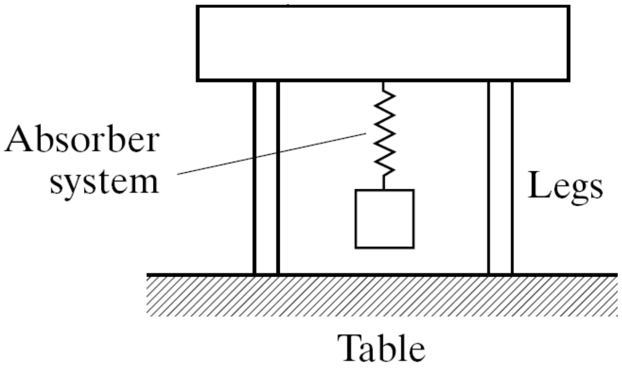
Design of Vibration Absorbers



Objective:

To reduce the vibration of a primary device by adding an absorber to the system

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Applications:

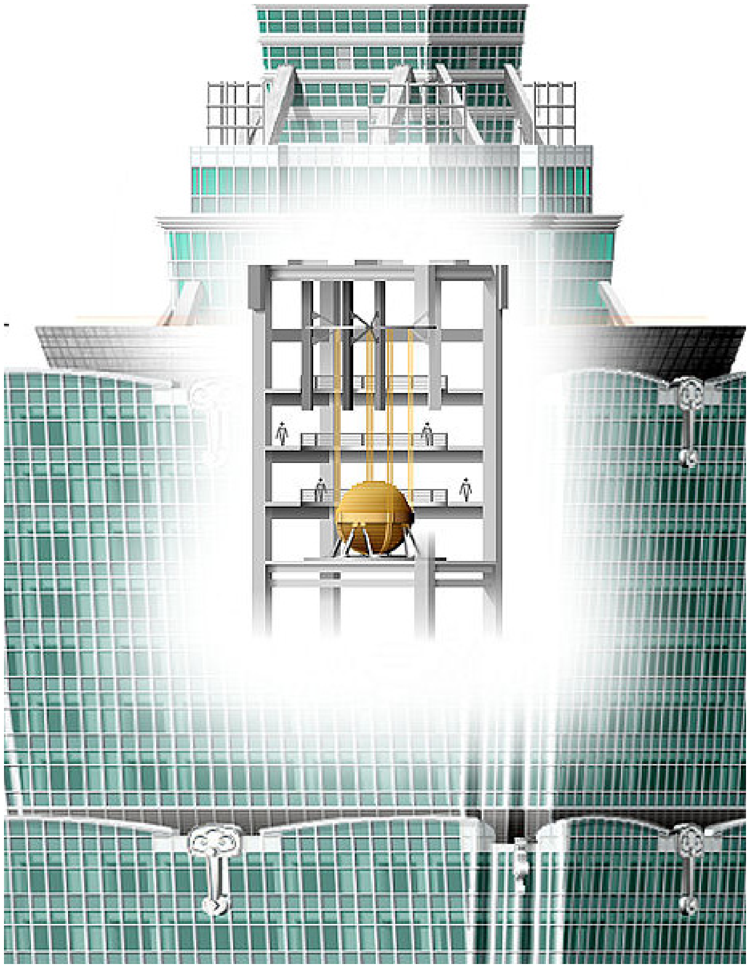
- Reciprocating machines
- Building excited by an earthquake
- Transmission lines or telephone lines excited by wind blowing

Applications (1)

Vibration absorber in the transmission lines

Tuned mass dampers beneath the bridge platform.

