

Fatigue Failure

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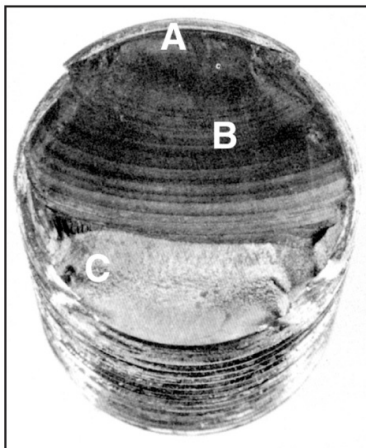
Introduction

- Fatigue failure occurs by machine parts are subjected to time varying loading.
- Failure probably occurs although the actual maximum stresses below the yield strength.
- Fatigue failure occur suddenly without warning, and hence dangerous.

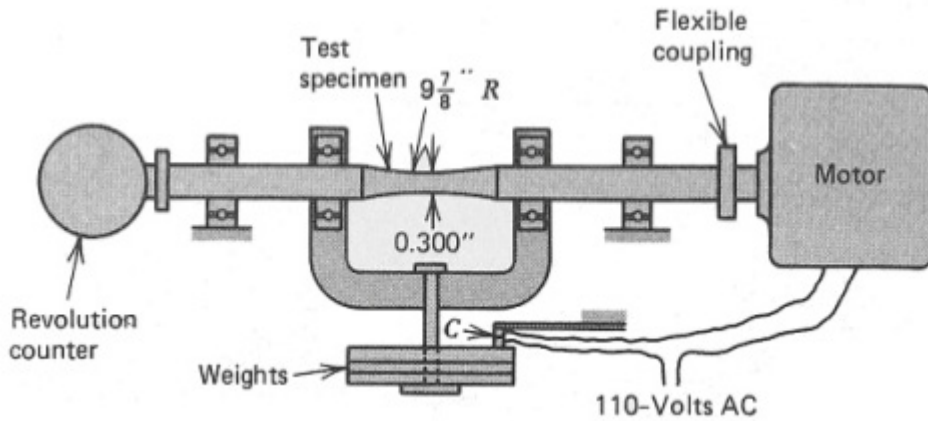
- The initiation of micro-cracks that are not normally discernible to the naked eye.
- frequently initiate at the discontinuity part that has high stress concentration.

Crack is enlarged by the repetitive load.

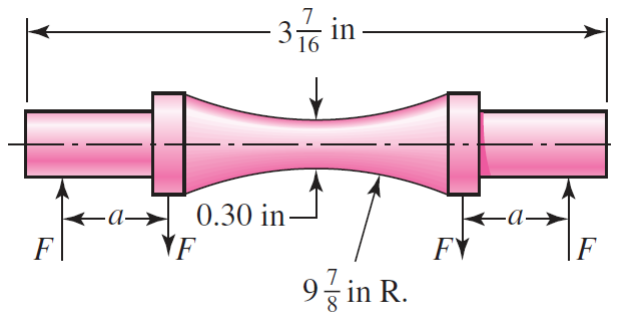
- Large crack
- The remaining material cannot support the loads, resulting in a sudden, fast fracture.



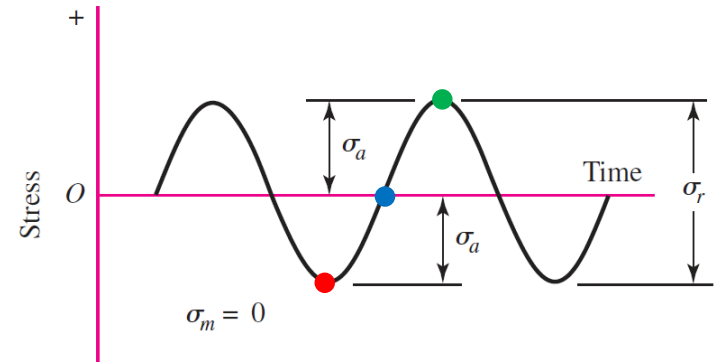
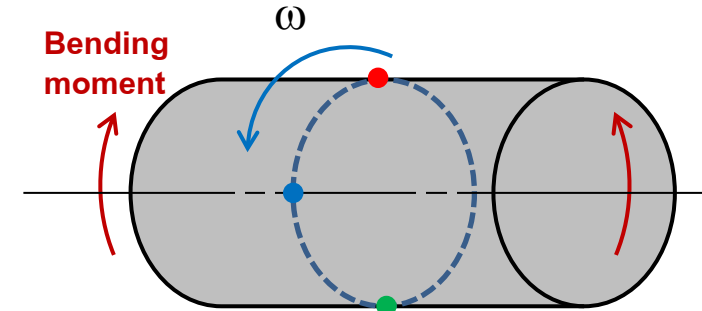
Fatigue testing



R.R. Moore rotating-beam machine

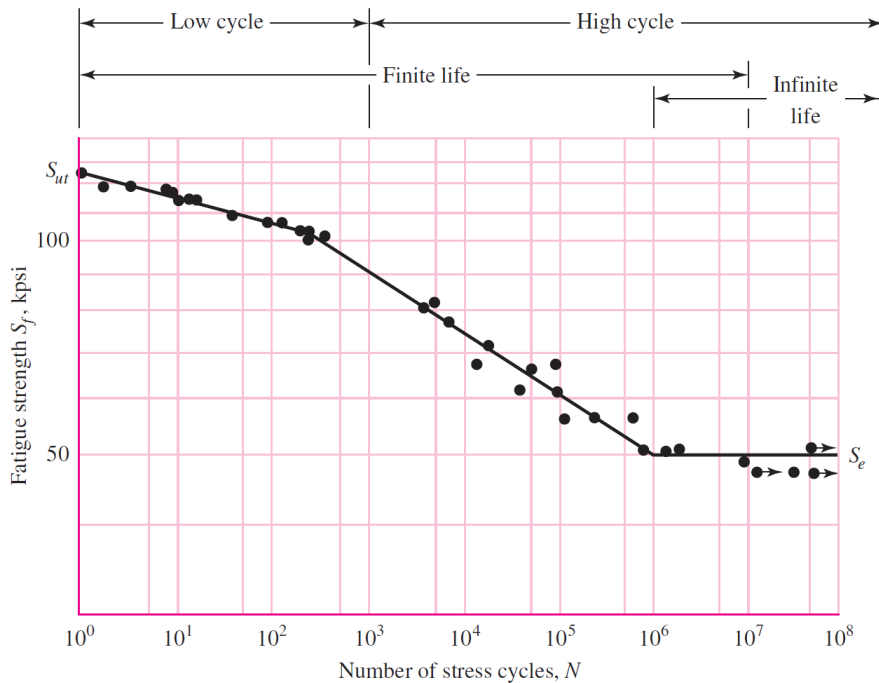


Test-specimen



The Endurance Limit (1)

When the material is subjected to the repetitive loading, the strength of material is reduce.

