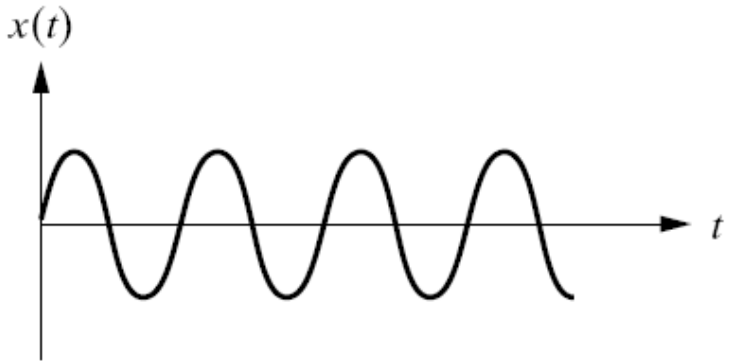
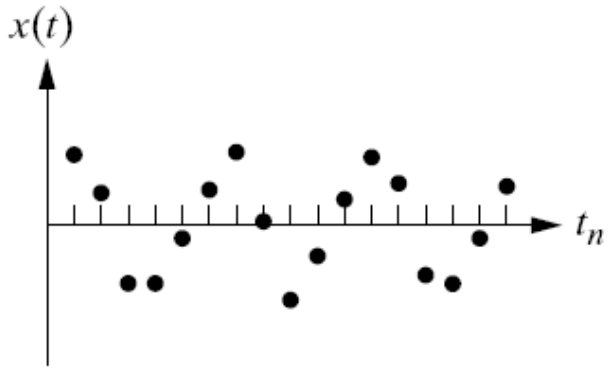
Digital signal processing

