

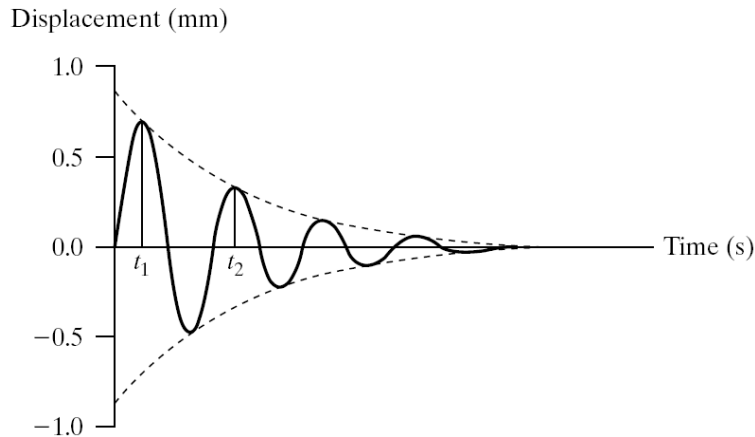
Logarithmic decrement (1)

m and k can be determined easily by static tests.

c ?



Determined by the concept of **logarithmic decrement**



Define logarithmic decrement δ

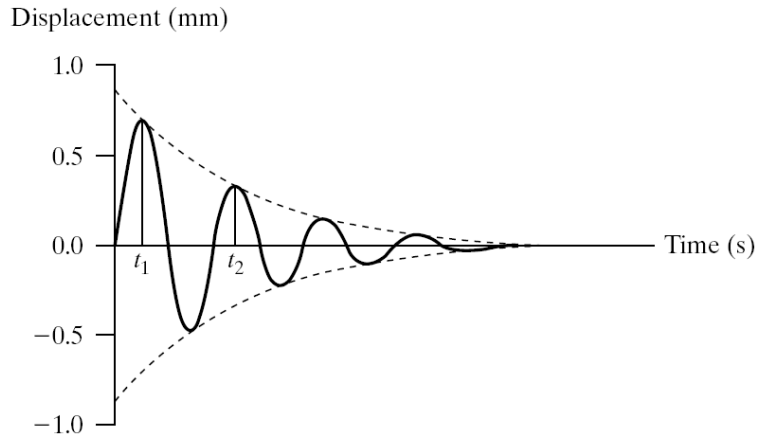
$$\delta = \ln \frac{x(t)}{x(t+T)}$$

Underdamped motion



$$\delta = \ln \frac{Ae^{-\zeta\omega_n t} \sin(\omega_d t + \phi)}{Ae^{-\zeta\omega_n (t+T)} \sin(\omega_d t + \omega_c T + \phi)}$$

Logarithmic decrement (2)



$$T = \frac{2\pi}{\omega_d} \quad \Rightarrow$$

$$\delta = \ln \frac{Ae^{-\zeta\omega_n t} \sin(\omega_d t + \phi)}{Ae^{-\zeta\omega_n(t+T)} \sin(\omega_d t + \omega_c T + \phi)}$$

$$\delta = \ln e^{\zeta\omega_n T} = \zeta\omega_n T$$

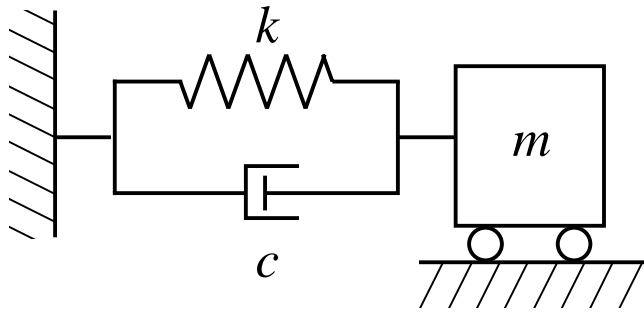
$$\delta = \zeta\omega_n \frac{2\pi}{\omega_n \sqrt{1-\zeta^2}} = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}}$$

Measurement done over any integer multiple of the period

$$\zeta = \frac{\delta}{\sqrt{4\pi^2 + \delta^2}}$$

$$\delta = \frac{1}{n} \ln \frac{x(t)}{x(t+nT)}$$

Example (measurement: c)