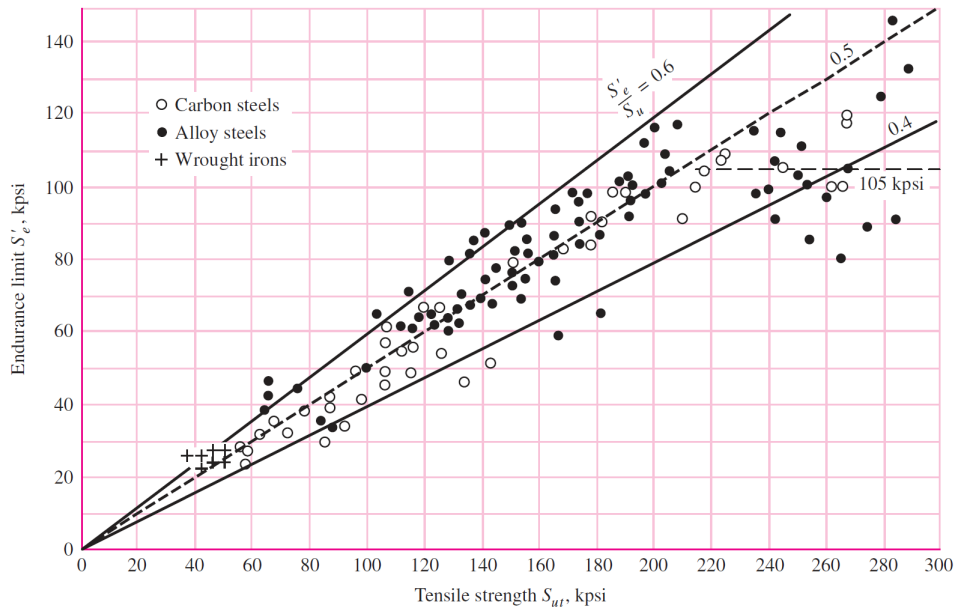


Tensile strength, S_{ut} : Maximum stress obtained from the tension test

Fatigue strength, S_f : The highest stress that a material can withstand for a given number of cycles without breaking

Endurance limit, S_e : the limiting value of stress at which failure occurs as the number of cycles become very large. For the stress below the endurance limit, failure never occurs.

The Endurance Limit (2)



Graph of endurance limits versus tensile strengths from actual test results.

$$S'_e = \begin{cases} 0.5(S_{ut}) & S_{ut} \leq 200 \text{ kpsi (1400 MPa)} \\ 100 \text{ kpsi (700 MPa)} & S_{ut} > 200 \text{ kpsi} \\ 700 \text{ Mpa} & S_{ut} > 1400 \text{ MPa} \end{cases}$$

S'_e is the endurance limit of the controlled laboratory specimen.
 Modifying factors will be added to account for difference between the specimen and the actual machine part.

Endurance Limit Modifying Factors

Endurance limit obtained from laboratory test is modified by multiplying many factors

$$S_e = k_a k_b k_c k_d k_e k_f S'_e$$

S_e : endurance limit at the critical location of a machine part

S'_e : rotary-beam test specimen endurance limit

k_a : surface condition modification factor

k_b : size modification factor

k_c : load modification factor

k_d : temperature modification factor

k_e : reliability factor

k_f : miscellaneous-effects modification factor

Surface factor, k_a

$$k_a = a S_{ut}^b$$

The surface of specimen is highly polished. For the other surface conditions, surface factor must be added.

Surface Finish	Factor a		Exponent b
	S_{ut} , kpsi	S_{ut} , MPa	
Ground	1.34	1.58	-0.085
Machined or cold-drawn	2.70	4.51	-0.265
Hot-rolled	14.4	57.7	-0.718
As-forged	39.9	272.	-0.995

Endurance Limit Modifying Factors

Size factor, k_b

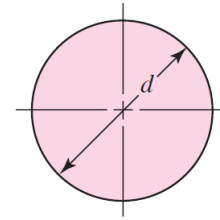
k_b is obtained from Bending and torsion experiments and can be calculated from

$$k_b = \begin{cases} (d/0.3)^{-0.107} & 0.11 \leq d \leq 2 \text{ in} \\ 0.91d^{-0.157} & 2 < d \leq 10 \text{ in} \\ (d/7.62)^{-0.107} & 2.79 \leq d \leq 51 \text{ mm} \\ 1.51d^{-0.157} & 51 < d \leq 254 \text{ mm} \end{cases}$$

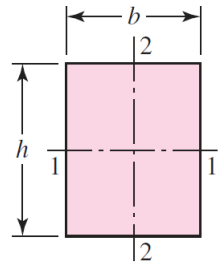
for axial loading, there is no size effect, $k_b = 1$

For nonrotating round bar and rectangular bar, use equivalent diameter d_e instead.

For nonrotating bar



$$d_e = 0.370d$$



$$d_e = 0.808\sqrt{hb}$$

Modifying Factors, k_b , k_c , k_d

Loading factor, k_c

Fatigue test was done with rotating bending load. For the other loading conditions the correction should be added.

$$k_c = \begin{cases} 1 & : \text{bending} \\ 0.85 & : \text{axial} \\ 0.59 & : \text{pure torsion} \end{cases}$$

torsion + bending, $k_c = 1$

Effect of operating temperature on the tensile strength of steel.

$$k_d = \frac{S_T}{S_{RT}}$$

S_T = Tensile strength at operating temp.

S_{RT} = Tensile strength at room temp.

Temperature factor, k_d

The tensile strength is changed with temperature.

This effect is corrected by k_d

$$k_d = 0.975 + 0.432(10^{-3})T_F - 0.115(10^{-5})T_F^2 + 0.104(10^{-8})T_F^3 - 0.595(10^{-12})T_F^4$$

$$70 \leq T_F \leq 1000^\circ\text{F}$$

Temperature, °C	S_T/S_{RT}	Temperature, °F	S_T/S_{RT}
20	1.000	70	1.000
50	1.010	100	1.008
100	1.020	200	1.020
150	1.025	300	1.024
200	1.020	400	1.018
250	1.000	500	0.995
300	0.975	600	0.963
350	0.943	700	0.927
400	0.900	800	0.872
450	0.843	900	0.797
500	0.768	1000	0.698
550	0.672	1100	0.567
600	0.549		

Modifying Factors, k_e , k_f

Reliability factor, k_e

Because of the scatter of the experimental data, reliability factor is added to compensate the uncertainty of endurance limit.

