Nonstationary Fay-Herriot Model for Small Area Estimation

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Overview of Presentation

- Background
- Linear mixed model (LMM) and the EBLUP of the small area mean
- Spatial nonstationary extension to the LMM and the NSEBLUP
- MSE estimation
- Empirical evaluations: model-based and design-based simulations
- Concluding remarks

Background

- Unit level **linear mixed models** with area specific random effects are now quite popular in small area estimation **(SAE)**

- The empirical best linear unbiased predictor **(EBLUP)** is widely used technique of SAE under these models and proven to be **efficient**, see **Rao (2003)**

- In many SAE problems, it is not always possible to use the unit level small area model simply because of the unavailability of the unit level data

- In such circumstances, SAE is carried out under **area level random effect models**

- The Fay–Herriot model is widely used area level model in SAE **(Fay and Herriot, 1979)** - one of the most popular methods of SAE

Background

- The **fixed effects** parameters are **spatially invariant**

- There are situations (e.g., agricultural, environmental data etc), where the relationship between *y* and *x* is **not constant** over the study area, that is, the **regression coefficients vary** spatially across the geography of interest, a phenomenon referred to as **spatial nonstationarity**

- The Fay–Herriot model **does not** account for the presence of spatial nonstationarity in the data

- **Geographical weighted regression (GWR)** approach - suitable for modelling spatial nonstationarity **(Fotheringham *et al.*, 2002)**

- We use the **geographically weighted** concept to fit Fay–Herriot model and consider SAE under this model

Area Level Model in SAE

- The simple area specific two stage model suggested by **Fay and Herriot (1979)** is described as

$$y_i = Y_i + e_i, \text{ and } Y_i = \mathbf{x}_i^T \beta + u_i (i = 1,\ldots,m)$$

- The first stage accounts for the **sampling variability** of the direct survey estimates y_i of true area values (e.g., population means) Y_i and the **second stage** links the true area values Y_i to a vector of known auxiliary variables $\mathbf{x}_i$

- We can express this model as an area level linear mixed model

$$y_i = \mathbf{x}_i^T \beta + u_i + e_i; i = 1,\ldots,m$$

- β is a p-vector of unknown fixed effect parameters

- u_i's are iid normal random errors with $E(u_i) = 0$ and $Var(u_i) = \sigma_u^2$

Area Level Model in SAE

- e_i's are independent sampling errors normally distributed with $E(e_i \mid Y_i) = 0$, $Var(e_i \mid Y_i) = \sigma_{ei}^2$, $u_i \perp e_i$
- Usually, σ_{ei}^2 is known and σ_u^2 is unknown and has to be estimated
- Aggregating *m*-area level model leads to linear mixed model of form

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{D}\mathbf{u} + \mathbf{e}$$

- $\mathbf{y} = (y_1, \ldots, y_m)^T; \mathbf{X} = (\mathbf{x}_1, \ldots, \mathbf{x}_m)^T; \mathbf{D} = (\mathbf{d}_1, \ldots, \mathbf{d}_m)^T = \mathbf{I}_m \; ; \mathbf{u} = (u_1, \ldots, u_m)^T$
- $\mathbf{u} \sim N(\mathbf{0}, \Sigma_u = \sigma_u^2 \mathbf{I}_m); \; \mathbf{e} = (e_1, \ldots, e_m)^T; \; \mathbf{e} \sim N(\mathbf{0}, \Sigma_e)$
- $\Sigma_e = diag\{\sigma_{ei}^2; 1 \le i \le m\}$ and $Var(\mathbf{y}) = \mathbf{V} = \mathbf{D}\Sigma_u\mathbf{D}^T + \Sigma_e$
- The parameter Σ_u (or σ_u^2) - referred to as the **variance component**
- Methods of estimating Σ_u include **ML** and **REML** under normality, the method of **fitting constants** without normality assumption **(Rao, 2003)**

- Let $\hat{\Sigma}_u$ denotes estimate of Σ_u and we define the plug-in estimator of covariance matrix $\hat{\mathbf{V}} = \mathbf{D}\hat{\Sigma}_u\mathbf{D}^T + \Sigma_e$