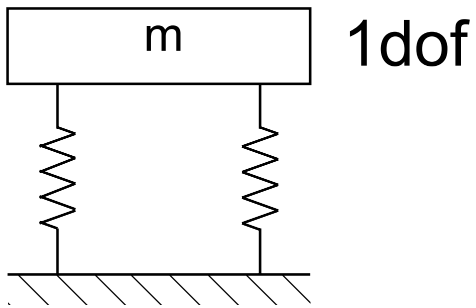
Applications (2)

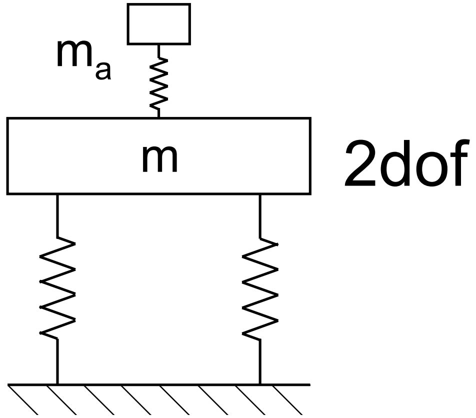


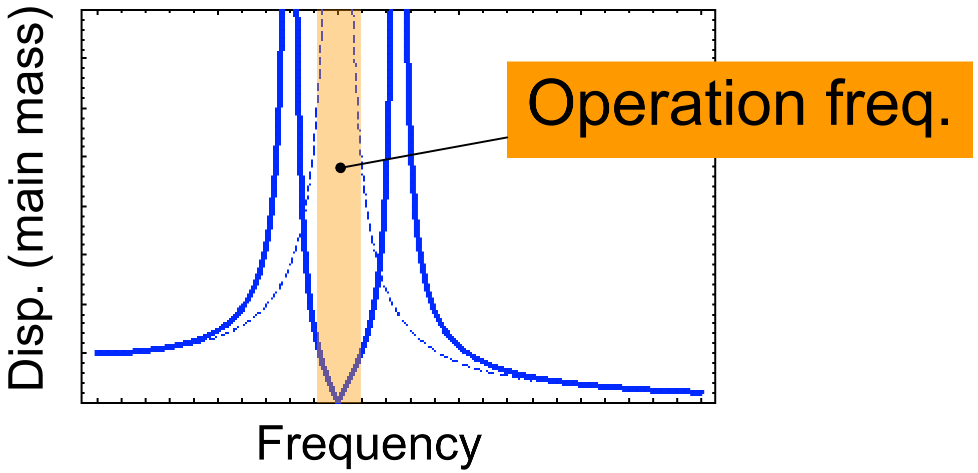
Tuned Mass Damper
in the building

How vibration absorber works

Vibration absorber is applied to the machine whose operation frequency meets its resonance frequency.

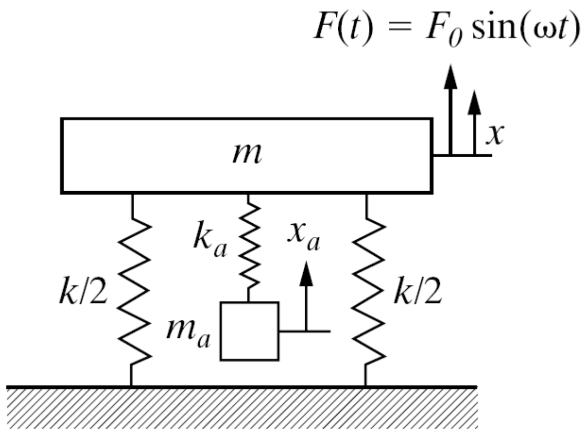






Vibration absorber is often used with machines run at constant speed or systems with const. excited freq., because the combined system has narrow operating bandwidth.

Principle (1)



Model

EOM

$$\begin{bmatrix} m & 0 \\ 0 & m_a \end{bmatrix} \begin{Bmatrix} \ddot{x} \\ \ddot{x}_a \end{Bmatrix} + \begin{bmatrix} k+k_a & -k_a \\ -k_a & k_a \end{bmatrix} \begin{Bmatrix} x \\ x_a \end{Bmatrix} = \begin{Bmatrix} F_0 \sin \omega t \\ 0 \end{Bmatrix}$$

Synchronous motion

$$\left. \begin{aligned} x(t) &= X \sin \omega t \\ x_a(t) &= X_a \sin \omega t \end{aligned} \right\}$$