Reliability, %	Transformation Variate z_α	Reliability Factor k_e
50	0	1.000
90	1.288	0.897
95	1.645	0.868
99	2.326	0.814
99.9	3.091	0.753
99.99	3.719	0.702
99.999	4.265	0.659
99.9999	4.753	0.620

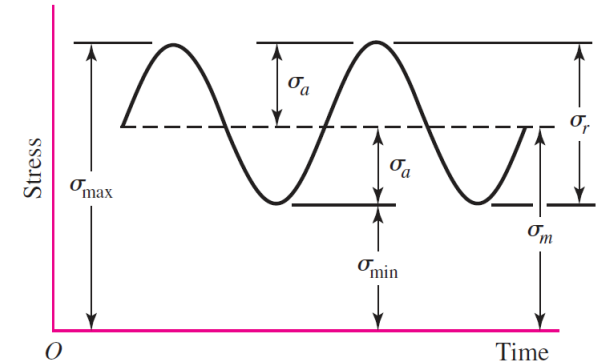
Reliability factor k_e corresponding to 8% standard deviation of the endurance limit.

Miscellaneous factor, k_f

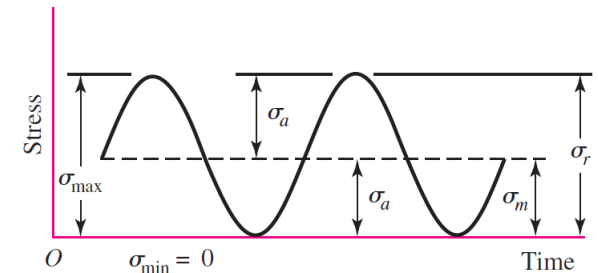
This factor is intended to account for the reduction in endurance limit due to all other effects such as the effect of manufacturing process, corrosion, frequency of loading. If there are no other effects use $k_f = 1$.

Fluctuating Stresses

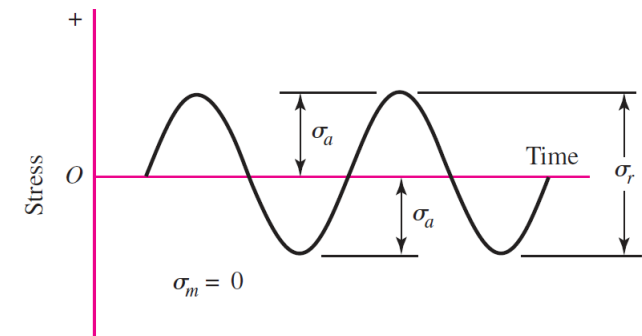
- Fluctuating stresses in machinery often take the form of a sinusoidal pattern because of the nature of rotating machinery.
- Other patterns are probable, but the shape of the wave is not important.
- The peaks values (maximum and minimum) are important.



(d)



(e)



(f)

Midrange component

$$\sigma_m = \frac{\sigma_{\max} + \sigma_{\min}}{2}$$

Alternating component

$$\sigma_a = \left| \frac{\sigma_{\max} - \sigma_{\min}}{2} \right|$$

Stress ratio

$$R = \frac{\sigma_{\min}}{\sigma_{\max}}$$

Amplitude ratio

$$A = \frac{\sigma_a}{\sigma_m}$$

Stress concentrations

- In the development of the basic stress equations, it was assumed that no geometric irregularities occurred in the member.
- The existence of irregularities (holes, grooves, notches) increases stresses significantly
- This phenomena is called stress concentration.

Stress concentration factor
(Normal stress)

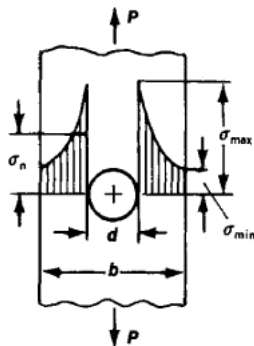
$$K_f = \frac{\sigma_{\max}}{\sigma_0}$$

Stress concentration factor
(Shear stress)

$$K_{fs} = \frac{\tau_{\max}}{\tau_0}$$

$\sigma_{\max}, \tau_{\max}$: actual maximum stress

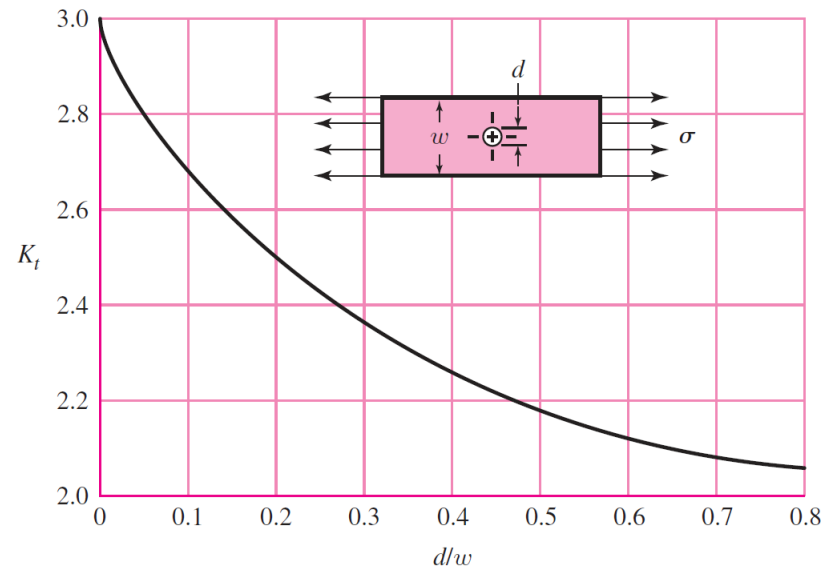
σ_0, τ_0 : stress calculated from basic equation