- Under LMM model, the **EBLUE** of β and the **EBLUP** of **u** are

$$\hat{\boldsymbol{\beta}} = (\mathbf{x}^T\hat{\mathbf{V}}^{-1}\mathbf{x})^{-1}\mathbf{x}^T\hat{\mathbf{V}}^{-1}\mathbf{y} \text{ and } \hat{\mathbf{u}} = \hat{\Sigma}_u\mathbf{D}^T\hat{\mathbf{V}}^{-1}(\mathbf{y} - \mathbf{X}\hat{\boldsymbol{\beta}})$$

- Using the estimated fixed and random effects, the **EBLUP** estimate for Y_i is **(Henderson, 1975 and Fay & Herriot, 1979)**

$$\hat{Y}_i^{EBLUP} = \mathbf{x}_i^T\hat{\boldsymbol{\beta}} + \mathbf{d}_i^T\hat{\Sigma}_u\mathbf{D}^T\hat{\mathbf{V}}^{-1}(\mathbf{y} - \mathbf{X}\hat{\boldsymbol{\beta}})$$

- $\mathbf{d}_i^T$ denotes the rows of $\mathbf{D}$ and take value 1 for area *i* and 0 otherwise

- In particular: $\hat{Y}_i^{EBLUP} = \mathbf{x}_i^T\hat{\boldsymbol{\beta}} + \hat{\gamma}_i(y_i - \mathbf{x}_i^T\hat{\boldsymbol{\beta}})$, where $\hat{\gamma}_i = \hat{\sigma}_u^2\left(\hat{\sigma}_u^2 + \sigma_{ei}^2\right)^{-1}$ is shrinkage effect for area *i*

Problem

- **LMM** - the fixed effect parameters β are **spatially invariant** i.e. the expected value of y_i given $\mathbf{x}_i$ is the same everywhere in the study area

- However, there are situations where the relationship between *y* and *x* is not constant over the study area, a phenomenon referred to as **spatial nonstationarity**

Solution

- **Geographical weighted regression (GWR)** is a method that is widely used for fitting data exhibiting spatial nonstationarity **(Brunsdon *et al*., 1998, Fotheringham *et al*., 2002)**

- We use the **GWR concept** to fit area level mixed model and consider SAE under this model

Geographically Weighted Area Level Model

- We define a **spatial nonstationary** or **geographical weighted regression** version of **Fay–Herriot model** suitable for such data

- Let l_i denote the coordinates of an arbitrary spatial location **(longitude and latitude)** in area *i*

- **Spatial nonstationary Fay–Herriot model** for area *i* is

  $y_i(l_i) = Y_i(l_i) + e_i$ and $Y_i(l_i) = \mathbf{x}_i^T \boldsymbol{\beta}(l_i) + u_i$

- $\boldsymbol{\beta}(l_i) = \boldsymbol{\beta} + \boldsymbol{\gamma}(l_i)$
- $\boldsymbol{\gamma}(l_i) = \left(\boldsymbol{\gamma}_1(l_i), \ldots, \boldsymbol{\gamma}_p(l_i)\right)^T$: **spatially correlated** vector-valued random process, with $E(\boldsymbol{\gamma}(l_i)) = \mathbf{0}$ and $\operatorname{cov}(\gamma_k(l_i), \gamma_l(l_j)) = c_{kl}\left(1 + L(l_i, l_j)\right)^{-1}$

- $L(l_i, l_j)$ is the **spatial distance** between locations l_i and l_j
- $\mathbf{C} = [c_{kl}]$ is unknown positive definite matrix

- A **Spatial nonstationary** version of **linear mixed model**

$$\begin{aligned} y_i(l_i) &= \mathbf{x}_i^T \boldsymbol{\beta}(l_i) + u_i + e_i \\ &= \mathbf{x}_i^T \boldsymbol{\beta} + \mathbf{x}_i^T \boldsymbol{\gamma}(l_i) + u_i + e_i \end{aligned}$$

