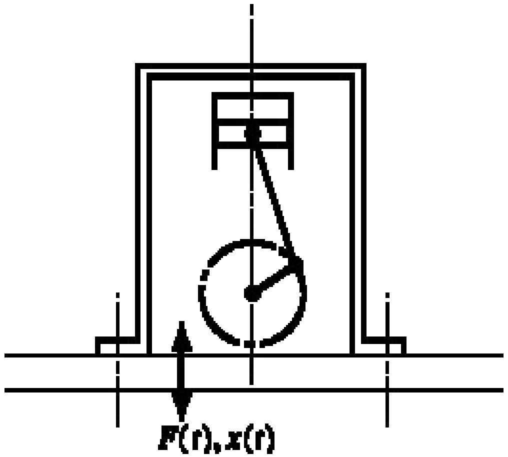
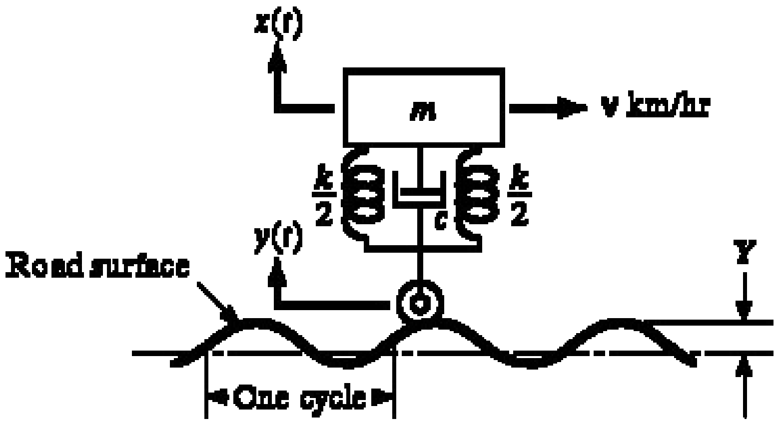
Forced vibration

A system is said to undergo forced vibration whenever external energy is supplied to the system during vibration.

External energy can be supplied through either an applied force or an imposed displacement excitation.