t = thickness

$$F = \sigma wt$$

$$\sigma_0 = \frac{F}{(w-d)t} = \frac{w}{(w-d)} \sigma$$



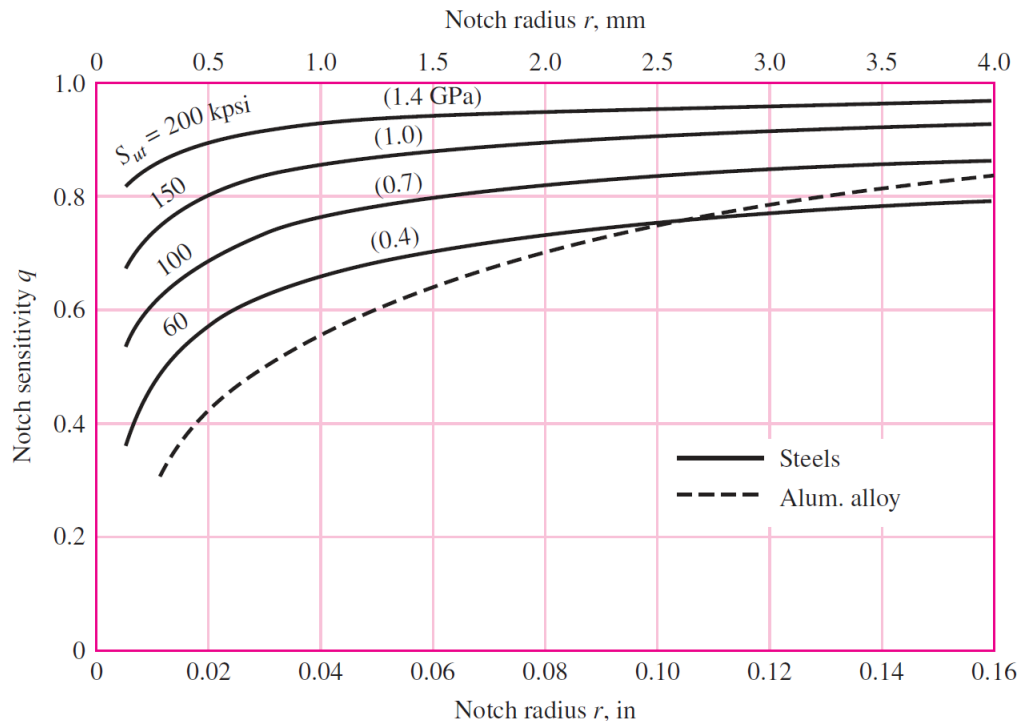
Notch-sensitivity, q

- K_t (theoretical stress concentration factor) in the figures depends only on the geometry of the part, but the stress concentration also be affected with the notch-sensitivity of the materials.
- K_f (fatigue stress concentration factor is used instead.

$$K_f = 1 + q(K_t - 1)$$



$$\sigma_{\max} = K_f \sigma_0$$



Notch-sensitivity charts for steel and UNS A92024-T wrought aluminum alloys subjected to reversed bending or reversed axial loads.

For larger notch radii, use the values of q corresponding to the $r = 0.16$ in (4 mm) ordinate.

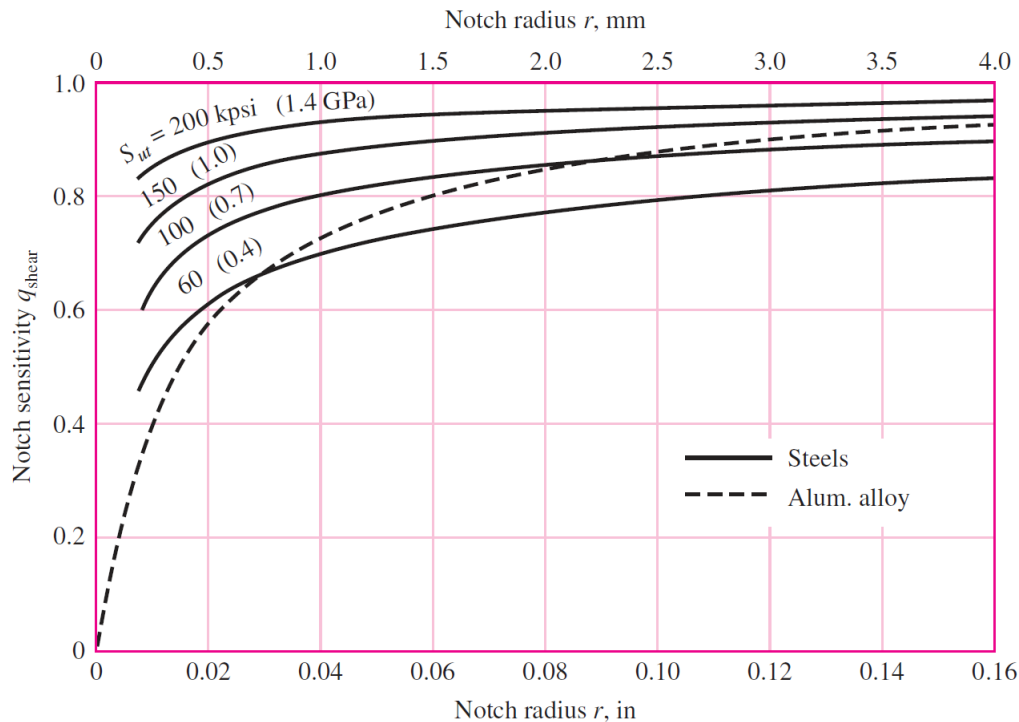
Notch-sensitivity, q_{shear}

For shear stress K_{ts} becomes K_{fs} .

$$K_{fs} = 1 + q_{\text{shear}} (K_{ts} - 1)$$



$$\tau_{\text{max}} = K_{fs} \tau_0$$

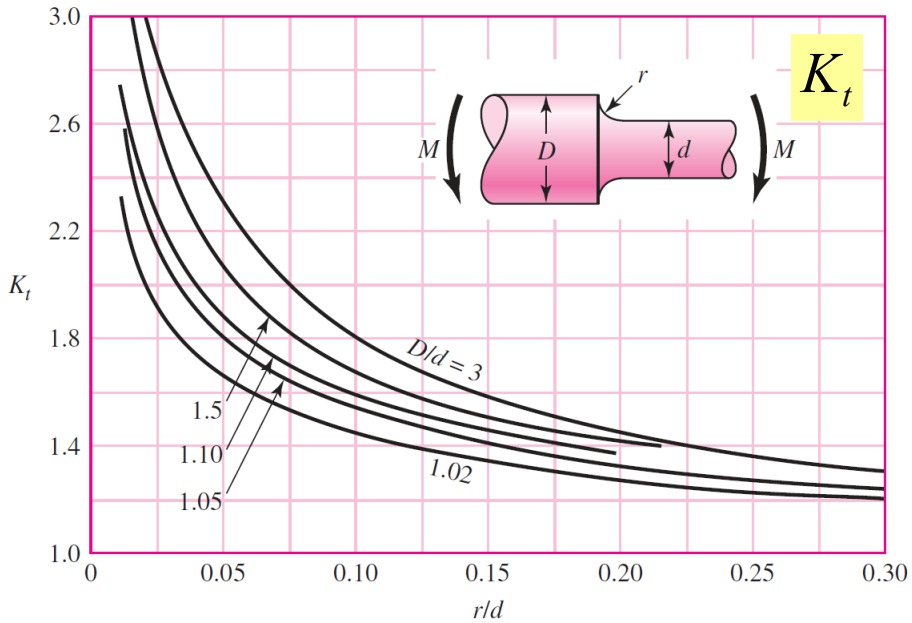


Notch-sensitivity charts for materials in reverse torsion.