Here,

- $E\left(y_i(l_i)\right) = \mathbf{x}_i^T \boldsymbol{\beta}$
- $Var\left(y_i(l_i)\right) = \mathbf{x}_i^T \mathbf{W}(l_i, l_i) \mathbf{x}_i + (\sigma_u^2 + \sigma_{ei}^2)$
- $Cov\left(y_i(l_i), y_j(l_j)\right) = \mathbf{x}_i^T \mathbf{W}(l_i, l_j) \mathbf{x}_j$
- $\mathbf{W}(l_i, l_j) = \left[Cov(\gamma_k(l_i), \gamma_l(l_j))\right] = c_{kl}\left(1 + L(l_i, l_j)\right)^{-1}$

In matrix form

$$\mathbf{y}(l) = \mathbf{X}\boldsymbol{\beta}(l) + \mathbf{D}\mathbf{u} + \mathbf{e} = \mathbf{X}\boldsymbol{\beta} + \mathbf{Z}\boldsymbol{\gamma}(l) + \mathbf{D}\mathbf{u} + \mathbf{e}$$

- $\mathbf{y}(l) = \left(y_1(l), \ldots, y_m(l) \right)^T$
- $\mathbf{Z} = \{diag(\mathbf{x}_1), \ldots\ldots, diag(\mathbf{x}_p)\}$ is the $m \times pm$ matrix
- $\boldsymbol{\gamma}(l)$ is a $pm \times 1$ vector is **spatial random effects**
- $\mathbf{D}$ is a $m \times m$ diagonal matrix; $\mathbf{u}$ is a vector of $m \times 1$ of **area random effects**, $\mathbf{e}$ is the vector of sampling errors

- $\mathbf{V} = Var(\mathbf{y}) = \mathbf{Z}\Sigma_\gamma \mathbf{Z}^T + \mathbf{D}\Sigma_u \mathbf{D}^T + diag(\sigma_{ei}^2; i = 1, \ldots., m)$

- $\Sigma_\gamma = \mathbf{C} \otimes \boldsymbol{\Omega}$ is $pm \times pm$ variance –covariance matrix
- $\boldsymbol{\Omega}$ is the distance matrix, with $\otimes$ denoting element-wise multiplication and $\Sigma_u = \sigma_u^2 \mathbf{I}_m$

Nonstationary EBLUP (NSEBLUP)

- Replacing the estimated values of variance components $\{\hat{\Sigma}_u, \hat{\Sigma}_\gamma\}$ of $\{\Sigma_u, \Sigma_\gamma\}$, the **EBLUE** of $\boldsymbol{\beta}$ and **EBLUPs** of $\boldsymbol{\gamma}(l)$ and $\mathbf{u}$ are

$$\hat{\boldsymbol{\beta}} = \left[\mathbf{X}^T \hat{\mathbf{V}}^{-1} \mathbf{X}\right]^{-1} \left[\mathbf{X}^T \hat{\mathbf{V}}^{-1} \mathbf{y}\right];$$

$$\hat{\boldsymbol{\gamma}}(l) = \hat{\boldsymbol{\Sigma}}_\gamma \mathbf{Z}^T \hat{\mathbf{V}}^{-1} \left(\mathbf{y} - \mathbf{X}\hat{\boldsymbol{\beta}}\right) \text{ and } \hat{\mathbf{u}} = \hat{\boldsymbol{\Sigma}}_u \mathbf{D}^T \hat{\mathbf{V}}^{-1} \left(\mathbf{y} - \mathbf{X}\hat{\boldsymbol{\beta}}\right)$$

- The **NSEBLUP** predictor of Y_i

$$\hat{Y}_i^{NSEBLUP}(l_i) = \mathbf{x}_i^T \hat{\boldsymbol{\beta}} + \mathbf{a}_i^T \mathbf{Z} \hat{\boldsymbol{\Sigma}}_\gamma \mathbf{Z}^T \hat{\mathbf{V}}^{-1} \left(\mathbf{y} - \mathbf{X}\hat{\boldsymbol{\beta}}\right) + \mathbf{a}_i^T \mathbf{D} \hat{\boldsymbol{\Sigma}}_u \mathbf{D}^T \hat{\mathbf{V}}^{-1} \left(\mathbf{y} - \mathbf{X}\hat{\boldsymbol{\beta}}\right)$$

