เฉลยแบบฝึกหัด 11.5

ข้อ 1

แทน $u = e^x$ ได้ $du = e^x dx$

$$\int e^x (\sin^4 e^x)(\cos^2 e^x) dx = \int (\sin^2 u)^2 \cos^2$$

$$= \int \left(\frac{1-\cos 2u}{2}\right)^2 \left(\frac{1+\cos 2u}{2}\right) du$$

$$= \frac{1}{8}\int (1-\cos 2u)(1-\cos^2 2u) du$$

$$= \frac{1}{8}\int (1-\cos^2 2u - \cos 2u + \cos^3 2u) du$$

$$= \frac{1}{8}\left(\int 1 du - \int \cos^2 2u du - \int \cos 2u du + \int \cos^3 2u du\right)$$

$$= \frac{1}{8}\left[u - \int \left(\frac{1+\cos 4u}{2}\right) du - \int \cos 2u du + \frac{1}{2}\int (\cos^2 2u) d(\sin 2u)\right]$$

$$= \frac{1}{8}\left[u - \frac{1}{2}u - \frac{\sin 4u}{8} - \frac{\sin 2u}{2} + \frac{1}{2}\int (1-\sin^2 2u) d(\sin 2u)\right]$$

$$= \frac{1}{8}\left[\frac{u}{2} - \frac{\sin 4u}{8} - \frac{\sin 2u}{2} + \frac{1}{2}\sin 2u - \frac{1}{2}\frac{\sin^3 2u}{3}\right] + C$$

$$= \frac{1}{16}u - \frac{1}{48}\sin^3(2u) - \frac{1}{64}(\sin 4u) + C$$

$$= \frac{1}{16}e^x - \frac{1}{48}\sin^3(2e^x) - \frac{1}{64}(\sin 4e^x) + C$$