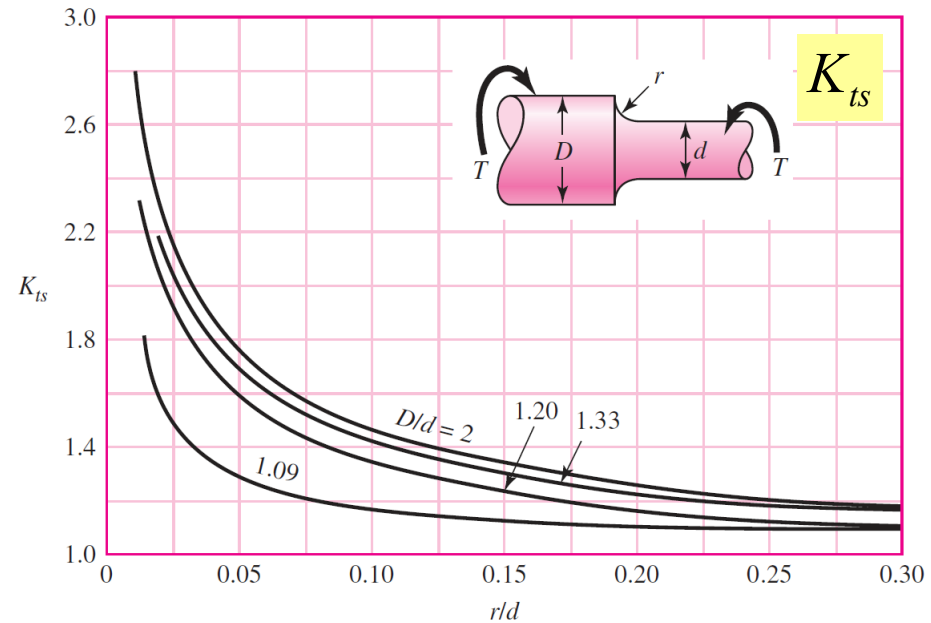
For larger notch radii, use the values of q_{shear} corresponding to the $r = 0.16$ in (4 mm).

Stress Concentrations (1)

Stress concentration factor K_t (bending) and K_{ts} (torsion) for round shaft with shoulder fillet



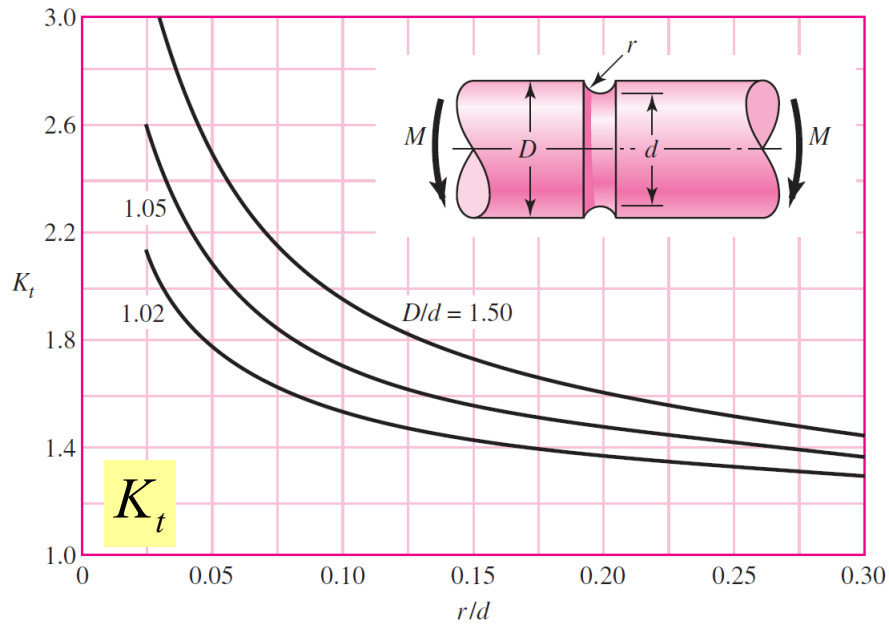
$$K_f = 1 + q(K_t - 1)$$



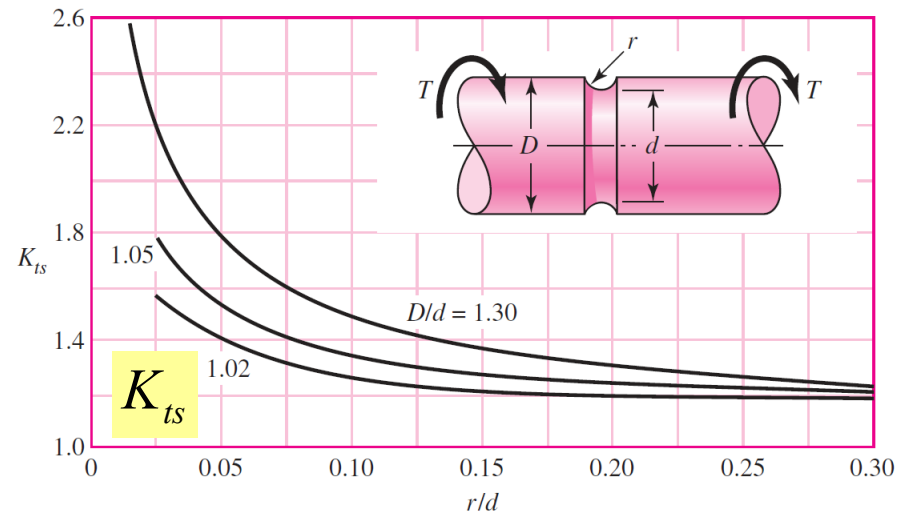
$$K_{fs} = 1 + q_{\text{shear}}(K_{ts} - 1)$$