- $\mathbf{a}_i^T = (0, \ldots, 0, 1, 0, \ldots, 0)$ is $1 \times m$ vector with 1 in position *i*-th

Parameter Estimation

- We assume a model where $\mathbf{C}$ is **diagonal**, so, assuming that the measurement variances σ_{ei}^2 are **known**, there are p+1 unknown parameters, $\{\sigma_u^2, c_{11}, c_{22}, \ldots, c_{pp}\}$

- In particular, we consider a model where the matrix $\mathbf{C} = [c_{kl}]$ is $\mathbf{C} = \lambda \mathbf{I}_p$, where $\lambda \geq 0$ reflects the **'intensity'** of spatial clustering in the data then

$$\mathbf{W}(l_i, l_j) = \left[Cov(\gamma_k(l_i), \gamma_l(l_j)) \right] = \left[\lambda \left(1 + L(l_i, l_j) \right)^{-1} \right]$$

- In this case there are just 2 parameters (λ and σ_u^2) that need to be estimated

- **REML** method

MSE Estimation

MSE of EBLUP predictor

- **MSE of EBLUP predictor: Prasad and Rao** MSE estimator **(Prasad and Rao,1990 and Datta *et al.*, 2005)**

MSE of NSEBLUP predictor

- **MSE of the NSEBLUP** is developed, similar to the approach described in **Opsomer *et al.* (2008)** for MSE of NPEBLUP

Nonstationary Synthetic Prediction (NSSYN)

- In real applications of SAE domains may be unplanned

- This may result in target small areas with zero sample sizes also referred to as **out of sample areas**

- Under LMM, the synthetic EBLUP **(SYN)** for out of sample area *i*

$$\hat{Y}_i^{SYN} = \mathbf{x}_i^T \hat{\boldsymbol{\beta}}$$

- The **Nonstationary** synthetic EBLUP (**NSSYN**) for out of sample area *i*

$$\hat{Y}_i^{NSSYN} = \left(\mathbf{x}_i^{OUT}\right)^T \hat{\boldsymbol{\beta}}(l_i)$$

$$= \left(\mathbf{x}_i^{OUT}\right)^T \hat{\boldsymbol{\beta}} + \mathbf{t}_i^T \mathbf{Z}^{OUT} \hat{\boldsymbol{\Sigma}}_{\gamma,OUT} \mathbf{Z}^T \hat{\mathbf{V}}^{-1} \left(\mathbf{y} - \mathbf{X}\hat{\boldsymbol{\beta}}\right)$$

- $\mathbf{t}_i^T = \left(0,\ldots,0,1,0,\ldots,0\right)$ is $1 \times m^{OUT}$ vector with 1 in position *i*-th, $i = 1,\ldots,m^{OUT}$

Empirical Evaluations

- Two types of simulation studies are carried out
- **Model based simulations** a synthetic population is generated at each simulation run under alternative model specifications and a sample is drawn from this population
- **Design based simulations** are based on realistic population structures obtained from real survey data
- The survey data are first used to generate a synthetic population. The synthetic population is then kept fixed and within area random samples of size equal to the area-specific sizes in the original sample, are drawn

Performance Measures

- **Relative Bias (RB)** and **Relative Root MSE (RRMSE)**

Estimators Investigated in Simulation Studies

Estimator	Model
EBLUP	FH Model
SYN	FH Model
NSEBLUP	NS version of FH Model
NSSYN	NS version of FH Model

Model Based Simulations

- Number of small areas m = 49, 100 and 196

- Use area level model to generate data with the procedure similar to one describe in **Datta *et al.* (2005)**