Sub. Into EOM

$$\begin{bmatrix} k+k_a-m\omega^2 & -k_a \\ -k_a & k_a-m_a\omega^2 \end{bmatrix} \begin{Bmatrix} X \\ X_a \end{Bmatrix} \sin \omega t = \begin{Bmatrix} F_0 \\ 0 \end{Bmatrix} \sin \omega t$$

$$\begin{Bmatrix} X \\ X_a \end{Bmatrix} = \begin{bmatrix} k+k_a-m\omega^2 & -k_a \\ -k_a & k_a-m_a\omega^2 \end{bmatrix}^{-1} \begin{Bmatrix} F_0 \\ 0 \end{Bmatrix}$$

Principle (2)

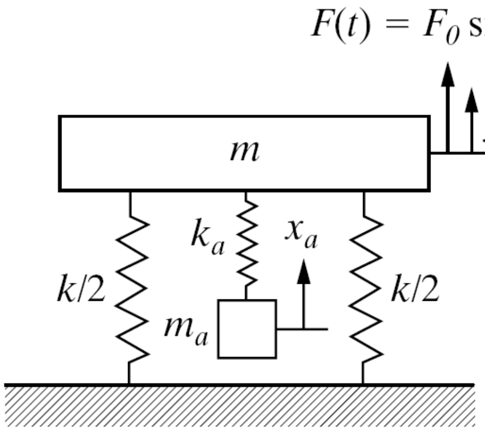
$$\begin{Bmatrix} X \\ X_a \end{Bmatrix} = \begin{bmatrix} k + k_a - m\omega^2 & -k_a \\ -k_a & k_a - m_a\omega^2 \end{bmatrix}^{-1} \begin{Bmatrix} F_0 \\ 0 \end{Bmatrix}$$

$$\begin{Bmatrix} X \\ X_a \end{Bmatrix} = \frac{1}{\Delta} \begin{bmatrix} k_a - m_a\omega^2 & k_a \\ k_a & k + k_a - m\omega^2 \end{bmatrix} \begin{Bmatrix} F_0 \\ 0 \end{Bmatrix} = \frac{1}{\Delta} \begin{Bmatrix} (k_a - m_a\omega^2)F_0 \\ k_a F_0 \end{Bmatrix}$$

where $\Delta = (k + k_a - m\omega^2)(k_a - m_a\omega^2) - k_a^2$

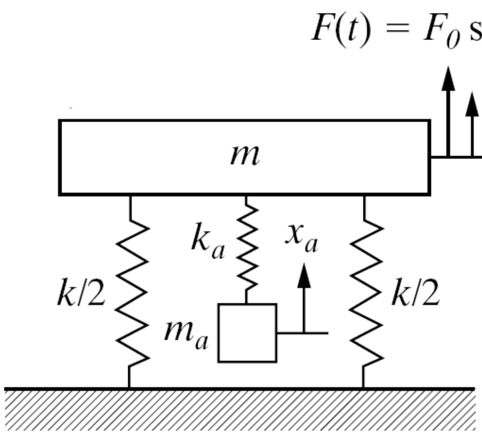
$$X = \frac{(k_a - m_a\omega^2)F_0}{\Delta}$$

$$X_a = \frac{k_a F_0}{\Delta}$$



Model

Principle (3)



$$X = \frac{(k_a - m_a\omega^2)F_0}{\Delta}$$

$$X_a = \frac{k_a F_0}{\Delta}$$

m_a and k_a can be chosen such that X = 0

$$\omega^2 = \frac{k_a}{m_a}$$

Motion of absorber mass: $x_a(t) = -\frac{F_0}{k_a}\sin\omega t$; $X_a = -\frac{F_0}{k_a}$

Force acting on the absorber mass: $k_a x_a = k_a(-F_0/k_a) = -F_0$

Force provided by m_a = – Disturbance force

→ Zero net force acting on the primary mass

Principle (4)