Stress Concentrations (2)

Stress concentration factor K_t (bending) and K_{ts} (torsion) for grooved round bar



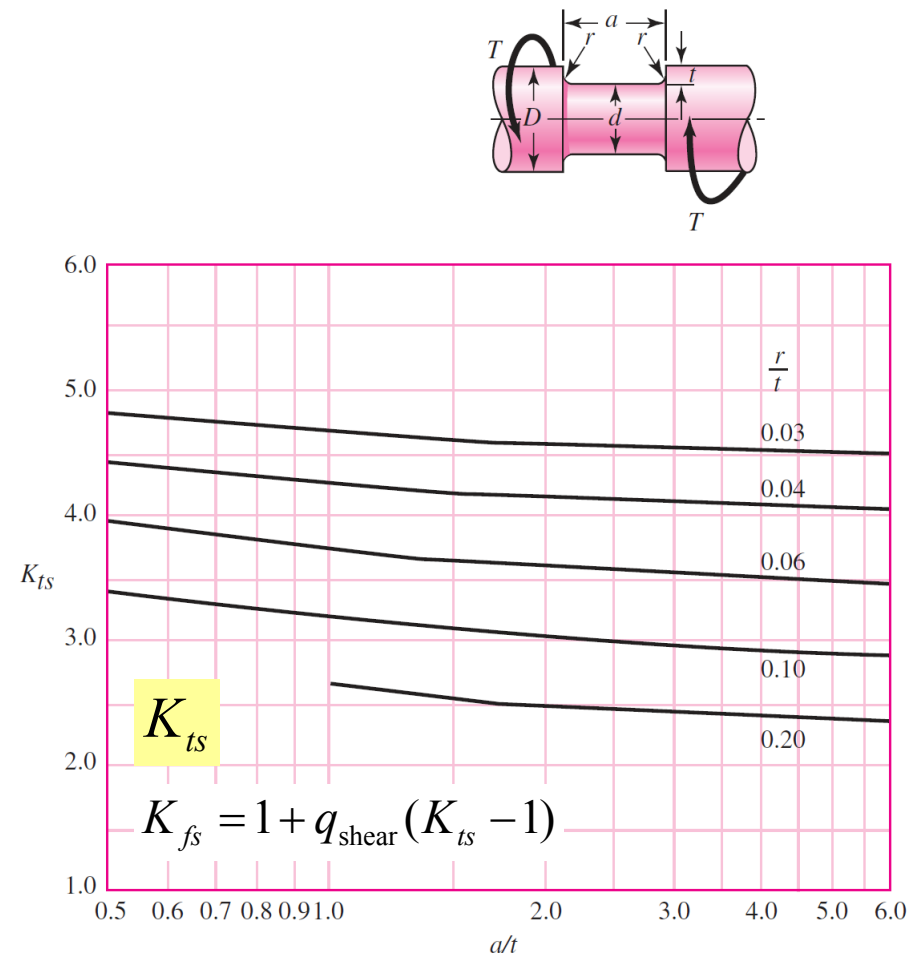
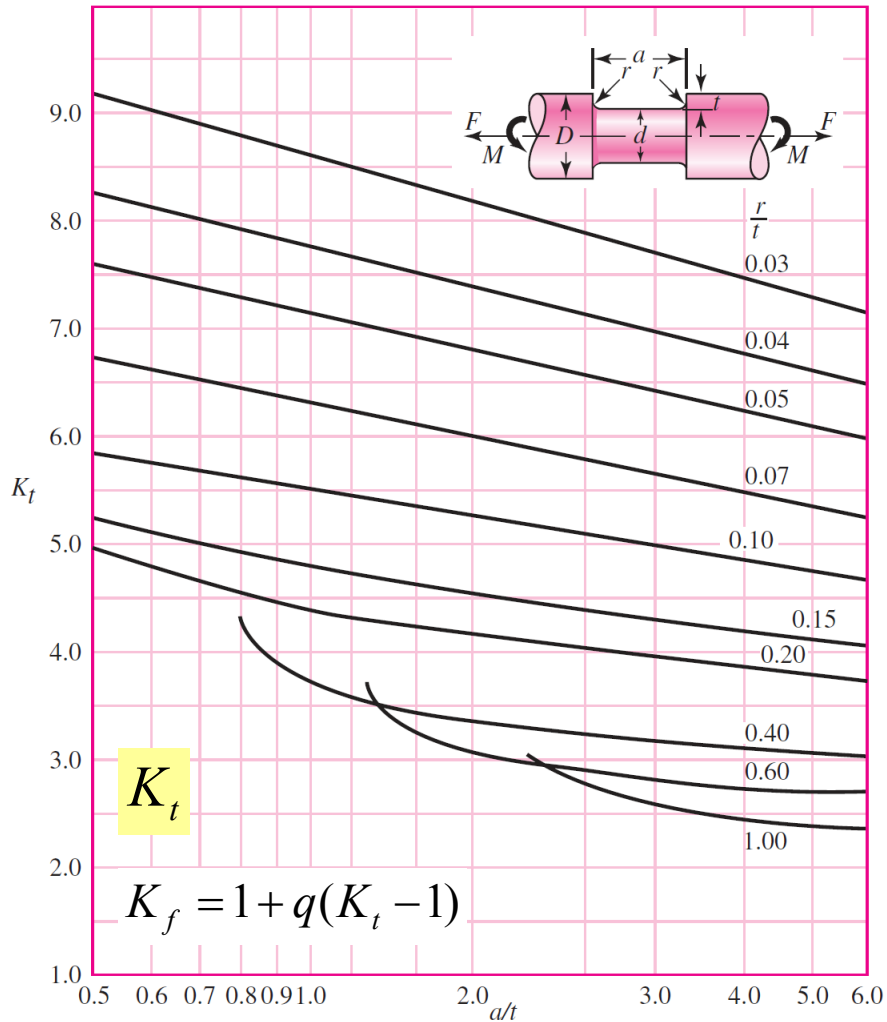
$$K_f = 1 + q(K_t - 1)$$