Stationary process

- **Regression parameters are spatially invariant**
  - $y_i = 10 + 2x_i + u_i + e_i; i = 1, \ldots, m$
  - $x_i \sim U[0,1];\ u_i \sim N\left(0, \sigma_u^2 = 1\right)$
  - $e_i \sim N\left(0, \sigma_{ei}^2\right),\ \sigma_{ei}^2\left(i = 1, \ldots, m\right)$ taking values 7, 6, 5, 4, 3 for equal number of areas, $u_i \perp e_i$

  - T = 1000 simulations

Nonstationary process -1

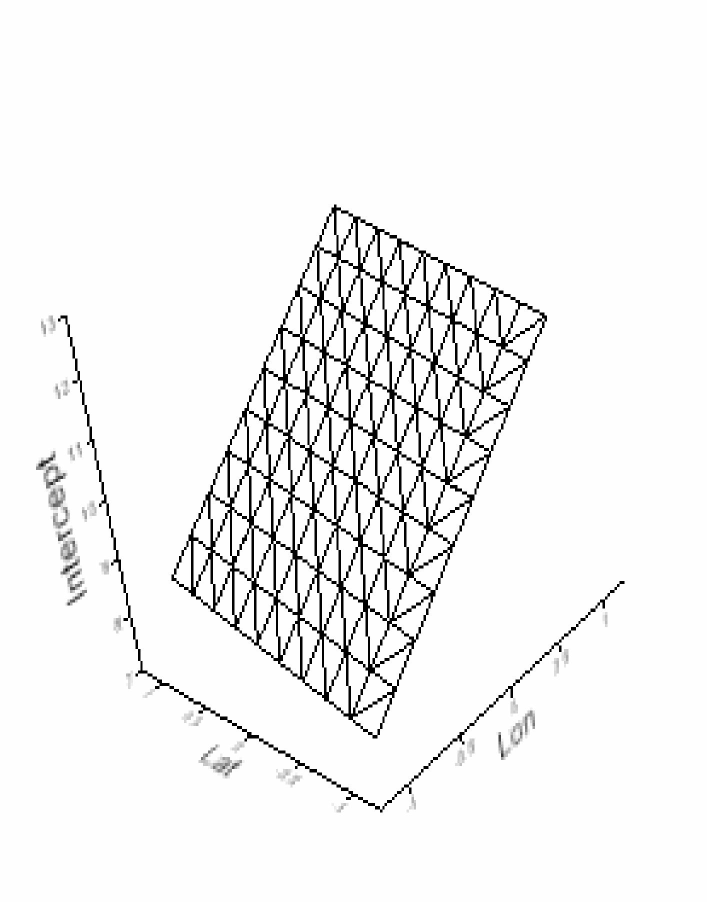
- **Regression parameters are spatially variant**
  - $y_i = \beta_{0i} + \beta_{1i} x_i + u_i + e_i, i = 1,\ldots,m$
  - $\beta_{0i} = 10 + 2 \times longitude_i + 0.5 \times latitude_i$
  - $\beta_{1i} = 4 \times \cos\left\{ sqrt\left\{ (1.2\pi \times longitude_i)^2 + (1.2\pi \times latitude_i)^2 \right\}\right\}$