From $$X = \frac{(k_a - m_a\omega^2)F_0}{(k + k_a - m\omega^2)(k_a - m_a\omega^2) - k_a^2}$$

Define $\omega_p = \sqrt{\frac{k}{m}}$ Original natural freq. of the primary system without the absorber

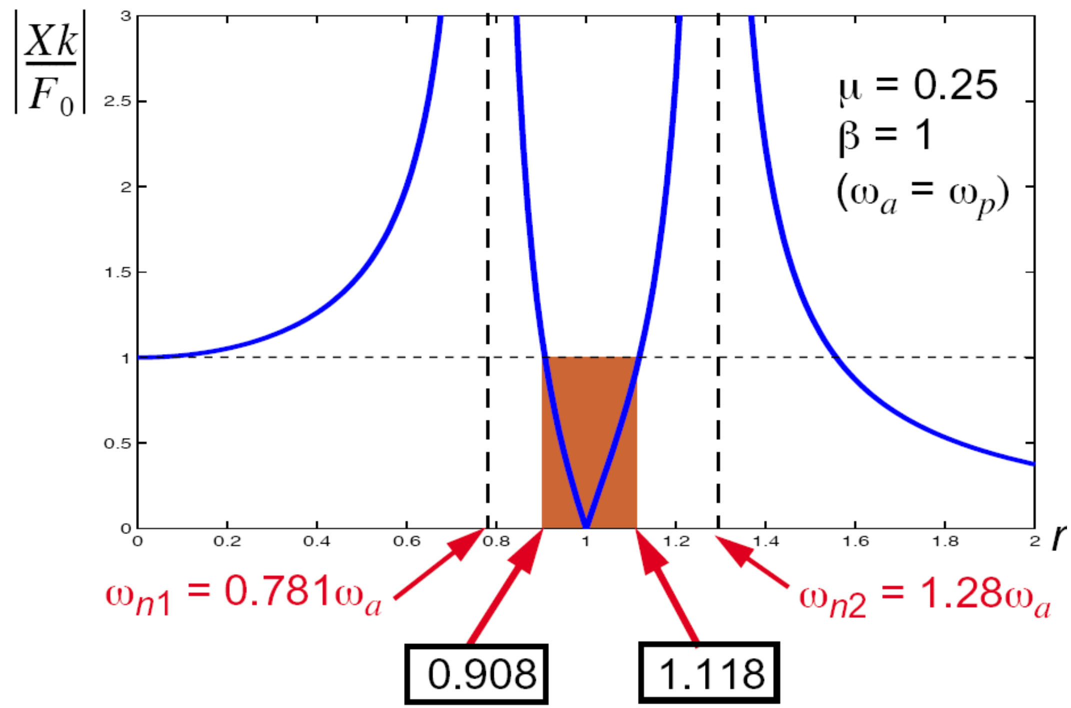
$\omega_a = \sqrt{\frac{k_a}{m_a}}$ Original natural freq. of the absorber before it is attached to the primary system

Normalize parameters $\mu = \frac{m_a}{m}$ $\beta = \frac{\omega_a}{\omega_p}$ $r = \frac{\omega}{\omega_a}$

Normalize disp. of the primary mass

$$\left|\frac{Xk}{F_0}\right| = \left|\frac{1 - r^2}{(1 + \mu\beta^2 - r^2)(1 - r^2) - \mu\beta^2}\right|$$