Applied force

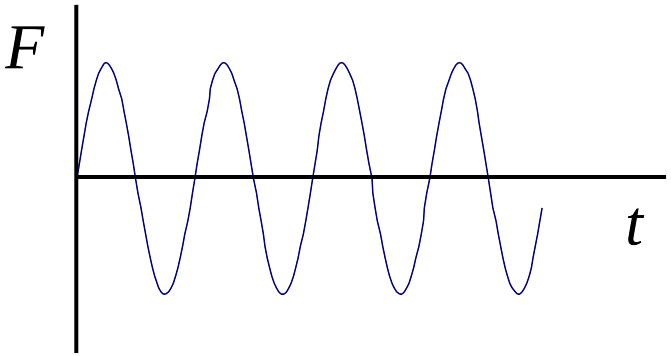


Displacement excitation

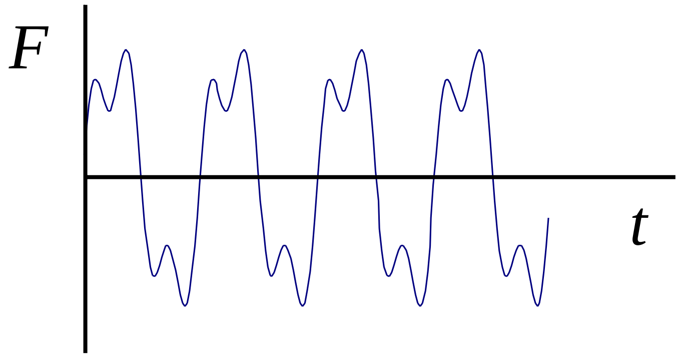
Types of forced vibration

Classified by input

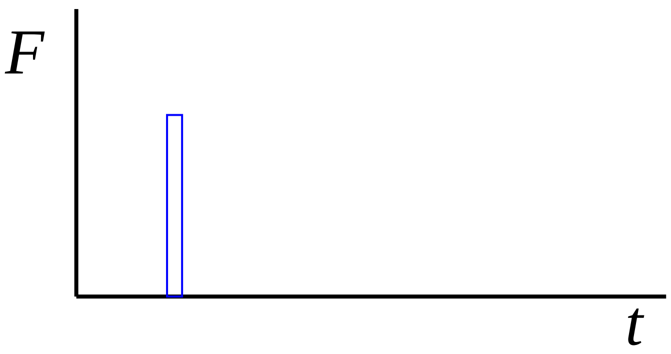
1. Harmonic (sinusoidal) input



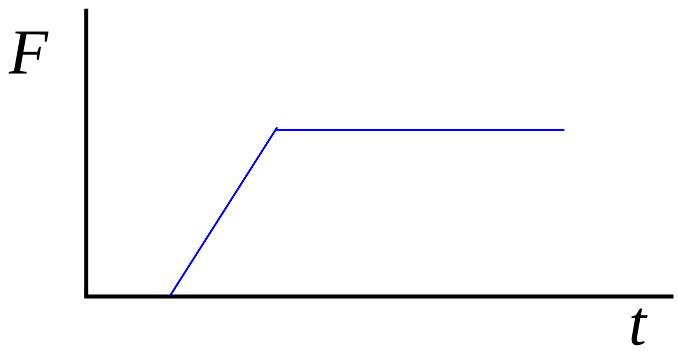
2. Arbitrary periodic input



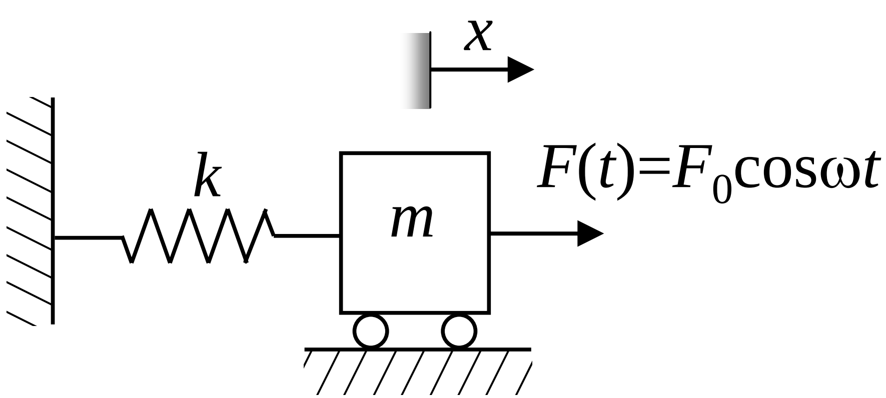
3. Impact

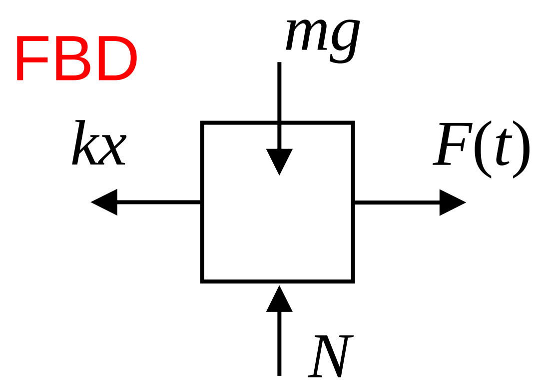


4. Arbitrary nonperiodic input



Harmonic excitation (undamped)





EOM: $m\ddot{x} + kx = F_0 \cos \omega t$

or $\ddot{x} + \omega_n^2 x = f_0 \cos \omega t$

The response $x(t)$ (solution) can be separated into 2 part;

1. Homogeneous solution $x_h(t)$ → $m\ddot{x} + kx = 0$
2. Particular solution $x_p(t)$ → $m\ddot{x} + kx = F_0 \cos \omega t$

$$x(t) = x_h(t) + x_p(t)$$

Response of harmonic excitation (1)

The homogeneous solution $x_h(t)$ is the solution of $m\ddot{x}+kx=0$

The homogeneous solution is given by:

$$x_h(t)=A_1\sin\omega_n t+A_2\cos\omega_n t$$

or

$$x_h(t)=A\sin(\omega_n t+\phi)$$

where $\omega_n=\sqrt{\dfrac{k}{m}}$ is the natural frequency

The homogeneous solution has the same form as free vibration response

Response of harmonic excitation (2)

The particular solution $x_p(t)$ is the solution of $m\ddot{x} + kx = F_0 \cos \omega t$

From physical observation, harmonic excitation causes vibration at its "driving frequency", ω.

