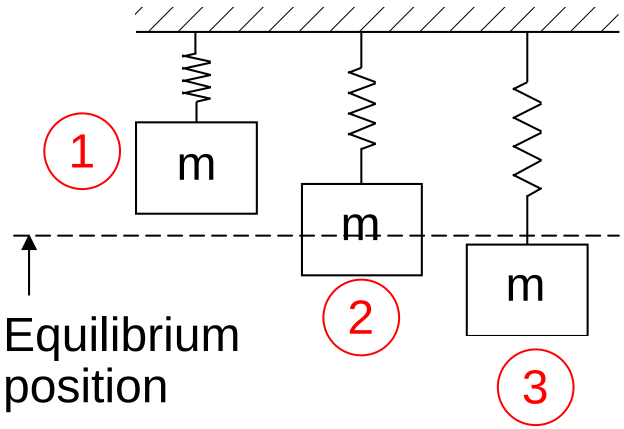
Modeling and energy methods (1)

In the conservative system (free vibration, undamped system), the total energy is constant.

Sum of kinetic and potential energy = constant



$$T + U = \text{constant}$$

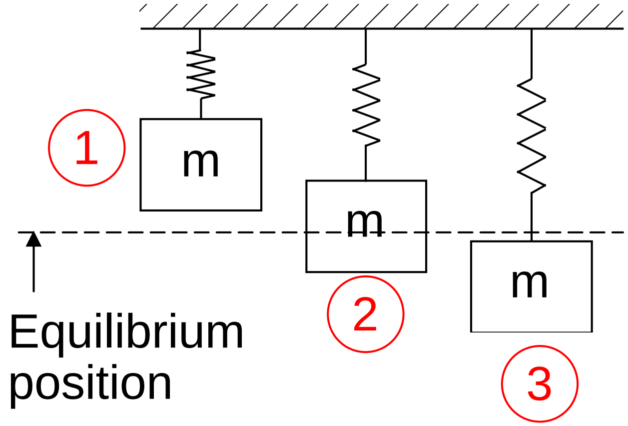
$$T_1 + U_1 = T_2 + U_2$$

where

T is kinetic energy

U is potential energy

Modeling and energy methods (2)



$$T + U = \text{constant}$$

$$T_1 + U_1 = T_2 + U_2$$

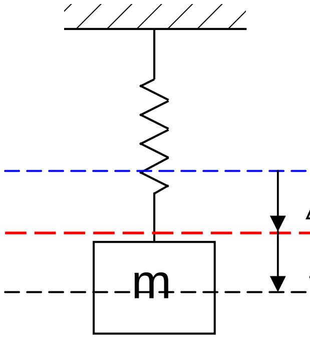
At max disp. $U = U_{max}$ $T = 0$

At equilibrium $U = 0$ $T = T_{max}$

Let 1 = position at max disp., 2 = position at equilibrium

$$U_{max} + 0 = 0 + T_{max} \Rightarrow U_{max} = T_{max}$$

Example (1)