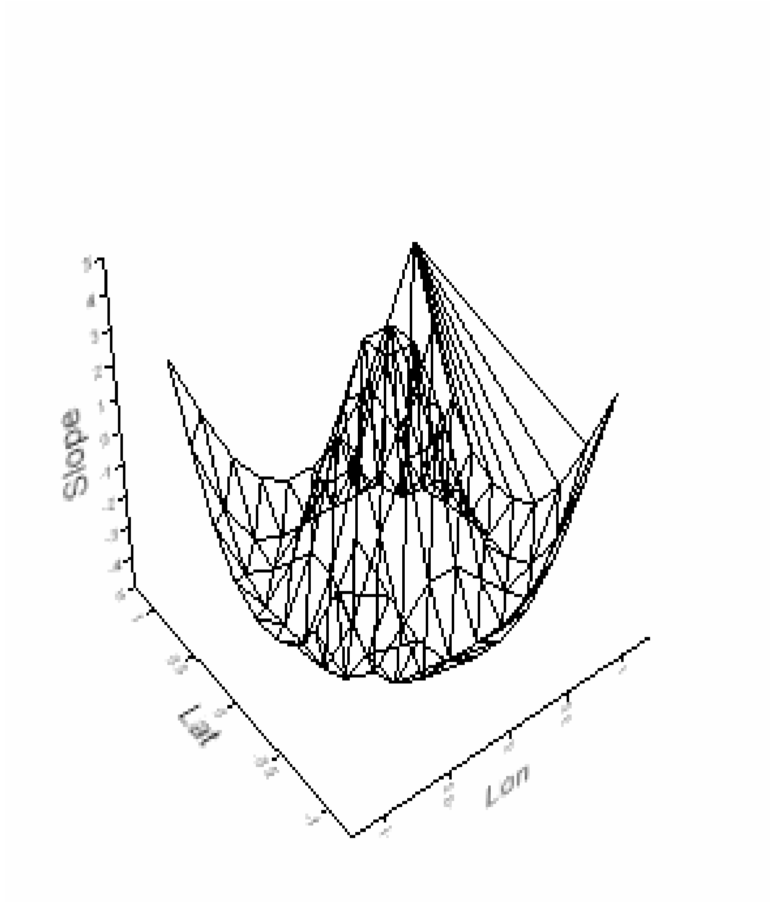
.

- We consider a regular lattice of spatial layout and assume that the observations are obtained from a uniform, two-dimensional grid consisting of $\sqrt{m} \times \sqrt{m}$ lattice points between -1 and 1 and with $2/\left(\sqrt{m} - 1\right)$ distance between any two neighboring points along the horizontal and vertical axes

- $\left\{ \left(latitude_i, longitude_i \right) = \left(k_1, k_2 \right);\ k_1, k_2 = \text{-1,-0.77,-0.55,-0.33,-0.11,0.11,0.33,0.55,0.77,1} \right\}$
- These *m* points are arranged in such a way that k_1 varies from -1 to 1 for each given k_2 varying from -1 to -1

The surface of regression coefficients for the nonstationary process (for $m = 100$)





Nonstationary process -2

- **Regression parameters are spatially variant**
  - $y_i(l_i) = \mathbf{x}_i^T \boldsymbol{\beta} + \mathbf{x}_i^T \boldsymbol{\gamma}(l_i) + u_i + e_i$
  - $\lambda = 6$
  - $u_i \sim N\left(0, \sigma_u^2 = 1\right)$
  - $e_i \sim N\left(0, \sigma_{ei}^2\right)$, $\sigma_{ei}^2\left(i = 1, \ldots, m\right)$ taking values 7, 6, 5, 4, 3 for equal number of areas, $u_i \perp e_i$

Model Based Simulation Results (Averaged over small areas)

	m =49		*m* =100		*m* =196	
Stationary process						
Indicator	EBLUP	NSEBLUP	EBLUP	NSEBLUP	EBLUP	NSEBLUP
RB,%	-0.13	-0.08	0.01	0.00	0.03	0.07
RRMSE,%	9.38	9.49	8.91	8.97	8.63	8.65
CR,%	93	95	90	92	92	94
TRMSE	1.03	1.04	0.98	0.99	0.95	0.95
ERMSE	1.06	1.20	0.97	1.04	0.93	0.98
Nonstationary process-1						
RB,%	2.82	1.40	2.51	0.88	2.62	0.63
RRMSE,%	17.78	15.61	17.21	14.12	17.01	13.20
CR,%	94	96	95	96	95	96
TRMSE	1.56	1.38	1.50	1.25	1.47	1.16
ERMSE	1.55	1.50	1.51	1.34	1.48	1.22
Nonstationary process-2 (lambda=6)						
RB,%	-0.08	-0.08	0.17	0.14	-0.12	-0.07
RRMSE,%	14.48	13.58	14.56	13.01	14.25	12.13
CR,%	95	95	95	95	95	95
TRMSE	1.61	1.51	1.60	1.43	1.57	1.33
ERMSE	1.61	1.58	1.59	1.44	1.57	1.35