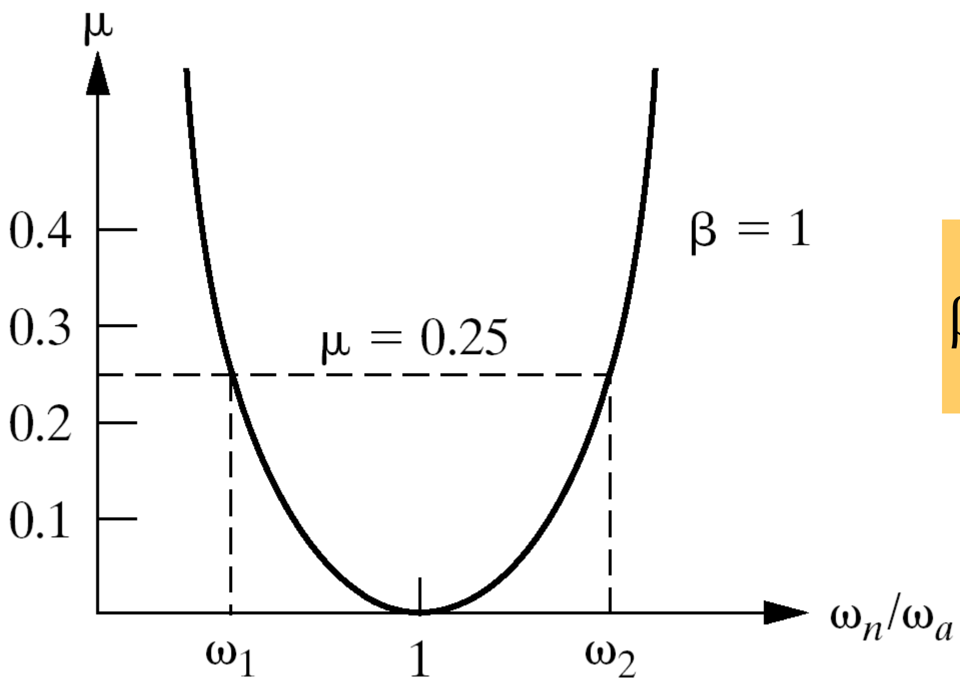
Principle (5)



$$\left|\frac{Xk}{F_0}\right| = \left|\frac{1-r^2}{(1+\mu\beta^2-r^2)(1-r^2)-\mu\beta^2}\right|$$

- Shade are is the useful operating bandwidth ($0.908\omega_a < \omega < 1.118\omega_a$)
- m_a and k_a are chosen such that r is within the bandwidth
- When r = 0.781 or 1.28, the combined system will experience the resonance and fail

