a vertical axis through point O. Derive the EOM.
[J.L.Meriam & L.G.Kraige /8.83]

<u>Given</u>: $J_G = (1/6)ml^2$



Example (6)

The uniform solid cylinder of mass m and radius r rolls without slipping during its oscillation on the circular surface of radius R. Derive the EOM. [J.L.Meriam & L.G.Kraige /8.93]