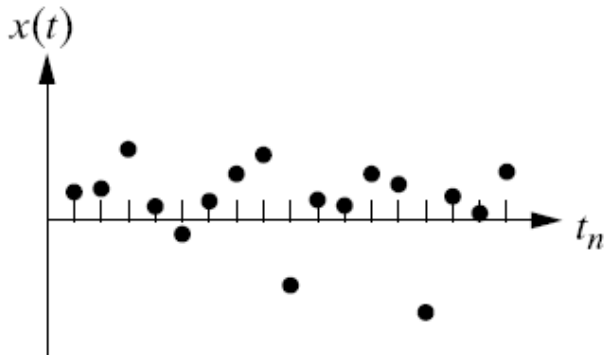
Sampling method



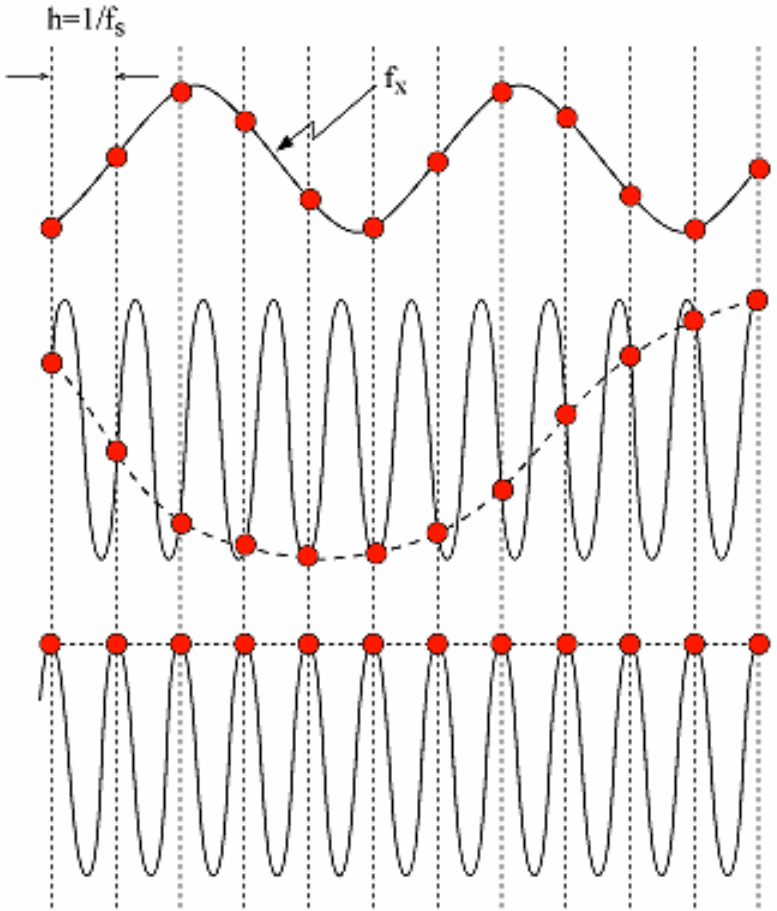
Time signal



Digital record



Sampling method (2)



Aliasing

If the sampling rate is too slow, the digital representation will cause high frequencies to appear as low frequencies.

- Problem is solved by choosing a sample rate of 2.5 times the largest frequency of interest
- Using an antialiasing filter (low-pass sharp-cutoff filter)

Fourier analysis (1)

Signals can be converted into frequency-domain by Fourier analysis

Fourier series of a periodic signal $F(t)$ of period T

$$F(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos n\omega_T t + b_n \sin n\omega_T t)$$

$$a_0 = \frac{2\pi}{T} \int_0^T F(t)dt$$

$$a_n = \frac{2}{T} \int_0^T [F(t) \cos n\omega_T t]dt$$

$$b_n = \frac{2}{T} \int_0^T [F(t) \sin n\omega_T t]dt$$

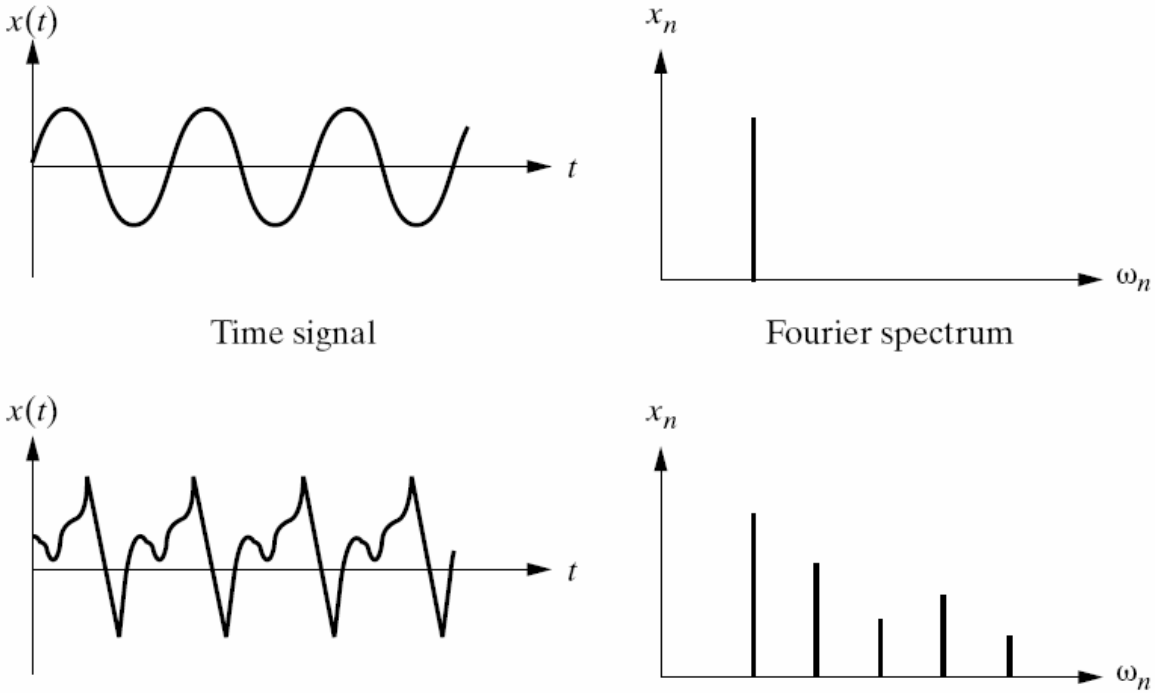
Amplitudes and phases of vibration components can be known from the coefficient of the Fourier series