$$U_{Spring} = \frac{1}{2}k(\Delta + x)^2$$

$$U_{Grav} = -mgx$$

$$T = \frac{1}{2}m\dot{x}^2$$

$$[T + U = const.]$$

$$\frac{1}{2}m\dot{x}^2 - mgx + \frac{1}{2}k(\Delta + x)^2 = const.$$

Differentiating

$$m\dot{x}\ddot{x} - mg\dot{x} + k(\Delta + x)\dot{x} = 0$$

$$(m\ddot{x} - mg + k\Delta + kx)\dot{x} = 0$$

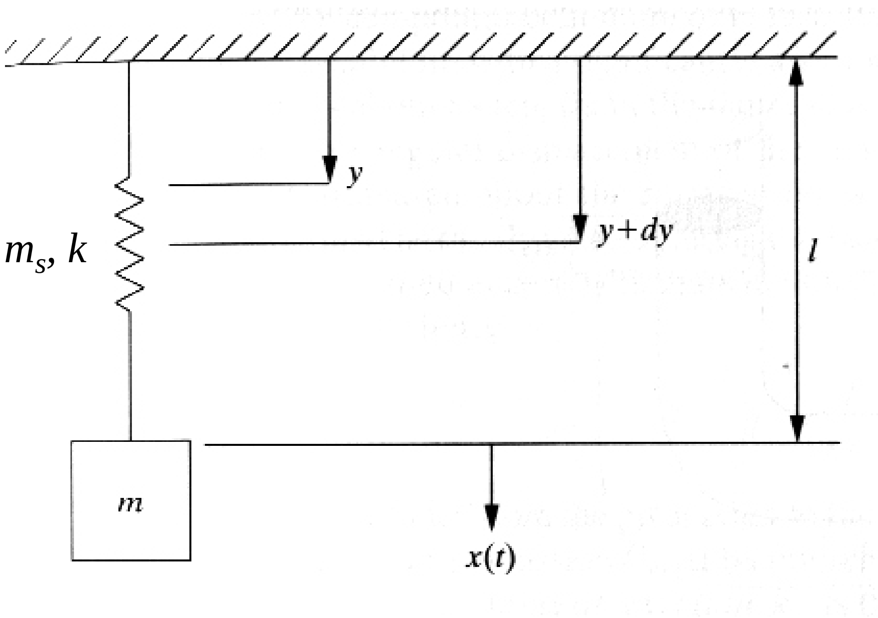
$mg = k\Delta$ (Equilibrium cond.)

$\dot{x} \neq 0$ (In general cases)

$$m\ddot{x} + kx = 0$$

Example (2)

Model the mass of the spring of the system shown in the figure. Derive EOM including the effect of the mass of the spring.



Example (3)

Derive the EOM of an airplane's steering-gear mechanism for the nose wheel of its landing gear. The mechanism is modeled as the single-degree-of-freedom system illustrated in the figure. [Inman/1.49]

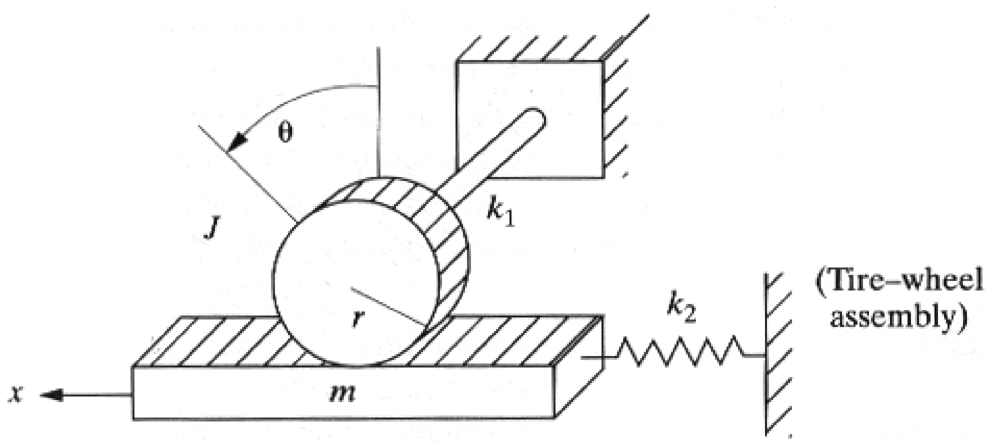
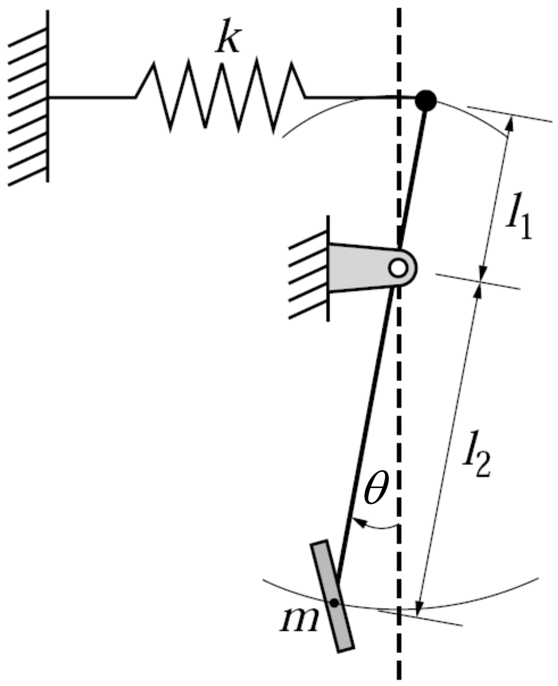