TRMSE and ERMSE are true and estimated value of Root MSE

Design Based Simulations

- The dataset comes from the U.S. Environmental Protection Agency's **Environmental Monitoring and Assessment Program (EMAP)** Northeast lakes survey **(Larsen *et al*., 2001)**

- The variable of interest is **Acid Neutralizing Capacity (ANC),** an indicator of the acidification risk of water bodies

- Elevation: **covariate** in the fixed part of the model

- **Small areas: 113 Hydrologic Unit Codes (HUCs)**, of which **64** have (<5) observations and **27** did not have any observations

- **Target:** estimation of small area mean of ANC for in **(86 areas)** and out **(27 areas)** of sample HUCs

Design Based Simulations

- Survey data used to generate nonparametrically a **synthetic population** of **21,026** ANC individual values by using a nearest-neighbour imputation algorithm that retains the spatial structure of the observed ANC values in the EMAP sample data **(Chandra *et al.*, 2012)**

- A total of **1000** independent random samples of lake locations are then taken from the population of **21,026** lake locations by randomly selecting locations in the **86 HUCs** that containing EMAP sampled lakes, with sample sizes in these HUCs set to the original EMAP sample size

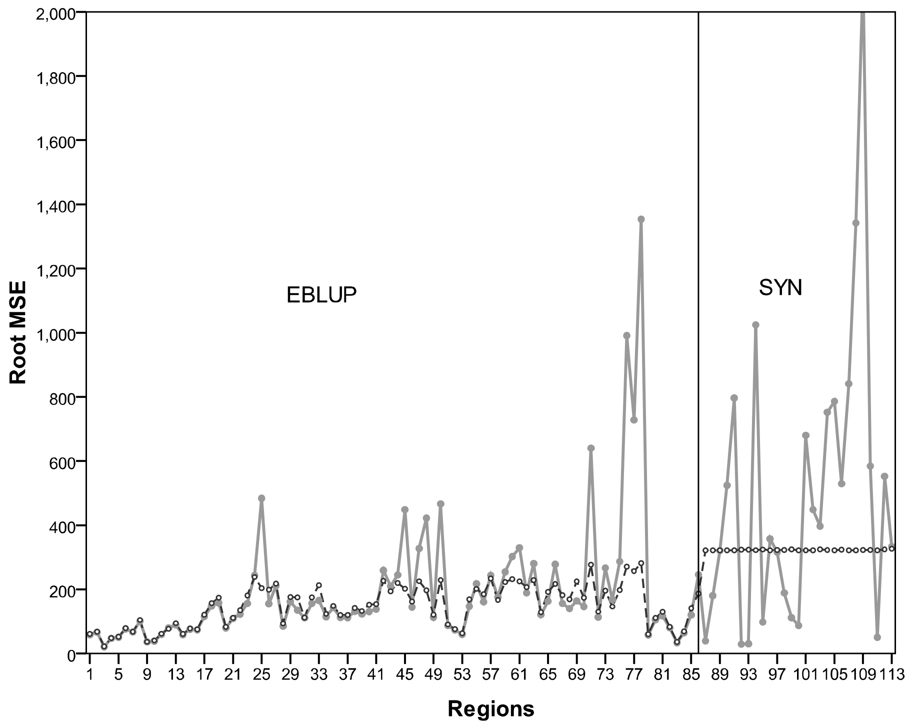
- Lakes in HUCs not sampled by EMAP are also not sampled in the simulation study