Fourier analysis (2)

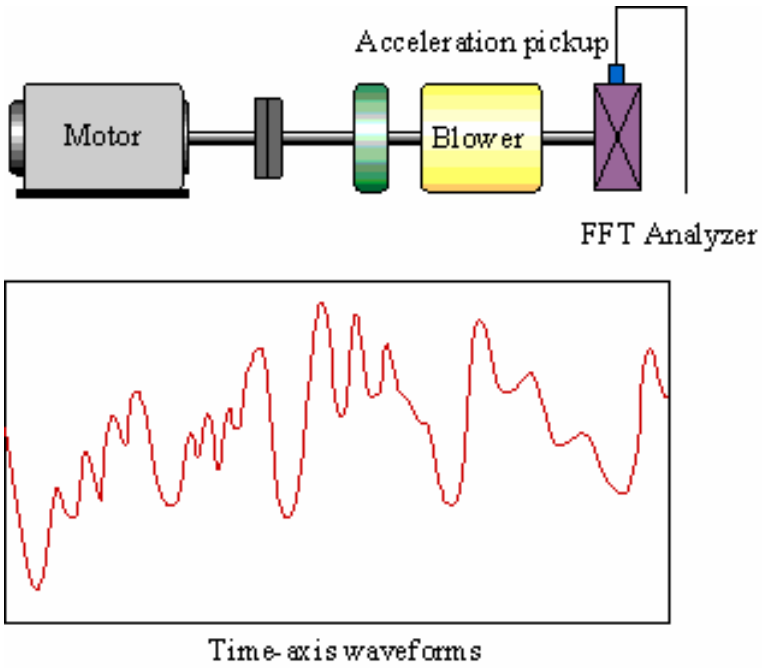
For any signal: Fourier transform indicates density of the amplitude of that signal at various frequency

Fourier transform of a signal $x(t)$ is

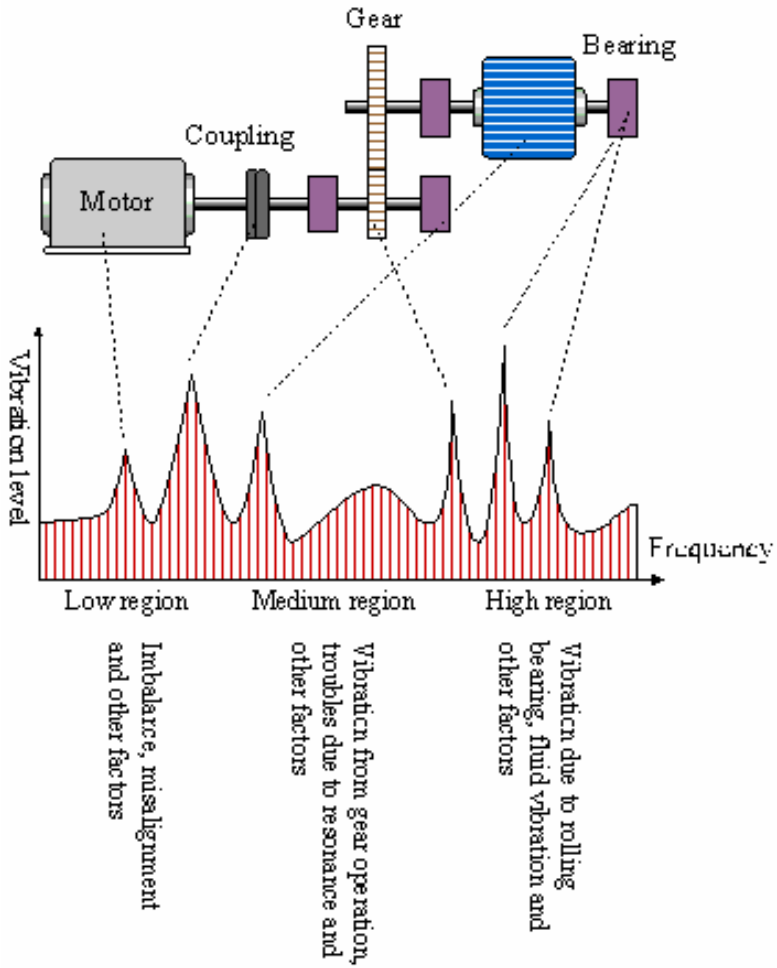
$$X(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$