Figure P1.49 Single-degree-of-freedom model of a steering mechanism.

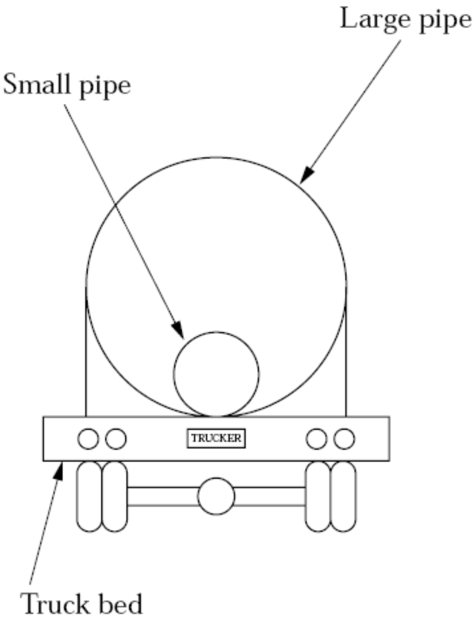
Example (4)

A control pedal of an aircraft can be modeled as the single-degree-of-freedom system shown in the figure. Consider the level as a massless shaft and the pedal as a lumped mass at the end of the shaft. Determine the EOM in θ. Assume the spring to be unstretched at $\theta = 0$. [Inman/1.50]

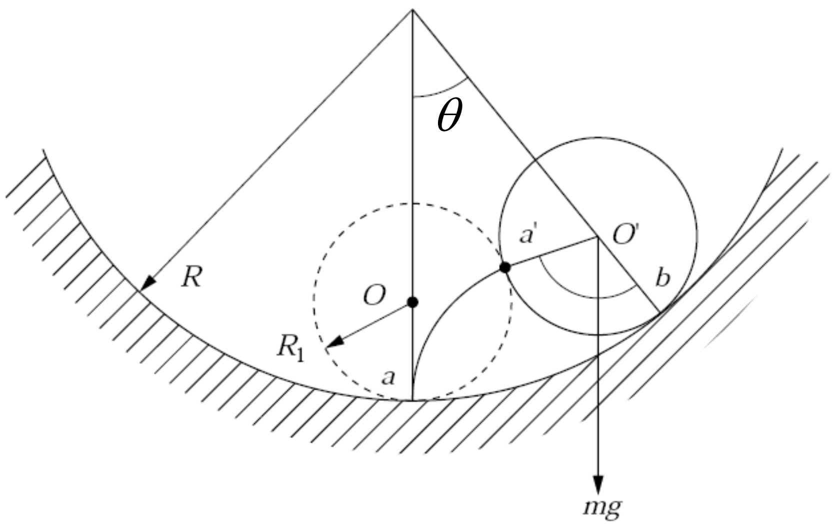


Example (5)

To save space, two large pipes are shipped one stacked inside the other as indicated in the figure. Derive the EOM of the smaller pipe (of radius R_1) rolling back and forth inside the large pipe (of radius R). Use the energy method and assume that the inside pipe rolls with out slipping and has a mass of m. [Inman/1.51]