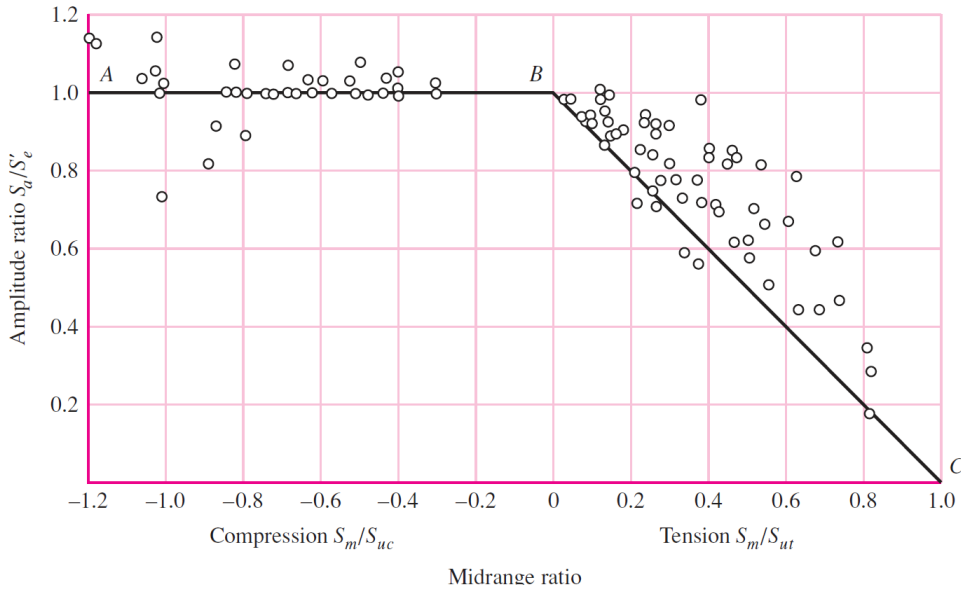
$$K_{fs} = 1 + q_{\text{shear}}(K_{ts} - 1)$$

Stress Concentrations (3)

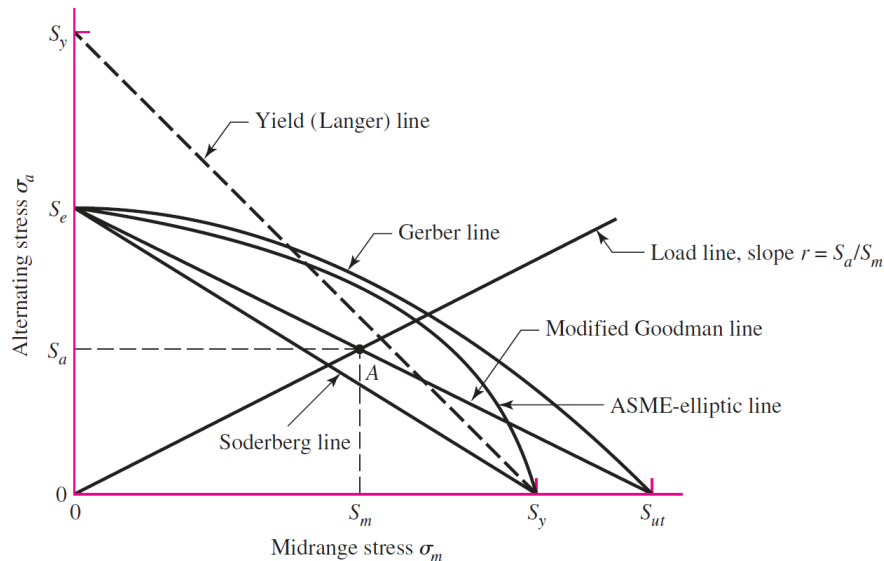
Stress concentration factor K_t (bending) and K_{ts} (torsion) of round shaft with flat-bottom groove



Fatigue Failure Criteria (1)



Plot of fatigue failures for midrange stresses in both tensile and compressive regions.



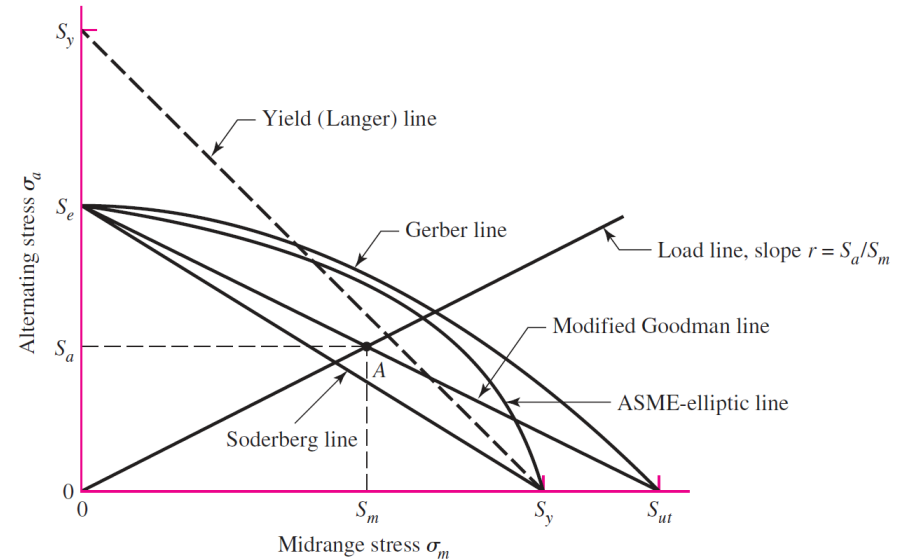
Fatigue diagram showing various criteria of failure. (tensile side)

Fatigue Failure Criteria (2)

Fatigue failure criteria	Equation
Soderberg	$\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_y} = \frac{1}{n}$
Modified-Goodman	$\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{1}{n}$
Gerber	$\frac{n\sigma_a}{S_e} + \left(\frac{n\sigma_m}{S_{ut}}\right)^2 = 1$
ASME-elliptic	$\left(\frac{n\sigma_a}{S_e}\right)^2 + \left(\frac{n\sigma_m}{S_y}\right)^2 = 1$
Langer static yield	$\sigma_a + \sigma_m = S_y/n$

Fatigue failure occurs when

- The left hand side > the right hand side (for n = 1)
- n from the calculation less than 1
- The maximum stress is larger than the static yield criteria