Application of Fourier analysis

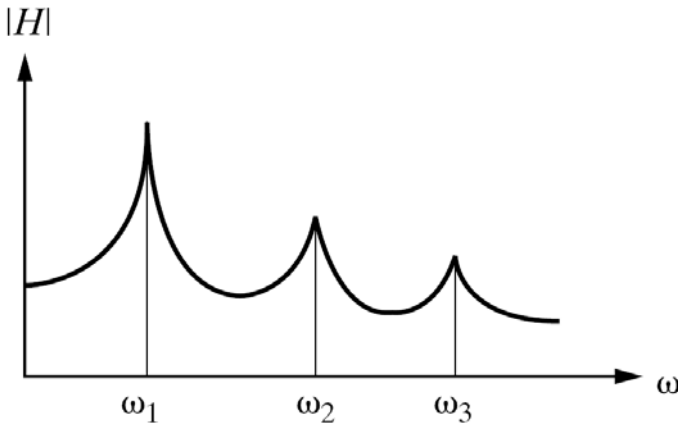


Time waveform is difficult to consider



Signal in freq. domain relates directly with operating speeds of machines

Modal data extraction (1)



3 peaks or resonance

3 dof model

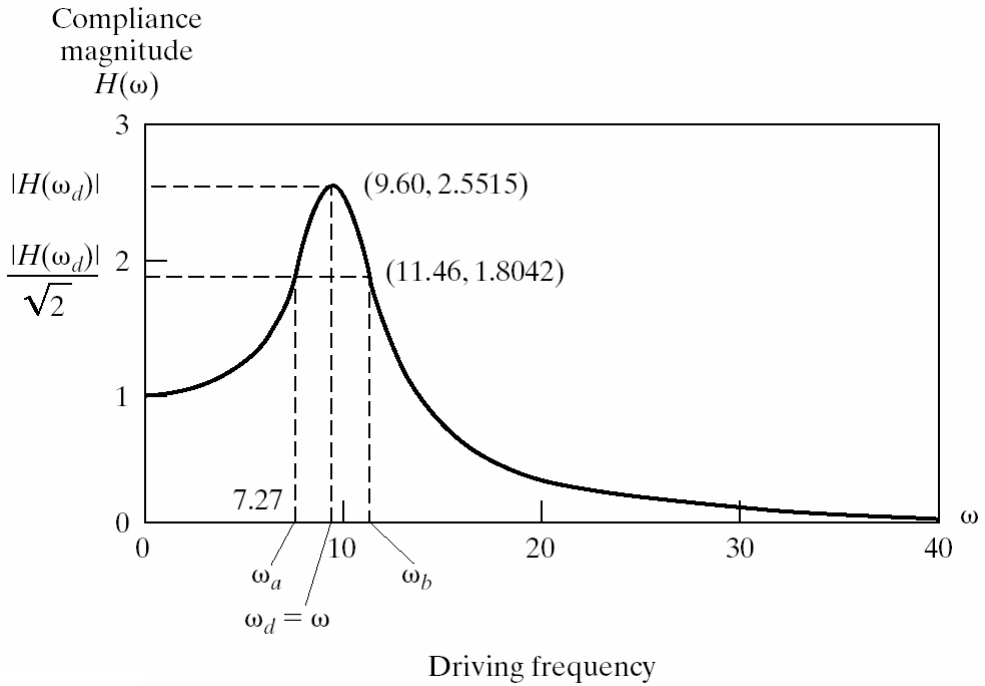
Single-degree-of-freedom curve fit (SDOF method)