The particular solution $x_p(t)$ can be written in the form:

$$x_p(t) = X \cos \omega t$$

Solve for X by substituting $x_p(t)$ into EOM

$$-\omega^2 mX \cos \omega t + kX \cos \omega t = F_0 \cos \omega t$$

$$X = \frac{F_0}{k - m\omega^2} \quad \Rightarrow \quad x_p(t) = \frac{F_0}{k - m\omega^2} \cos \omega t$$

Response of harmonic excitation (3)

$$x(t) = x_h(t) + x_p(t) = \underbrace{A_1 \sin\omega_n t + A_2 \cos\omega_n t}_{x_h(t)} + \underbrace{\frac{F_0}{k - m\omega^2}\cos\omega t}_{x_p(t)}$$

A_1 and A_2 can be solved by using initial condition

$$x(t=0) = x_0 \quad \text{and} \quad \dot{x}(t=0) = \dot{x}_0 = v_0$$

$$A_1 = \frac{v_0}{\omega_n} \quad , \quad A_2 = x_0 - \frac{F_0}{k - m\omega^2}$$

$$x(t) = \frac{v_0}{\omega_n}\sin\omega_n t + \left(x_0 - \frac{F_0}{k - m\omega^2}\right)\cos\omega_n t + \frac{F_0}{k - m\omega^2}\cos\omega t$$

Response of harmonic excitation (4)

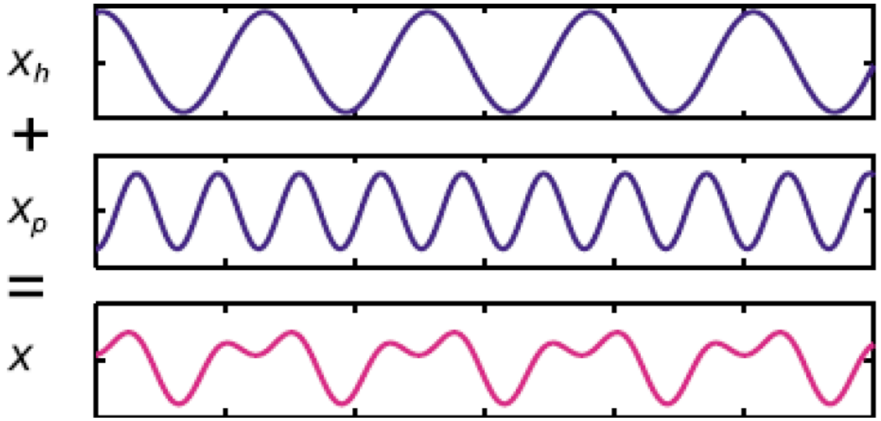
For $x(t=0)=x_0$ and $\dot{x}(t=0)=\dot{x}_0=v_0$

$$x(t)=\frac{v_0}{\omega_n}\sin\omega_n t+\left(x_0-\frac{F_0}{k-m\omega^2}\right)\cos\omega_n t+\frac{F_0}{k-m\omega^2}\cos\omega t$$

or

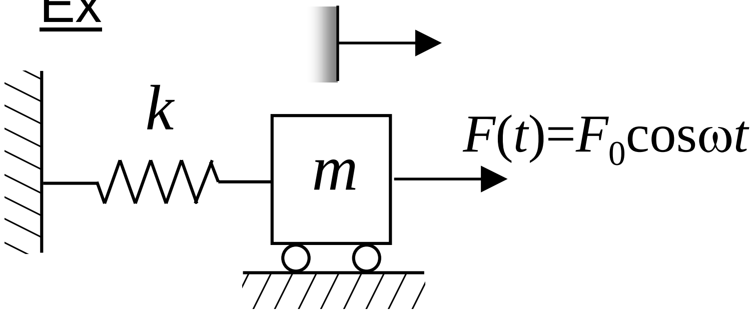
$$x(t)=\frac{v_0}{\omega_n}\sin\omega_n t+\left(x_0-\frac{f_0}{\omega_n^2-\omega^2}\right)\cos\omega_n t+\frac{f_0}{\omega_n^2-\omega^2}\cos\omega t$$