σ_a : Alternating stress

σ_m : Midrange stress

S_e : endurance limit

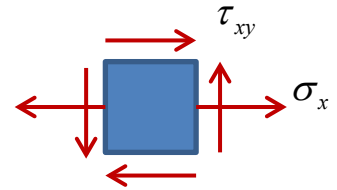
S_y : yield strength

S_{ut} : tensile strength

n : factor of safety

Combinations of Loading Modes

Ex. Shaft subjected to bending
+ torsional shear stress



1. Calculate σ_m, τ_m and σ_a, τ_a

2. Consider Stress concentration factor

3. Calculate von Mises stress σ'_m and σ'_a

$$\sigma' = \frac{1}{\sqrt{2}} [(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2)]^{1/2}$$

$$\sigma' = (\sigma_x^2 + 3\tau_{xy}^2)^{1/2}$$

$$\sigma'_m = ((K_f \sigma_m)^2 + 3(K_{fs} \tau_m)^2)^{1/2}$$

$$\sigma'_a = ((K_f \sigma_a)^2 + 3(K_{fs} \tau_a)^2)^{1/2}$$

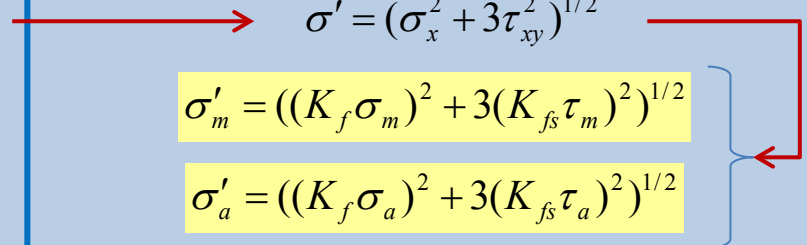
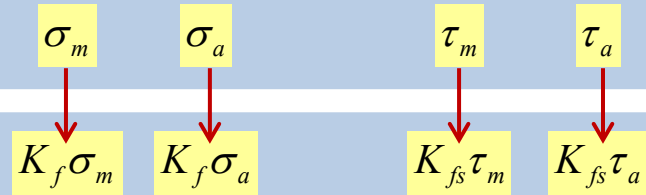
4. Substitute von Mises stress in fatigue criteria

$$\frac{\sigma'_a}{S_e} + \frac{\sigma'_m}{S_y} = \frac{1}{n}$$

Ex. Soderberg

5. Check for static yield

$$\sigma'_a + \sigma'_m = S_y / n$$



Example

A rotating shaft is made of 42×4 mm AISI 1018 cold-drawn steel tubing and has a 6 mm diameter hole drilled transversely through it. Estimate the factor of safety guarding against fatigue and static failure using Gerber and Langer failure criteria for the following loading conditions:

- (a) The shaft is subjected to a completely reversed torque of 120 Nm in phase with a completely reversed bending moment of 150 Nm.
- (b) The shaft is subjected to pulsating torque fluctuating from 60 to 160 Nm and a steady bending moment of 150 Nm.

(Shigley's Mechanical Engineering Design, Ninth Edition, Example 6-14)

Given: $S_{ut} = 440$ MPa, $S_y = 370$ MPa

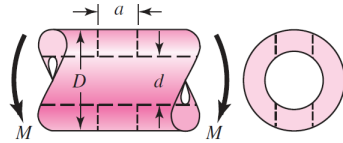
Equations for calculate stress and K_t , K_{ts} are given in Table A-16

Example

Table A-16

Approximate Stress-Concentration Factor K_t for Bending of a Round Bar or Tube with a Transverse Round Hole

Source: R. E. Peterson, *Stress-Concentration Factors*, Wiley, New York, 1974, pp. 146, 235.



The nominal bending stress is $\sigma_0 = M/Z_{\text{net}}$ where Z_{net} is a reduced value of the section modulus and is defined by

$$Z_{\text{net}} = \frac{\pi A}{32D}(D^4 - d^4)$$

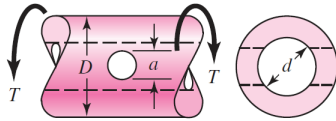
Values of A are listed in the table. Use $d = 0$ for a solid bar

a/D	d/D					
	0.9		0.6		0	
	A	K_t	A	K_t	A	K_t
0.050	0.92	2.63	0.91	2.55	0.88	2.42
0.075	0.89	2.55	0.88	2.43	0.86	2.35
0.10	0.86	2.49	0.85	2.36	0.83	2.27
0.125	0.82	2.41	0.82	2.32	0.80	2.20
0.15	0.79	2.39	0.79	2.29	0.76	2.15
0.175	0.76	2.38	0.75	2.26	0.72	2.10
0.20	0.73	2.39	0.72	2.23	0.68	2.07
0.225	0.69	2.40	0.68	2.21	0.65	2.04
0.25	0.67	2.42	0.64	2.18	0.61	2.00
0.275	0.66	2.48	0.61	2.16	0.58	1.97
0.30	0.64	2.52	0.58	2.14	0.54	1.94

Example

Table A-16 (Continued)

Approximate Stress-Concentration Factors K_{ts} for a Round Bar or Tube Having a Transverse Round Hole and Loaded in Torsion Source: R. E. Peterson, *Stress-Concentration Factors*, Wiley, New York, 1974, pp. 148, 244.



The maximum stress occurs on the inside of the hole, slightly below the shaft surface. The nominal shear stress is $\tau_0 = TD/2J_{\text{net}}$, where J_{net} is a reduced value of the second polar moment of area and is defined by

$$J_{\text{net}} = \frac{\pi A(D^4 - d^4)}{32}$$

Values of A are listed in the table. Use $d = 0$ for a solid bar.

a/D	0.9		0.8		0.6		0.4		0	
	A	K_{ts}	A	K_{ts}	A	K_{ts}	A	K_{ts}	A	K_{ts}
0.05	0.96	1.78							0.95	1.77
0.075	0.95	1.82							0.93	1.71
0.10	0.94	1.76	0.93	1.74	0.92	1.72	0.92	1.70	0.92	1.68
0.125	0.91	1.76	0.91	1.74	0.90	1.70	0.90	1.67	0.89	1.64
0.15	0.90	1.77	0.89	1.75	0.87	1.69	0.87	1.65	0.87	1.62
0.175	0.89	1.81	0.88	1.76	0.87	1.69	0.86	1.64	0.85	1.60
0.20	0.88	1.96	0.86	1.79	0.85	1.70	0.84	1.63	0.83	1.58
0.25	0.87	2.00	0.82	1.86	0.81	1.72	0.80	1.63	0.79	1.54
0.30	0.80	2.18	0.78	1.97	0.77	1.76	0.75	1.63	0.74	1.51
0.35	0.77	2.41	0.75	2.09	0.72	1.81	0.69	1.63	0.68	1.47
0.40	0.72	2.67	0.71	2.25	0.68	1.89	0.64	1.63	0.63	1.44