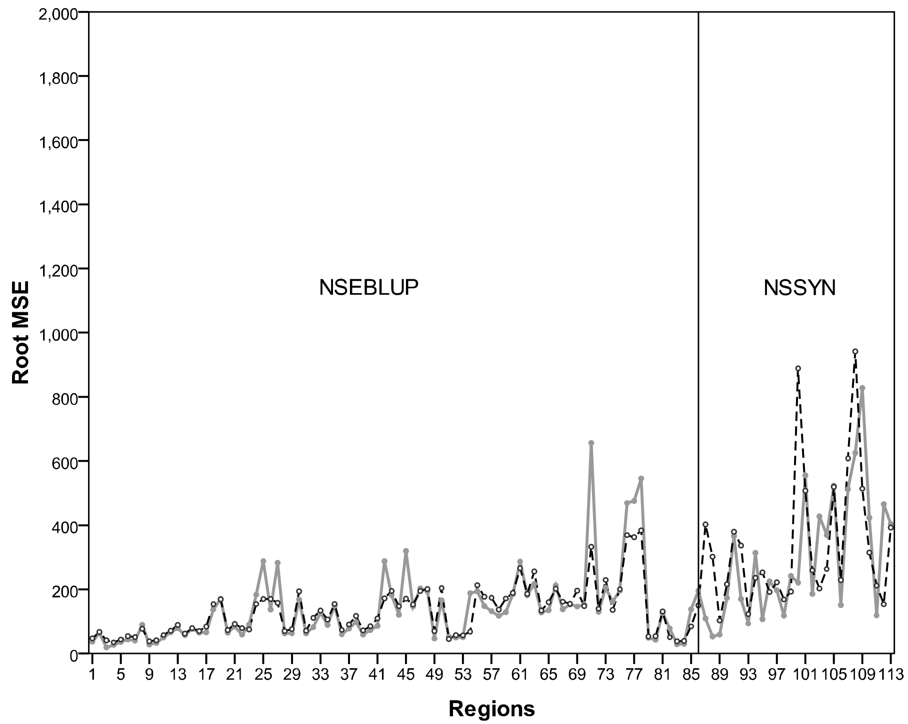
- **ANOVA test** of **Brundson *et al.* (1999)** rejected the null hypothesis of stationarity of the model parameters when the model was fitted to the EMAP data $\Rightarrow$ **model parameters are nonstationary**

Design Based Simulation Results

Predictor	Indicator	Summary of across areas distribution (%)					
		Min	Q1	Median	Mean	Q3	Max
86 sampled HUCs							
EBLUP	RB	-59.84	-18.02	3.24	-3.32	8.52	26.05
	RRMSE	4.89	25.77	30.77	33.69	40.59	68.36
NSEBLUP	RB	-60.11	-10.72	-0.71	-2.51	8.53	38.31
	RRMSE	4.88	17.8	24.3	26.9	33.19	61.07
27 non-sampled HUCs							
SYN	RB	-81.06	-59.89	-46.03	-22.54	1.95	184.93
	RRMSE	6.23	33.39	53.09	50.82	62.08	185.51
NSSYN	RB	-68.95	-34.44	-8.78	-11.48	7.79	43.76
	RRMSE	13.06	18.82	29.82	32.79	45.79	69.12

Region-specific values of actual RMSE (solid line, ●) and estimated Root MSE (dashed line, ○) for the EMAP data





Concluding Remarks

- We examine a **nonstationary extension** of the popular EBLUP, which we refer to as the **NSEBLUP**
- The empirical results show that the proposed **NSEBLUP** can be used for efficiently **borrowing strength over space** in the presence of **spatial nonstationarity** in the data
- The **NSEBLUP** can significantly improve **synthetic estimation** for out of sample areas
- The MSE of the proposed NSEBLUP works well
- We also explored a parametric **bootstrap** approach for MSE estimation

- For small number of areas, bootstrap approach based MSE estimator appears to be slightly more stable than the analytical MSE

- The **nonparametric** spline-based models **(Opsomer *et al.*, 2008; Giusti *et al.*, 2012)** and **spatial models** that assume dependence between areas via simultaneous autoregression **(Pratesi and Salvati, 2008 and Singh *et al.*, 2005)** are other alternative to incorporate the spatial structure of the data in small area models

- We also examined the performance of SAE methods based on these two models, i.e., **NPEBLUP** and **SEBLUP** respectively

- In our empirical evaluations, the proposed **NSEBLUP** emerged as best performing method of SAE when compared with **NPEBLUP** and **SEBLUP**

Thank you

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