Example of a total response



Amplitude of the response

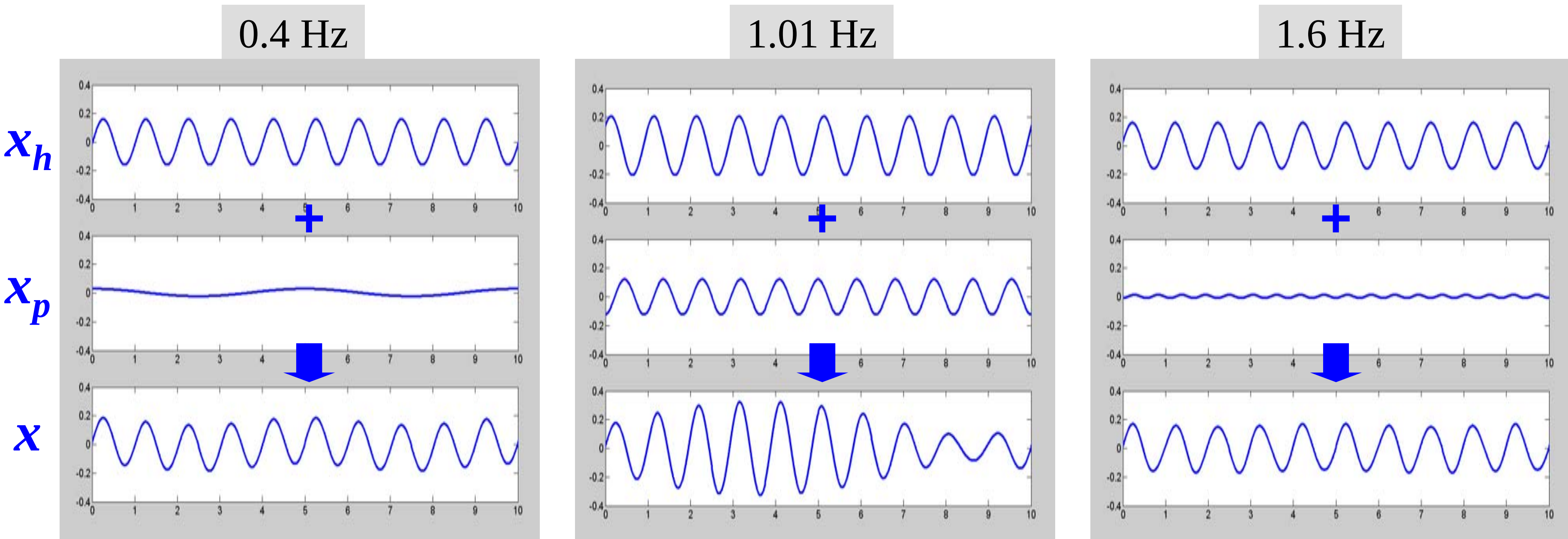
Ex



Natural freq. f_n = 1 Hz
Excited force/mass f_0 = 1 N/kg
Excited freq. f = 0.4, 1.01, 1.6 Hz
x_0 = 0.01 m
v_0 = 1 m/s

