

# Discussion

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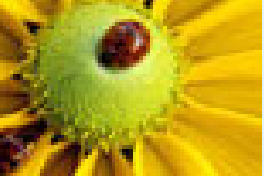
An important observation is made that the MSE depends only on the 2nd & 4th moments of the random effects and errors.



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errors often occur in the process of derivation, and computer programming.





## New era statistics?



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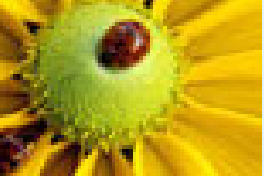
New era? As long as one is able to compute, statistics is in business.

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In fact, even if an analytic solution is possible to obtain, a computational solution may still have some practical advantages.



Some practical/theoretical issues:



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1. Sometimes the moment-matching fails to produce a distribution.

a. 3-point distribution: sometimes the 4th moment estimator  $= \hat{\sigma}^2$ ; as a result, the probability  $p = 1$ , resulting a degenerate distribution.

b. Pearson family: in some cases the estimated 4th moment is  $\leq 3$ , hence does not have a degree of freedom (d.f.):

$$z_4 = 3 \left( \frac{r - 2}{r - 4} \right),$$

. where  $r$  is the d.f.

c.  $t$ -distribution: only works if the estimates kurtosis is positive, as noted by Hall & Maiti (2006).



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In this regard, some recent work of Lahiri and coauthors on adjusted (restricted) maximum likelihood are interesting, but so far their method only applies to the Fay-Herriot model.



# Prof. Partha Lahiri's Presentation

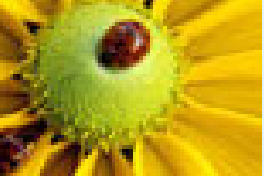
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Data involves 23 hospitals (out of a total of 219 hospitals) that had at least 50 kidney transplants during a 27 month period.



The  $Y_i$ 's are graft failure rates for kidney transplant operations, that is,  $y_i =$  number of graft failures/ $n_i$ , where  $n_i$  is the number of kidney transplants at hospital  $i$  during the period of interest.



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The  $D_i$  is approximated by the binomial variance, that is,  $(0.2)(0.8)/n_i$ , where 0.2 is the observed failure rate for all of the hospitals.

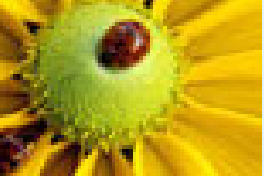


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This brings up the issue about another extension of the FH model, where the  $D_i$ 's are unknown, but current-data information is available about the sampling variation.



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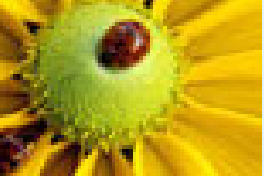
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More specifically, I'd like to see the answer to the following question: Consider the Winsorized EBLUP, which is non-smooth, under the standard FH model (with normality, etc.). Is the MSPE a smooth function of the parameters? (I hope the question is already answered.)



It has been found that the EBLUP is not robust to model misspecification, and some alternative has been suggested that is more robust to model misspecifications, e.g., the observed best prediction (OBP; Jiang *et al.* 2011).



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Open problem: can someone solve this problem for me, please? (Of course, it is not just for me.)





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The most general model, which is unlikely to be misspecified, is  $y_i = \mu_i + v_i + e_i$ , where the  $\mu_i$ 's are completely unknown constants. The OBP can be produced under this general model, but it is very difficult, if possible at all, to produce the 2nd-order unbiased MSPE est. that is  $\geq 0$ .



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For example, the parametric bootstrap is unlikely to work, because there are too many parameters.

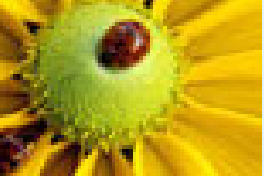


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Let  $\mathcal{D}$  denote the space of  $E(y) = [E(y_i)]_{1 \leq i \leq m}$ , and  $|\mathcal{D}|$  the dimension of  $\mathcal{D}$ . Under the most general model,  $|\mathcal{D}| = m$ ; under the assumed linear model,  $|\mathcal{D}| = p$ , where  $p$  is the dimension of  $x_i$ .



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Find the condition on how fast  $|\mathcal{D}|$  can increase with  $m$  so that one can obtain a 2nd-order unbiased, nonnegative, MSPE estimator.





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Depending on the answer, it could be a complete solution rather than a partial solution.