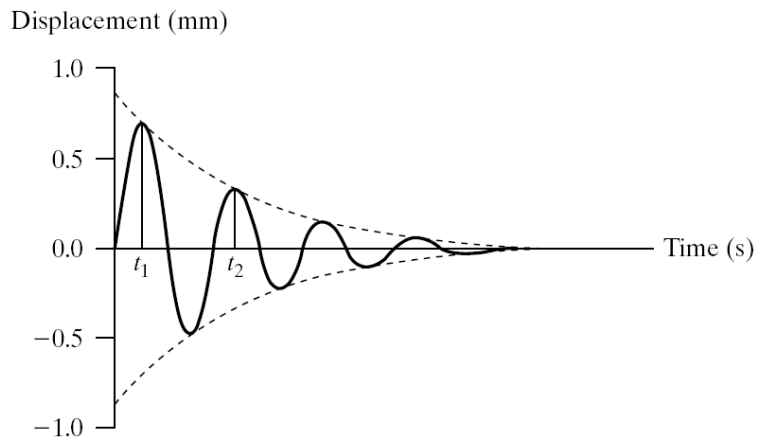
$$m = 2\text{kg}$$

$$k = 1.5 \times 10^3 \text{ N/m}$$

Displacement at $t_1 = 9 \text{ mm}$

at $t_2 = 1 \text{ mm}$

Calculate the damping coefficient

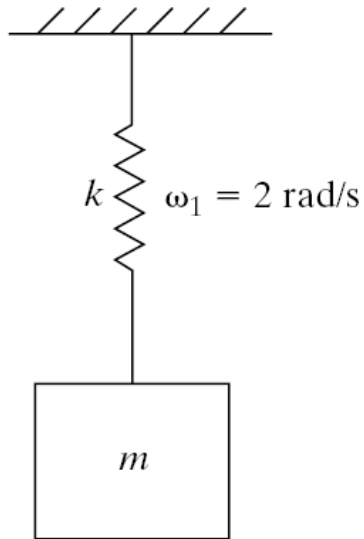


Example (measurement: m, k)

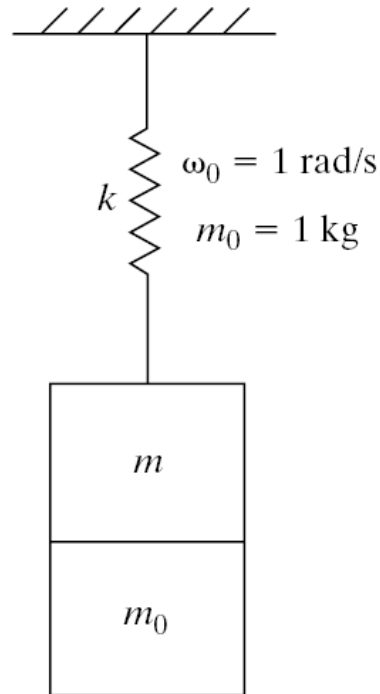
m and k are unknown. $\omega_1 = 2$ rad/s

Add m_0 1kg, $\omega_0 = 1$ rad/s

Determine m and k



(a)



(b)

Design consideration

Design in vibration: adjusting the physical parameters of a device to cause its vibration response to meet a specified shape or performance criterion.

Example

Design a m-c-k system to have the desired response.

underdamped, overdamped, critically damped

Design a system that has a given natural frequency.

Select connection of springs (series or parallel)

Use elastic elements as springs

Consider acceptable static deflection

Design consideration (Example)

Consider modeling the vertical suspension system of a small sports car, as a single-DOF system of the form

$$m\ddot{x} + c\dot{x} + kx = 0$$

m = the mass of the automobile 1361 kg

Static deflection = 0.05 m

calculate c and k of the suspension system

If m_0 290 kg are in the car, how does this affect the effective damping ratio?

Design consideration (Example)

For a m - c - k system, $m = 5$ kg, $k = 5000$ N/m. During free vibration, the amplitude decays to 0.25 of its initial value after five cycles. Calculate the viscous damping coefficient, c .

Choose a dashpot's viscous damping value such that when placed in parallel with the spring reduces the frequency of oscillation to 9 rad/s.

[Inman 1.72,1.73]