From

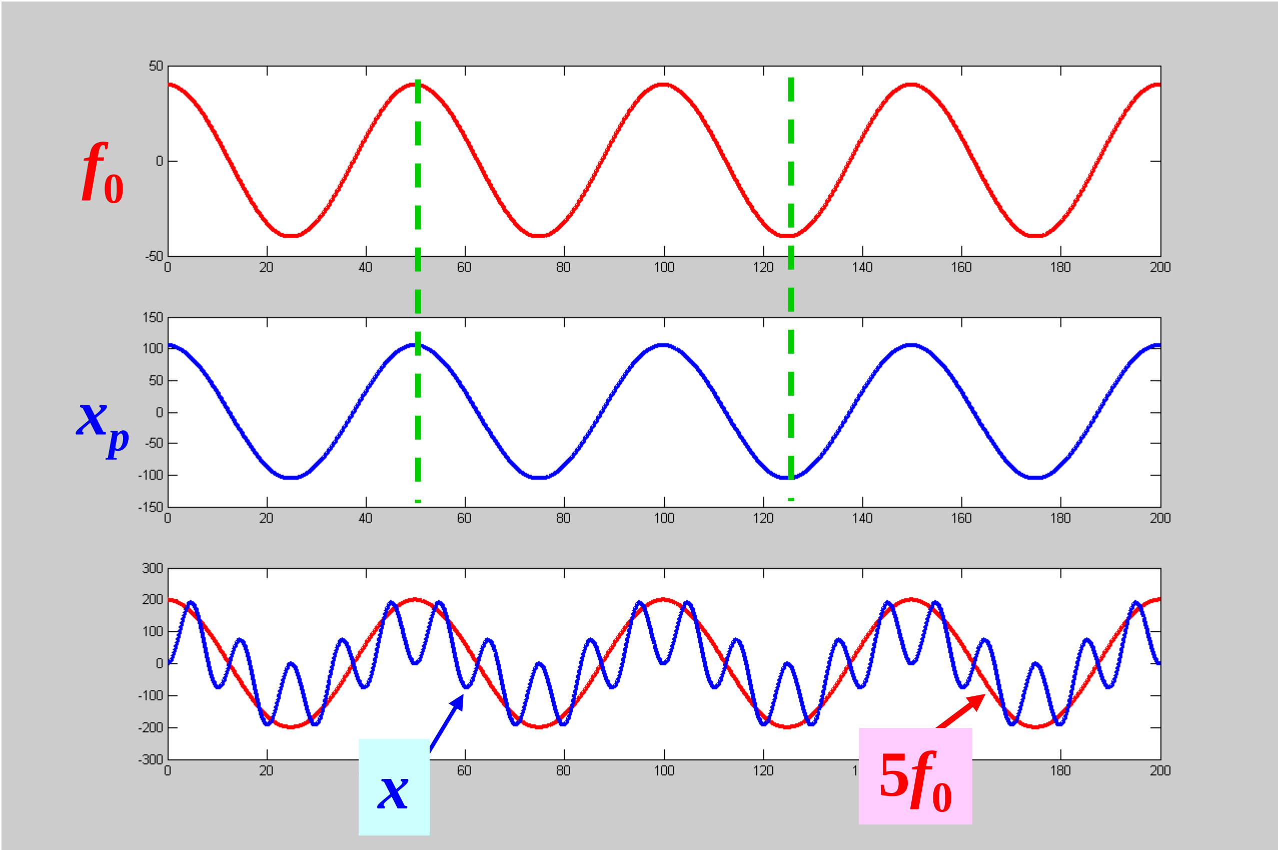
$$x(t) = \frac{v_0}{\omega_n}\sin\omega_n t + \left(x_0 - \frac{f_0}{\omega_n^2 - \omega^2}\right)\cos\omega_n t + \frac{f_0}{\omega_n^2 - \omega^2}\cos\omega t$$



Phase of the response (1)

Natural freq. f_n = 0.1 Hz
Excited force/mass f_0 = 40 N/kg
Excited freq. f = 0.02 Hz
x_0 = 0.1 m
v_0 = 0 m/s