FRF is sectioned off into freq ranges bracketing each successive peak.

Each peak is assumed that it is the FR of SDOF system.

(resonance frequencies are far apart)

Modal data extraction (2)



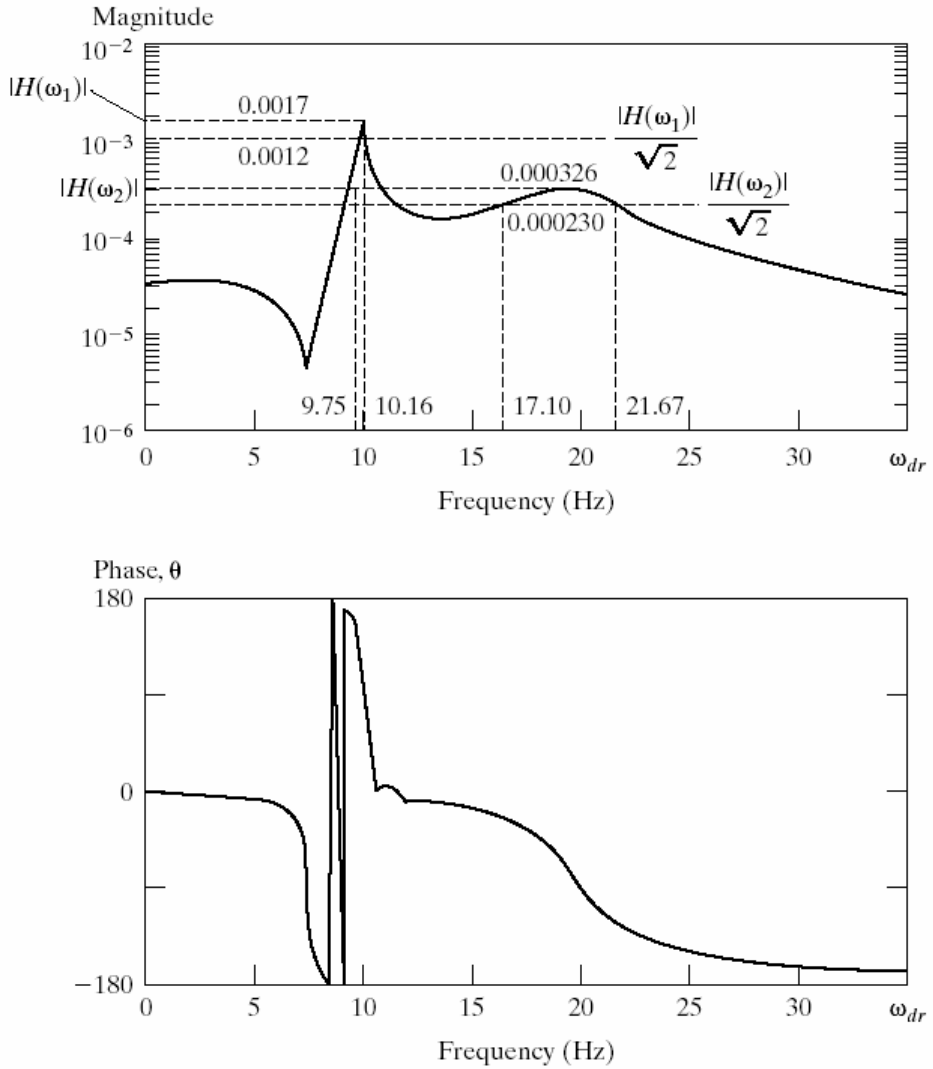
Modal dampings

$$\left|H(\omega_a)\right| = \left|H(\omega_b)\right| = \frac{\left|H(\omega_d)\right|}{\sqrt{2}}$$

then $\omega_b - \omega_a = 2\zeta\omega_d$

$$\zeta = \frac{\omega_b - \omega_a}{2\omega_d}$$

Example (1)