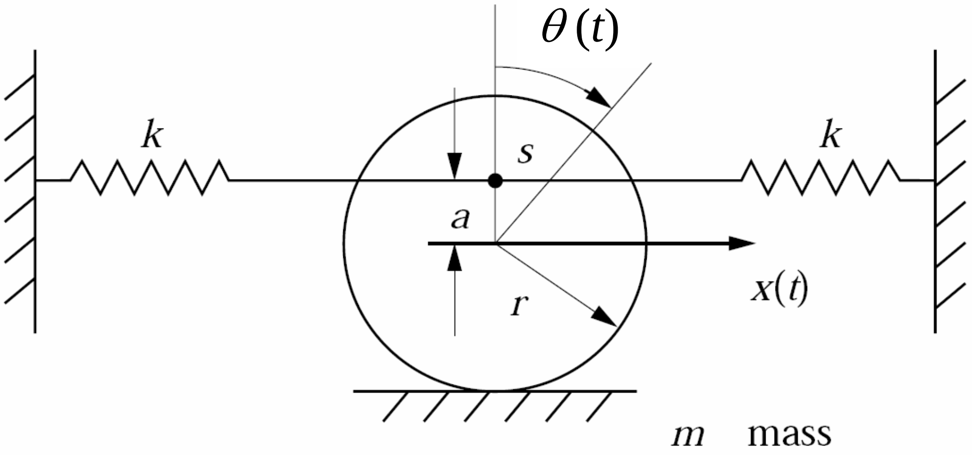
(a)



(b)

Example (6)

Consider the disk of the figure connected to two springs. Derive EOM for small angle $\theta(t)$. [Inman/1.82]



Energy method for multi-DOF

Eom of the multi-DOF can be derived using Lagrange's equations as follow.

Lagrange's equations

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right) - \frac{\partial T}{\partial q_i} + \frac{\partial U}{\partial q_i} = Q_i$$

$i = 1, 2, \ldots, n$

coordinate i,
(according to DOF)

Where q_i is the generalized coordinate

$\dot{q}_i$ is the generalized velocity

Q_i is the nonconservative forces corresponding to q_i

Force for q_i = translation coordinate
Moment for q_i = rotation coordinate

Example (Lagrange's equations)

Derive the EOM of the system shown in the figure using the lagrange's equations.

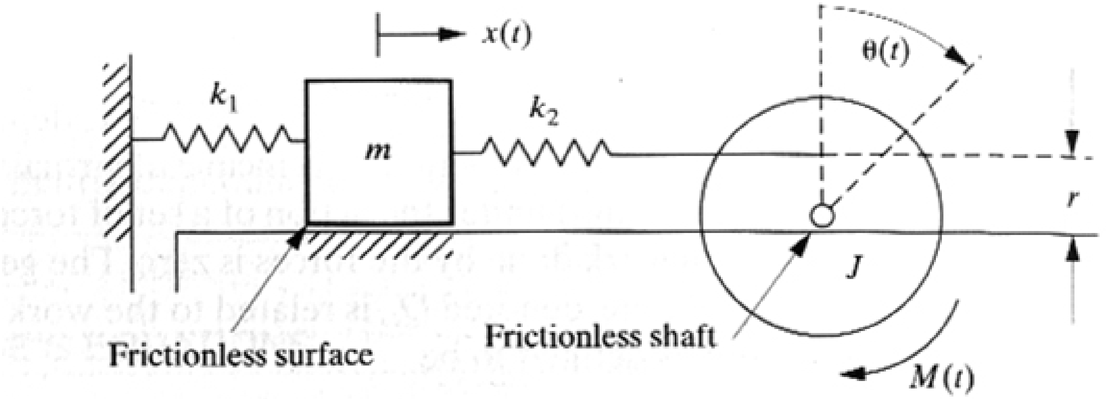


Figure 4.18 Vibration model of a simple machine part. The quantity $M(t)$ denotes an applied moment. The disk rotates without translation.