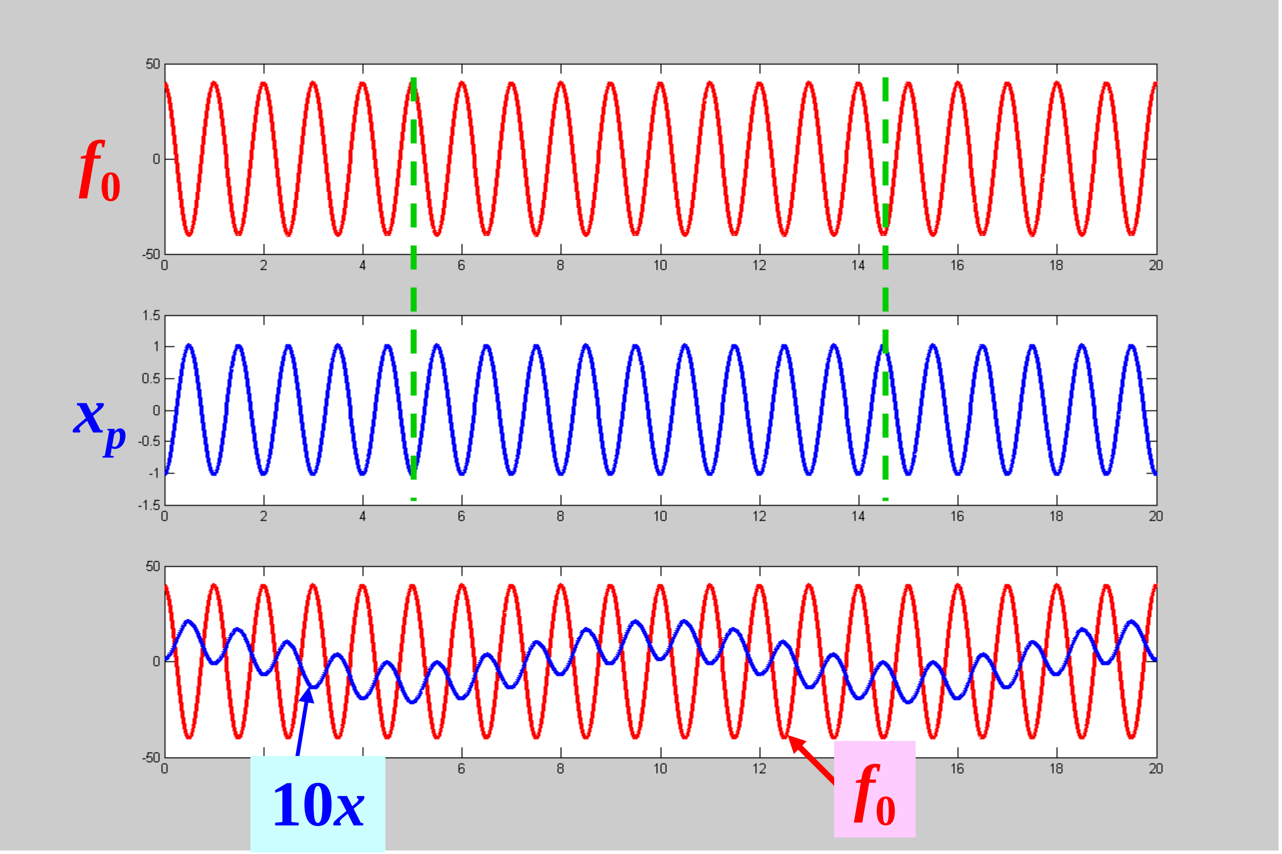
$\omega/\omega_n = 0.2$



Phase of the response (2)

Natural freq. f_n = 0.1 Hz
Excited force/mass f_0 = 40 N/kg
Excited freq. f = 1 Hz
x_0 = 0.1 m
v_0 = 0 m/s

$\omega/\omega_n = 10$



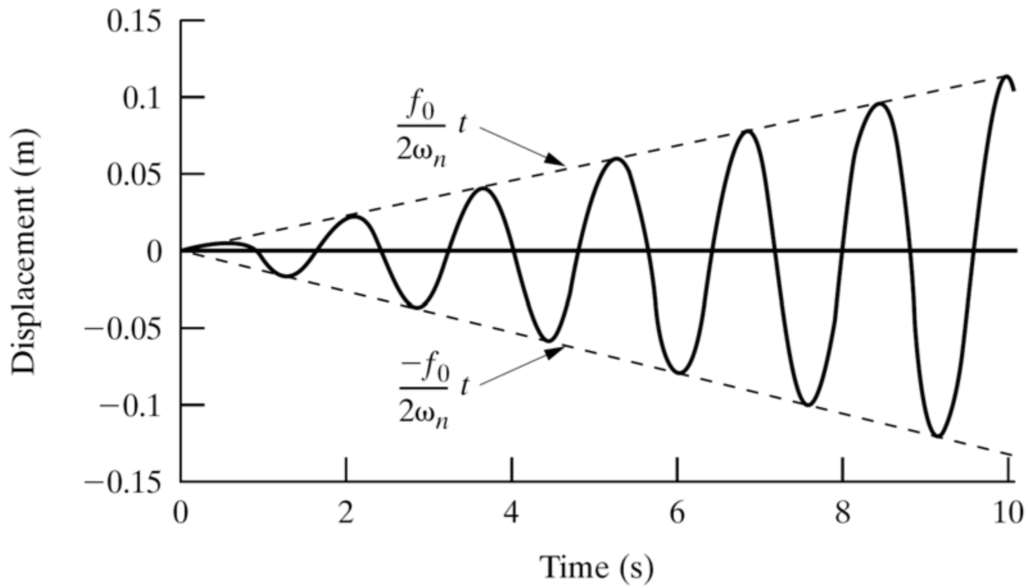
Resonance

When forcing frequency is equal to the natural frequency of the system, the response grows without bound. This phenomenon is called resonance.