Bandwidth of operating frequency

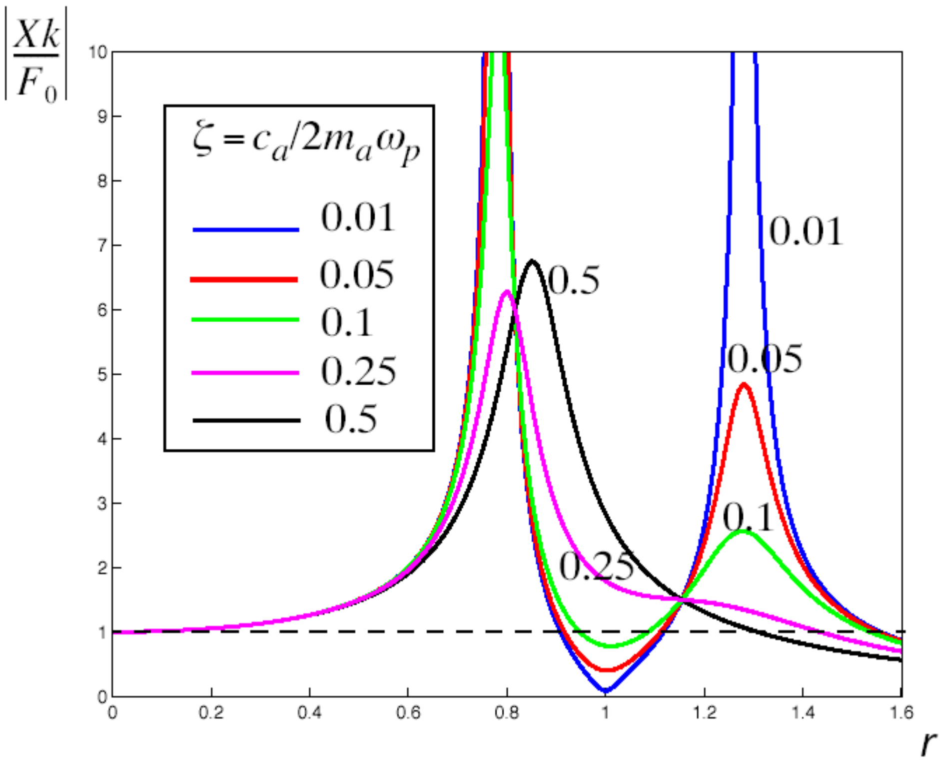


$$\mu = \frac{m_a}{m} \qquad \beta = \frac{\omega_a}{\omega_p}$$

$$\beta^2\left(\frac{\omega_n^2}{\omega_a^2}\right)^2 - [1+\beta^2(1+\mu)]\left(\frac{\omega_n^2}{\omega_a^2}\right) + 1 = 0$$

- As μ is increased, ω_n split farther apart, and farther from the operating point $\omega = \omega_a$
- $0.05 < \mu < 0.25$ (recommend)
- Very large μ $\longrightarrow$ large m_a $\longrightarrow$ stress and fatigue problems

Damping in vibration absorption



$\beta = 1$

$\mu = 0.25$

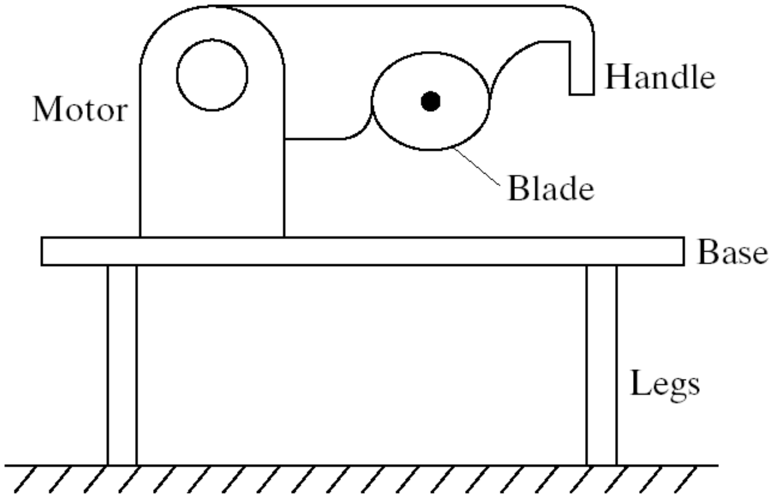
- Damping can reduce the resonance amplitude of the system
- Amplitude at operating point increase with increasing damping

Design procedure

- Select ω which will be tuned to zero amplitude
- Relation between k_a and m_a is obtained from $\omega^2 = k_a/m_a$
- Select m_a and k_a (consider restrictions: force, motion of absorber mass)
- Check the ratio $\mu = m_a/m$ (recommended value: $0.05<\mu<0.25$)

Example 5.3.1

A radial saw base has a mass of 73.16 kg and is driven by a motor. The motor runs at constant speed and produces a 13-N force at 180 cpm. The manufacturer wants a vibration absorber designed to drive the table oscillation to zero. Design the absorber assuming that the stiffness provided by the table legs is 2600 N/m. Absorber has a maximum deflection of 0.2 cm.



Example

A diesel engine, weighting 3000 N, is supported on a pedestal mount. The engine induces vibration through its pedestal mount at an operating speed of 6000 rpm. Determine the parameters of the vibration absorber what will reduce the vibration when mounted on the pedestal. The magnitude of the exciting force is 250 N, and the amplitude of motion of the absorber mass is to limited to 2 mm.

Example

A pipe carrying steam through a section of a factory vibrates violently when the driving pump hits a speed of 232 rpm. In an attempt to design an absorber, a trial 1 kg absorber tuned to 232 rpm was attached. By changing the pump speed, it was found that the pipe-absorber system has a resonance at 198 rpm. Redesign the absorber so that the natural frequencies are less than 160 rpm and more than 320 rpm.