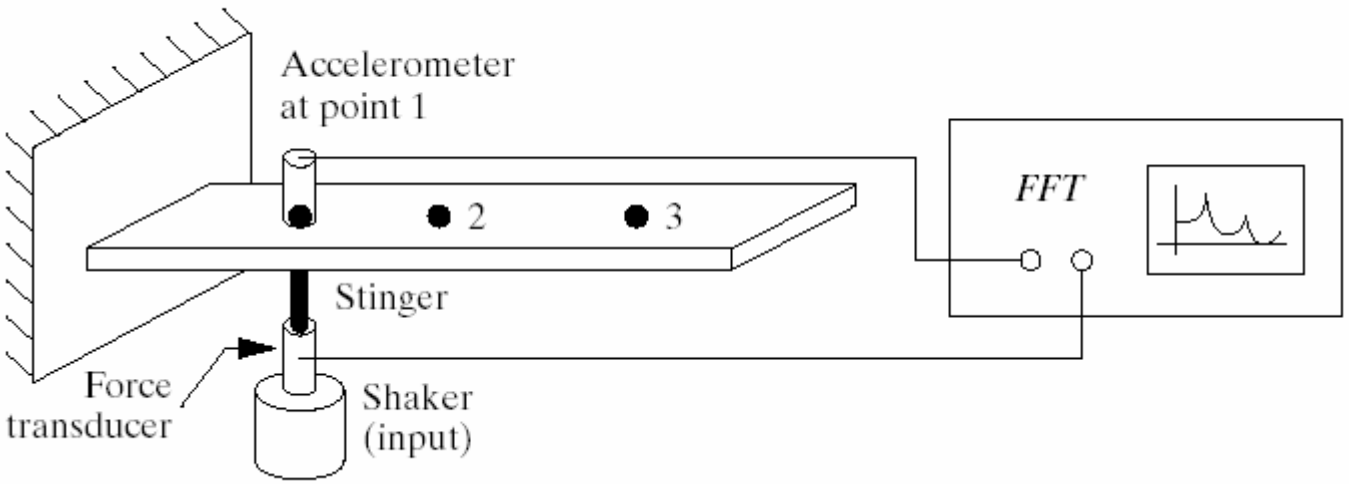
From measured FRF

DOF

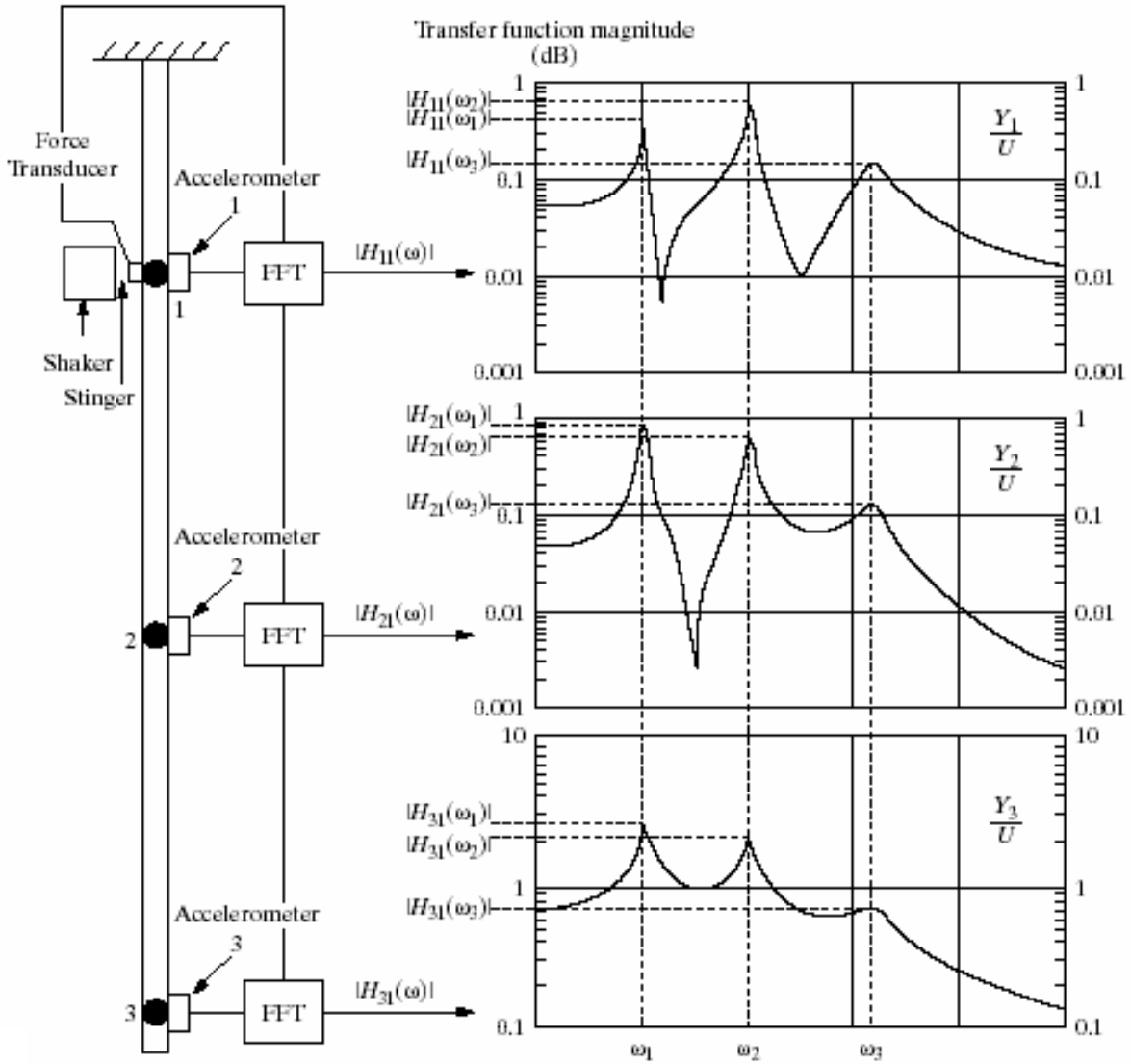
Natural frequencies

Damping ratios

Example (2-1)


Accelerometer
at point 1
2
3
FFT
Stinger
Force
transducer
Shaker
(input)

Example (2-2)


Force
Transducer
Accelerometer
1
FFT
$|H_{11}(\omega)|$
Shaker
Stinger
Accelerometer
2
FFT
$|H_{21}(\omega)|$
Accelerometer
3
FFT
$|H_{31}(\omega)|$
Transfer function magnitude
(dB)
$|H_{11}(\omega_2)|$
$|H_{11}(\omega_1)|$
$|H_{11}(\omega_3)|$
$\frac{Y_1}{U}$
$|H_{21}(\omega_1)|$
$|H_{21}(\omega_2)|$
$|H_{21}(\omega_3)|$
$\frac{Y_2}{U}$
$|H_{31}(\omega_1)|$
$|H_{31}(\omega_2)|$
$|H_{31}(\omega_3)|$
$\frac{Y_3}{U}$
1
0.1
0.01
0.001
10
ω_1
ω_2
ω_3