In this case $\omega = \omega_n$ → $x_p(t) = tX \cos \omega t$

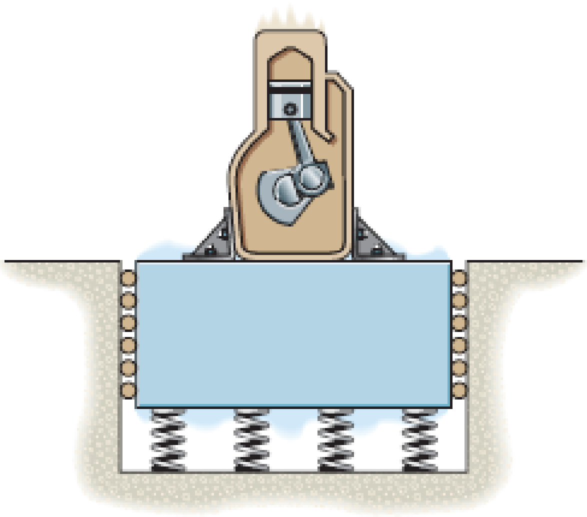
The total response is

$$x(t) = \frac{v_0}{\omega} \sin \omega t + x_0 \cos \omega t + \frac{f_0}{2\omega} t \sin \omega t$$



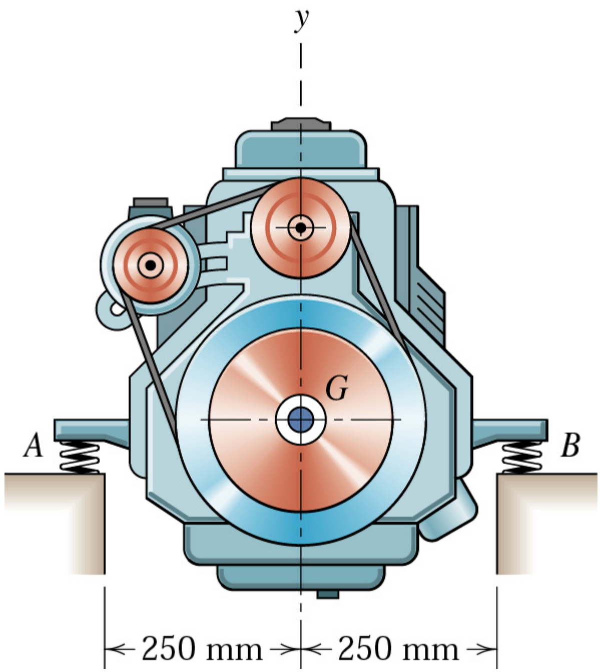
Example 1

The engine is mounted on a foundation block which is spring-supported. Describe the steady-state vibration of the system if the block and engine have a total weight of 7500 N (≈ 750 kg) and the engine, when running, creates an impressed force $F = 250\sin(2t)$ N, where t is in seconds. Assume that the system vibrates only in the vertical direction, with the positive displacement measured downward, and that the total stiffness of the springs can be represented as k = 30 kN/m. Also determine the rotational speed ω of the engine which will cause resonance. [R. C. Hibbeler 22-55,56]



Example 2

A 220-kg engine is mounted on a test stand with spring mounts at A and B, each with a stiffness of 105 kN/m. The radius of gyration of the engine about its mass center G is 115 mm. With the motor not running, calculate the natural frequency $(f_n)_y$ of vertical vibration and $(f_n)_\theta$ of rotation about G. [J. L. Meriam & L. G. Kraige 8/113]



Example 3

Determine the amplitude of vertical vibration of the spring-mounted trailer as it travels at a velocity of 25 km/h over the road whose contour may be expressed by a sinusoid. The mass of the trailer is 500 kg and that of the wheels alone may be neglected. During loading, each 75 kg added to the load caused the trailer to sag 3 mm on its springs. Assume that the wheels are in contact with the road at all times and neglect damping. At what critical speed v_c is the vibration of the trailer greatest? [J. L. Meriam & L. G. Kraige 8/71]